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Evaluating the Stolt stretch parameter

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ABSTRACT

The Stolt migration extension to a varying velocity case (Stolt stretch) implies describing a vertical heterogeneity by a constant parameter (W). This paper exploits the connection between modified dispersion relations and traveltime approximations to derive an explicit expression for W. The expression provides theoretically the highest possible accuracy within the Stolt stretch framework. Applications considered include optimal partitioning of the velocity distribution for the cascaded migrations and extension of the Stolt stretch method to transversally isotropic models.

INTRODUCTION

Stolt migration is regarded as the fastest post-stack migration method of all the known algorithms. A known price for that speed is the constant velocity assumption. The time-stretching trick proposed in Stolt's classic paper (1978) provides an approximate extension of the method to a variable velocity case. Stolt stretch implicitly transforms reflection traveltime curves to fit an approximate constant velocity pattern (Levin, 1983, 1985; Claerbout, 1985). In other words, the wave equation with variable velocity is transformed by a particular stretch of the time axis to an approximate differential equation with constant coefficients. The two constant coefficients are an arbitrarily chosen frame velocity and a specific nondimensional parameter (W in Stolt's original notation). In the constant velocity case W is equal to 1, and the transformed equation coincides with the exact constant velocity wave equation. In variable velocity media, W is generally assumed to lie between 0 and 1. As shown by Beasley et al. (1988), the cascaded *f–k* migration approach can move the value of W for each migration in a cascade closer to 1, thus increasing the accuracy of the Stolt stretch approximation.

The W factor was defined by Stolt (1978) as an approximate average of a complicated function. Stolt's definition cannot be used directly for computation because it includes a combined dependence on both time and space coordinates. Therefore, in practice, the estimation of this factor is always replaced by a heuristic guess. That's why Levin (1983) called the W parameter "infamous" (joking, of course), and Beasley et al. (1988) called it it "esoteric."

This paper develops a method to evaluate the Stolt stretch parameter explicitly. The main idea is to constrain the parameter by fitting the exact and approximated traveltime functions. In the case of isotropic interpretation, the W parameter is connected to the "parameter of het-

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erogeneity" (Malovichko, 1978; Castle, 1988; de Bazelaire, 1988). In the case of anisotropic (transversally isotropic) interpretation, it can be related to the "parameter of anellipticity" (Muir and Dellinger, 1985; Dellinger et al., 1993).

STOLT STRETCH THEORY

In order to simplify the references, I will begin with the textbook definitions of the Stolt migration method. The reader familiar with Stolt stretch theory can skip this section and go on to a new piece of theory in the next one.

Post-stack seismic migration is theoretically a two-stage process consisting of wavefield downward continuation in depth z based on the wave equation

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial z^2} = \frac{1}{v^2(x,z)} \frac{\partial^2 P}{\partial t^2} \tag{1}$$

and the imaging condition $t = 0$ (here the velocity v is twice as small as the actual wave velocity). Stolt time migration performs both stages in one step, applying the frequency-domain operator

$$\tilde{P}_0(k_x, \omega_0) = \tilde{P}_v(k_x, \omega_v(k, \omega_0)) \left| \frac{d\omega_v(k, \omega_0)}{d\omega_0} \right| , \tag{2}$$

where

$$\begin{aligned} \tilde{P}_v(k_x, \omega_v) &= \int\int P_v(x, t_v) \exp(i\omega_v t_v - i k_x x) \, dt_v \, dx \ , \\ \tilde{P}_0(k_x, \omega_0) &= \int\int P_0(x, t_0) \exp(i\omega_0 t_0 - i k_x x) \, dt_v \, dx \ , \end{aligned}$$

$P_0(x, t_0)$ stands for the initial zero-offset (stacked) seismic section defined on the surface $z = 0$, $P_v(x, t_v)$ is the time-migrated section, and t_v is the vertical traveltime

$$t_v = \int_0^z \frac{dz}{v(x,z)} \ . \tag{3}$$

The function $\omega_v(k, \omega_0)$ in (2) corresponds to the dispersion relation of the wave equation (1) and in the constant velocity case has the explicit expression

$$\omega_v(k, \omega_0) = \operatorname{sign}(\omega_0) \sqrt{\omega_0^2 - v^2 k^2} \ . \tag{4}$$

The choice of the sign in (4) is essential to distinguish between upgoing and downgoing waves. It is the upgoing part of the wave field that is used in migration.

