T-SQUARED GAIN FOR MARINE SEISMOGRAMS

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ABSTRACT

Kjartansson's all-purpose gain function t^2 when adapted for deep water