For the case of a varying velocity Stolt (1978) suggested the following change of the time variable (referred to in the literature as *Stolt stretch*):

$$s(t) = \left(\frac{2}{v_0^2} \int_0^t \eta \, d\tau \right)^{1/2} , \tag{5}$$

where v_0 is an arbitrarily chosen constant velocity, and η is a function defined by the parametric expressions

$$\eta(\zeta) = \int_0^\zeta v(x,z)\,dz\;,\; \tau(\zeta) = \int_0^\zeta \frac{dz}{v(x,z)}\;. \tag{6}$$

With the stretch (5), seismic time migration can be related to the transformed wave equation

$$\frac{\partial^2 P}{\partial x^2} + W\,\frac{\partial^2 P}{\partial \hat{z}^2} + 2\,\frac{(1-W)}{v_0}\,\frac{\partial^2 P}{\partial \hat{z}\partial \hat{t}} = \frac{(2-W)}{v_0^2}\,\frac{\partial^2 P}{\partial \hat{t}^2}\;. \tag{7}$$

Here $\hat{z}$ and $\hat{t}$ are the transformed depth and time coordinates that possess the following property: if $\hat{z}=0$, $\hat{t}=s\,(t_0)$, and if $\hat{t}=0$, $\hat{z}=v_0 s\,(t_v)$. W is a varying coefficient defined as

$$W = a^2 + 2b\,(1-a^2)\;, \tag{8}$$

where

$$b = \frac{\eta(z)}{\eta(\zeta)}\;,\; a = \frac{s(\tau)\,v_0\,v(x,z)}{\eta(\zeta)}\;,\; \tau = \int_0^\zeta \frac{dz}{v(x,z)} = t + \int_0^z \frac{dz}{v(x,z)}\;.$$

Stolt's idea was to replace the slowly varying parameter W with its average value. Thus equation (7) is approximated by an equation with constant coefficients, which has the dispersion relation

$$\widehat{\omega}_v\,(k,\widehat{\omega}_0) = \left(1-\frac{1}{W}\right)\widehat{\omega}_0 + \frac{\text{sign}\,(\widehat{\omega}_0)}{W}\sqrt{\widehat{\omega}_0^2 - W\,v_0^2 k^2}\;. \tag{9}$$

Stolt's approximate method for migration in heterogeneous media consists of the following steps:

1. stretching the time variable according to (5),
2. interpolating the stretched time to a regular grid,
3. double Fourier transform,
4. *f–k* time migration by operator (2) with the dispersion relation (9),
5. inverse Fourier transform,
6. inverse stretching (shrinking) the vertical time variable on the migrated section.

The value of W must be chosen prior to migration. According to Stolt's original definition (8), the depth variable z gradually changes in the migration process from zero to ζ, causing the coefficient b in (8) to change monotonically from 0 to 1. If the velocity v monotonically increases with depth, then $\eta''(z) = \frac{\partial v}{\partial z} \geq 0$, and the average value of b is

$$\bar{b} = \frac{1}{\zeta\,\eta(\zeta)}\int_0^\zeta \eta(z)dz \leq \frac{1}{\zeta\,\eta(\zeta)}\int_0^\zeta \eta(\zeta)\frac{z}{\zeta}dz = \frac{1}{2}\;. \tag{10}$$

As follows from (8) and (10), in the case of monotonically increasing velocity, the average value of W has to be less than 1 (W equals 1 in a constant velocity case). Analogously, in the case of a monotonically decreasing velocity, W is always greater than 1. In practice, W is included in migration routines as a user-defined parameter, and its value is usually chosen to be somewhere in the range of 1/2 to 1.

In this paper I will describe a straightforward way to determine the most appropriate value of W for a given velocity distribution.

A useful tool for that purpose is Stewart Levin's formula for the traveltime curve. Levin (1985) applied the stationary phase technique to the dispersion relation (9) to obtain an explicit formula for the summation curve of the integral migration operator analogous to the Stolt stretch migration. The formula evaluates the summation path in the stretched coordinate system, as follows:

$$s\,(t_0) = \left(1 - \frac{1}{W}\right) s\,(t_v) + \frac{1}{W} \sqrt{s^2\,(t_v) + W\,\frac{(x-x_0)^2}{v_0^2}}\;. \tag{11}$$

Here x_0 is the midpoint location on a zero-offset seismic section, and x is the space coordinate on the migrated section. Formula (11) shows that, with the stretch of the time coordinate, the summation curve has the shape of a hyperbola with the apex at $\{x, s\,(t_v)\}$ and the center (the intersection of the asymptotes) at $\left\{x, \left(1-\frac{1}{W}\right) s\,(t_v)\right\}$. In the case of homogeneous media, $W = 1$, $s(t) \equiv t$, and (11) reduces to the well-known hyperbolic diffraction traveltime curve. It is interesting to note that inverting formula (11) for $s\,(t_v)$ determines the impulse response of the migration operator, which can be interpreted as the wavefront from a point source in the $\{x, \hat{z}, \hat{t}\}$ domain of equation (7):

$$\hat{z} - \hat{z_0} = \left(\frac{1}{Q} - 1\right) R \pm \frac{1}{Q} \sqrt{R^2 - Q\,(x-x_0)^2}\;, \tag{12}$$

where $R = v_0 \hat{t}$, and $Q = 2 - W$. According to equation (12), wavefronts from a point source in the stretched coordinates for $W < 2$ have an elliptic shape, with the center of the ellipse at $\{x, \hat{z_0} + \left(\frac{1}{Q} - 1\right) R\}$ and the semi-axes $a_x = \frac{R}{\sqrt{Q}}$ and $a_z = \frac{r}{Q}$. The ellipses stretch differently for $W < 1$ and $W > 1$ (Figure 1). In the upper part that corresponds to the upgoing waves, they look nearly spherical, since the radius of the front curvature at the top apex equals the distance from the source.

EVALUATING THE *W* PARAMETER AND STOLT STRETCH ACCURACY

Formula (11) belongs to the three-parameter class of traveltime approximations. The key result of this paper uses a remarkable formal similarity between (11) and Malovichko's approximation for the reflection traveltime curve in vertically inhomogeneous media (Malovichko, 1978; Castle, 1988; de Bazelaire, 1988) defined by

$$t_0 = \left(1 - \frac{1}{S\,(t_v)}\right) t_v + \frac{1}{S\,(t_v)} \sqrt{t_v^2 + S\,(t_v)\,\frac{(x-x_0)^2}{v_{rms}^2\,(t_v)}}\;, \tag{13}$$

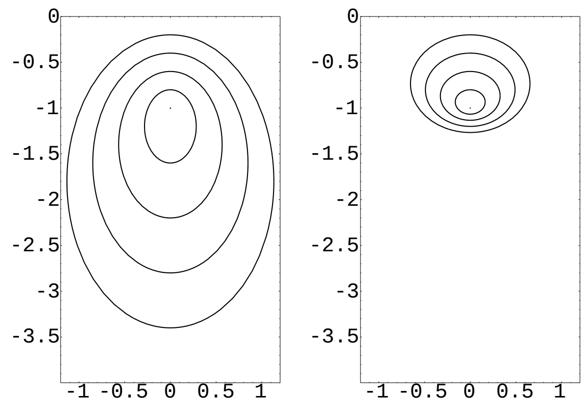

Figure 1: Wavefronts from a point source in the stretched coordinate system. Left: velocity decreases with depth (W=1.5). Right: velocity increases with depth (W=0.5). stoltst-stofro [CR]

where v_{rms} stands for the effective (root mean square) velocity along the vertical ray

$$v_{rms}^2\,(t_v) = \frac{\eta(z)}{t_v} = \frac{1}{t_v}\int_0^{t_v} v^2\,dt_v\;, \tag{14}$$

and S is the *parameter of heterogeneity*:

$$S\,(t_v) = \frac{1}{v_{rms}^4 t_v}\int_0^{t_v} v^4\,dt_v\;. \tag{15}$$

In terms of the S parameter, the variance of the squared velocity distribution along the vertical ray is

$$\sigma^2 = \frac{1}{t_v}\int_0^{t_v} v^4\,dt_v - v_{rms}^4 = v_{rms}^4(S-1)\;. \tag{16}$$

As follows from equality (16), $S \geq 1$ for any type of velocity distribution (S equals 1 in a constant velocity case). For most of the distributions occurring in practice, S ranges between 1 and 2.

Malovichko's formula (13) is known as the most accurate three-parameter approximation of the NMO curve in vertically inhomogeneous media. Since reflection from a horizontal reflector in that class of media is kinematically equivalent to diffraction from a point, formula (13) can be similarly regarded as an approximation of the summation path of the post-stack Kirchhoff-type migration operator. In this case, it has the same meaning as formula (11). An important difference between the two formulae is the fact that equation (13) is written in the initial coordinate system and includes coefficients varying with depth, while equation (11) applies the transformed coordinate system and constant coefficients. Using this fact, the rest of this section compares the accuracy of the approximations and relates Stolt's W factor to Malovichko's parameter of heterogeneity.

Equations (11) and (13) both approximate the traveltime curve in the neighborhood of the vertical ray. Therefore, to compare their accuracy, it is appropriate to consider series expansion of the diffraction traveltime in the vicinity of the vertical ray[2]:

$$t_0(l) = t_0|_{l=0} + \frac{1}{2}\left.\frac{d^2t_0}{dl^2}\right|_{l=0} l^2 + \frac{1}{4!}\left.\frac{d^4t_0}{dl^4}\right|_{l=0} l^4 + \cdots \ , \tag{17}$$

where $l = x - x_0$. Expansion (17) contains only even powers of l because of the obvious symmetry of t_0 as a function of l.

The special choice of parameters t_v, v_{rms}, and S allows Malovichko's formula (13) to provide correct values for the first three terms of expansion (17):

$$t_0|_{l=0} = t_v \ ; \tag{18}$$

$$\left.\frac{d^2t_0}{dl^2}\right|_{l=0} = \frac{1}{t_v v_{rms}^2(t_v)} \ ; \tag{19}$$

$$\left.\frac{d^4t_0}{dl^4}\right|_{l=0} = -\frac{3\,S(t_v)}{t_v^3 v_{rms}^4(t_v)} \ . \tag{20}$$

Considering Levin's formula (11) as an implicit definition of the function $t_0(t_v)$, we can iteratively differentiate it following the rules of calculus:

$$\left.\frac{ds}{dl}\right|_{l=0} = s'(t_0)\left.\frac{dt_0}{dl}\right|_{l=0} = 0 \ ;$$

$$\left.\frac{d^2s}{dl^2}\right|_{l=0} = \left.\left(s'(t_0)\frac{d^2t_0}{dl^2} + s''(t_0)\left(\frac{dt_0}{dl}\right)^2\right)\right|_{l=0} = s'(t_v)\left.\frac{d^2t_0}{dl^2}\right|_{l=0} = \frac{1}{v_0^2\,s(t_v)} \ ; \tag{21}$$

$$\left.\frac{d^3s}{dl^3}\right|_{l=0} = \left.\left(3s''(t_0)\frac{dt_0}{dl}\frac{d^2t_0}{dl^2} + s'(t_0)\frac{d^3t_0}{dl^3} + s'''(t_0)\left(\frac{dt_0}{dl}\right)^3\right)\right|_{l=0} = 0$$

$$\begin{aligned}\left.\frac{d^4s}{dl^4}\right|_{l=0} &= \Bigg(6s'''(t_0)\left(\frac{dt_0}{dl}\right)^2\frac{d^2t_0}{dl^2} + 3s''(t_0)\left(\frac{d^2t_0}{dl^2}\right)^2 + 4s''(t_0)\frac{dt_0}{dl}\frac{d^3t_0}{dl^3} + \\ &\quad \left. + s'(t_0)\frac{d^4t_0}{dl^4} + s^{IV}(t_0)\left(\frac{dt_0}{dl}\right)^4\Bigg)\right|_{l=0} = \\ &= \left.\left(s''(t_v)\left(\frac{d^2t_0}{dl^2}\right)^2 + s'(t_v)\frac{d^4t_0}{dl^4}\right)\right|_{l=0} = -\frac{3\,W}{v_0^4\,s^3(t_0)} \ .\end{aligned} \tag{22}$$

Substituting the definition of Stolt stretch transform (5) into (21) produces an equality similar to (19), which means that approximation (11) is theoretically accurate in depth-varying velocity media up to the second term in (17). It is this remarkable property that proves the validity

[2]Though a power series of the type (17) is not the best possible representation of the traveltime curve, it is quite suitable for comparing different approximations in the vicinity of the vertical ray. In the post-stack migration problem, those approximations imply that the reflector dips have zero mean value. If we assumed that the mean dip value on a particular seismic section were different from zero, we could apply expansions different from expansion (17). That curious option is beyond the scope of this paper.

of the Stolt stretch method (Levin, 1983; Claerbout, 1985). Formula (11) will be accurate up to the third term if the value of the fourth-order traveltime derivative in (22) coincides with (20). Substituting equation (20) into (22) transforms the latter to the form

$$\frac{1-W}{v_0^2\,s^2\,(t_v)} = \frac{v^2\,(t_v) - S\,(t_v)\,v_{rms}^2\,(t_v)}{v_{rms}^4\,(t_v)\,t_v^2}\;. \tag{23}$$

It is now easy to derive from equation (23) the desired explicit expression for the Stolt stretch parameter W, as follows:

$$W = 1 - \frac{v_0^2\,s^2\,(t_v)}{v_{rms}^2\,(t_v)\,t_v^2}\left(\frac{v^2\,(t_v)}{v_{rms}^2\,(t_v)} - S\,(t_v)\right)\;. \tag{24}$$

Expression (24) is derived so as to provide the best possible value of W for a given depth (vertical time t_v). To get a constant value for a range of depths one should take an average of the right hand side of (24) in that range. The error associated with Stolt stretch can be approximately estimated from (17) as the difference between the fourth-order terms:

$$\delta = \frac{l^4}{8}\,\frac{W\,(t_v) - W}{t_v s^2\,(t_v)\,v_{rms}^2\,(t_v)\,v_0^2}\;, \tag{25}$$

where $W\,(t_v)$ is the right-hand side of (24), and W is the constant value of W chosen for Stolt migration. To estimate the best possible accuracy that the Stolt stretch method can achieve, we must take into account the sixth-order term in (17) related to the sixth-order derivative of the traveltime curve. For the true traveltime curve, the expression for the sixth-order derivative in the vicinity of the vertical ray is known from the literature (Bolshyh, 1956; Taner and Koehler, 1969) to be

$$\left.\frac{d^6 t_0}{dl^6}\right|_{l=0} = \frac{45}{t_v^5\,v_{rms}^6}\left(2\,S^2\,(t_v) - \frac{1}{t_v v_{rms}^6\,(t_v)}\int_0^{t_v} v^6\,dt_v\right)\;. \tag{26}$$

First, let us estimate the error of Malovichko's approximation (13). Differentiating (13) six times and setting the offset l to zero yields

$$\left.\frac{d^6 t_0}{dl^6}\right|_{l=0} = \frac{45\,S^2\,(t_v)}{t_v^5\,v_{rms}^6}\;. \tag{27}$$

The estimated error is proportional to the difference between (27) and (26):

$$\delta_M = \frac{l^6}{6!}\left[\frac{45}{t_v^5\,v_{rms}^6}\left(\frac{1}{t_v v_{rms}^6\,(t_v)}\int_0^{t_v} v^6\,dt_v - S^2\,(t_v)\right)\right]\;. \tag{28}$$

It is interesting to note that replacing the parameter of heterogeneity S by its definition (15) changes the expression in the round brackets to the following form:

$$\frac{1}{t_v\,v_{rms}^6}\int_0^{t_v} v^6\,dt_v - S^2 = \frac{1}{t_v^2\,v_{rms}^6}\left(\int_0^{t_v} v^2\,dt_v \int_0^{t_v} v^6\,dt_v - \left(\int_0^{t_v} v^4\,dt_v\right)^2\right)\;. \tag{29}$$

According to the Schwarz inequality from calculus (also known as the Cauchy–Bunyakovski inequality), the value of expression (29) can never be less than zero; hence $\delta_M \geq 0$ for any velocity distribution. This conclusion indicates that Malovichko's approximation tends to increase the traveltime at large offsets beyond its true value.

Differentiating (22) twice and eliminating terms that vanish at $l = 0$ produces

$$\left.\frac{d^6 s}{dl^6}\right|_{l=0} = \left.\left(15\, s'''(t_v) \left(\frac{d^2 t_0}{dl^2}\right)^3 + 15\, s''(t_v)\, \frac{d^2 t_0}{dl^2}\, \frac{d^4 t_0}{dl^4} + s'(t_v)\, \frac{d^6 t_0}{dl^6}\right)\right|_{l=0} = \\ = \frac{45\, W^2}{s\,(t_v)^5\, v_0^6}\,. \tag{30}$$

Evaluating the sixth-order traveltime derivative from (30) and subtracting (26), we get a somewhat lengthy but explicit expression for the error associated with Stolt stretch approximation in the case of the best possible choice of W:

$$\delta_L = \delta_M + \\ + \frac{l^6}{6!}\left[\frac{45\,(1-W)}{t_v^3\, v_{rms}^4(t_v)\, s^2(t_v)\, v_0^2}\left(\frac{v^2(t_v)}{v_{rms}^2(t_v)} - \frac{t_v^2\, v_{rms}^2(t_v)}{s^2(t_v)\, v_0^2}\right) - \frac{30\, v(t_v)\, v'(t_v)}{t_v^4\, v_{rms}^8(t_v)}\right]. \tag{31}$$

ISOTROPIC HETEROGENEITY VERSUS ANELLIPTIC ANISOTROPY

A controversial issue associated with the topic of this paper is whether the non-hyperbolicity of the traveltime curves is caused mainly by heterogeneity or by anisotropy. To find a connection between the two different descriptions of media, we can consider an alternative three-parameter traveltime approximation (the anelliptic anisotropic moveout formula), proposed by Muir and Dellinger (1985) :

$$t_0 = \frac{t_v^4 + (f+1)\, t_v^2\, \frac{(x-x_0)^2}{v_{rms}^2} + f^2\, \frac{(x-x_0)^4}{v_{rms}^4}}{t_v^2 + f\, \frac{(x-x_0)^2}{v_{rms}^2}}\,. \tag{32}$$

Here f is the *parameter of anellipticity*. Differentiating (32) four times, setting $l = x - x_0$ to zero, and equating the result with (20) results in the following formal relationship between f and Malovichko's parameter of heterogeneity:

$$S = 1 + 4\, f - 4\, f^2\,. \tag{33}$$

Equation (33) clearly demonstrates the uncertainty between the anisotropic and heterogeneous isotropic interpretations. Both of them can explain the cause of the nonhyperbolicity of traveltime curves. An important difference is that the parameter of heterogeneity is uniquely determined by the velocity distribution according to (15), while the f parameter is assumed to be an independent functional. The definition (15), applied in combination with (24), is suitable for calculating the Stolt stretch factor in an isotropic model for a given velocity function.

If the correction parameter is measured experimentally by a non-hyperbolic velocity analysis in the form of either equation (13) or equation (32), it accumulates both heterogeneous and anisotropic factors and can be used for an explicit determination of W in (24) independently of the preferred explanation. In the case of the anisotropic moveout velocity analysis, we merely need to substitute the connection formula (33) into (24) to find W. An alternative approach to Stolt-type migration in transversally isotropic media was proposed recently by Ecker and Muir (1993). However, Stolt stretch migration is superior to that method in its ability to cope with varying rms velocities.

EXAMPLES

A simple analytic example of isotropic heterogeneity is the case of a constant velocity gradient. In this case the velocity distribution can be described by the linear function $v(z) = v(0)(1 + \alpha z)$. Stolt stretch transform is found from (5) as

$$s(t) = \left(\frac{e^{2\alpha v(0) t} - 1 - 2\alpha v(0)\, t}{2\alpha^2 v_0^2} \right)^{1/2} . \tag{34}$$

Let κ be the logarithm of the velocity change $v(z)/v(0)$. Then an explicit expression for W factor follows from (24):

$$W = \frac{2\kappa}{e^{2\kappa} - 1} = \frac{v^2(0)}{v_{rms}^2(\kappa)} . \tag{35}$$

For $\kappa \to 0$ (a small depth or a small velocity gradient), $W \approx 1 - \kappa$. For $\kappa \to \infty$ (a large positive change of velocity) W monotonically approaches zero. Formula (35) can be a useful rule of thumb for a rough estimation of W.

Numerical example of the Stolt stretch parameter computation is illustrated in Figures 2 and 3. The left side of Figure 2 shows a smoothed interval velocity curve from the Gulf of Mexico. The corresponding optimal values of the W factor as a function of vertical time (in the isotropic model) are shown on the right. Though the velocity function is smooth, substantial changes in W occur, making its mean value for the times $t_v \le 6$ sec equal to 0.631.

The theory of cascaded migrations (Larner and Beasley, 1987; Beasley et al., 1988) proves that Stolt-type *f-k* migration for a nonuniform velocity $v(t_v)$ can be performed as a cascaded process consisting of migrations with the smaller velocities $v_i(t_v)$, $i = 1, 2, \dots, n$, such that $v_1^2 + v_2^2 + \dots + v_n^2 = v^2$. As shown by Larner and Beasley (1987), it is important to partition the velocity so that for each particular t_v all the velocities in the cascade, except maybe the last one, are constant. The advantage of the cascaded *f–k* migration method is based on the fact that each small velocity v_i describes a more homogeneous medium than the initial $v(t_v)$ function. Therefore, the W factor for each migration in a cascade is closer to 1, and the Stolt stretch approximation is more accurate. This fact is illustrated in Figure 3, which shows an optimal partitioning of the velocity and the corresponding values of the W factor. In accordance with the empirical conclusions of Beasley et al. (1988), a cascade of only four migrations was sufficient to increase the value of W to more than 0.8. With a further increase of the number of cascaded migrations, the method becomes as accurate with respect to vertical velocity variations as phase-shift migration. Theoretically, this limit corresponds to the velocity continuation concept (Fomel, 1994). Note that the theory of cascaded *f–k* migration is strictly valid for isotropic models. The anisotropic interpretation does not support it, since the intrinsic anisotropy factor is not supposed to change with the velocity partitioning.

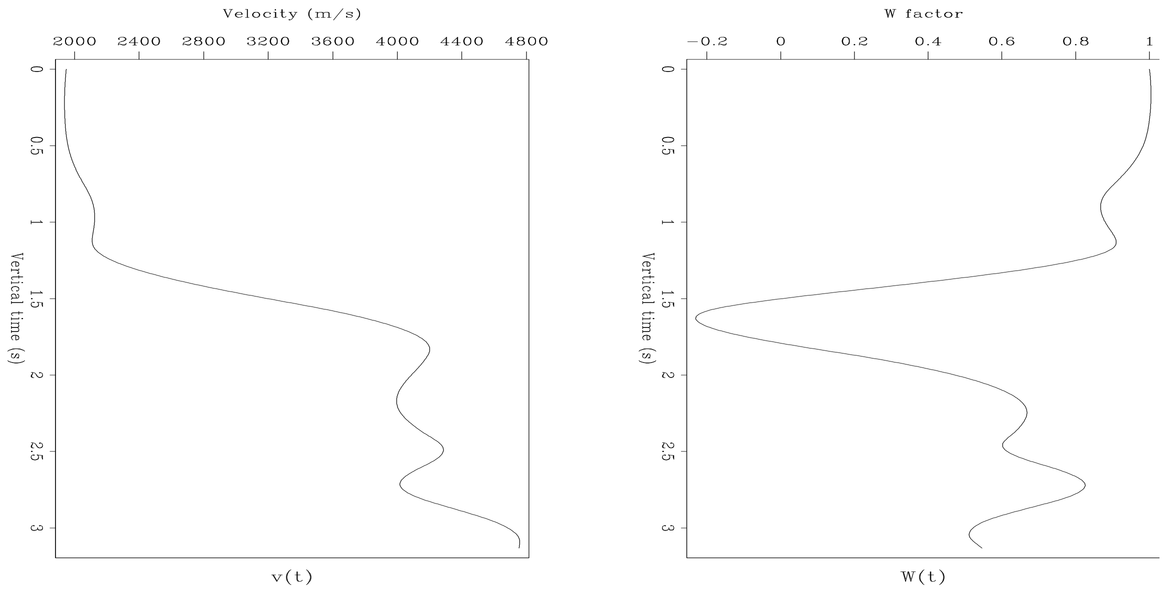

Figure 2: Smoothed interval velocity distribution from the Gulf of Mexico (left) and the corresponding W factor as a function of vertical time (right). The mean value of W is 0.631. stoltst-stovwt [ER]

CONCLUSIONS

The main result of this paper is an analytic explicit expression (24) that allows us to choose the most appropriate value for the Stolt stretch factor. Possible applications include the optimal design of interval velocities partitioning for the method of cascaded *f–k* migrations and extension of the Stolt stretch method to a transversally isotropic model.

Nowadays the topic of this paper seems to be out of fashion. When everyone is interested in prestack depth migration in the time-space domain, it is difficult to attract any attention to post-stack time migration in the frequency domain. Nevertheless, I believe the art of approximation demonstrated by Robert Stolt in his famous paper to be a good example to follow when working on many different problems, which was the main reason for this research.

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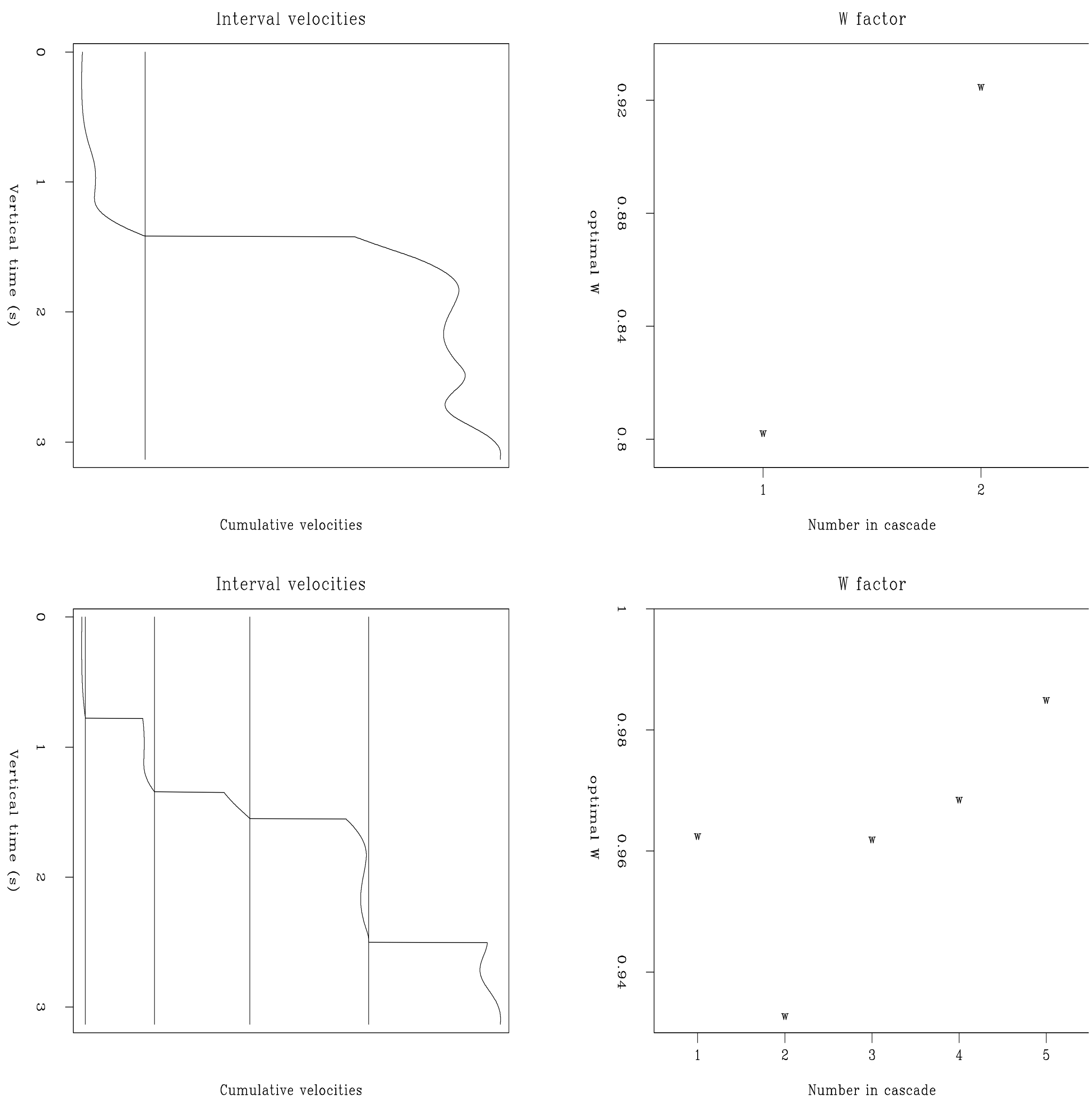

Figure 3: Left: Optimal partitioning of the velocity function for the method of cascaded migrations. Right: corresponding mean values of W. Top: four cascaded migrations. Bottom: seven cascaded migrations. stoltst-stocvw [ER]

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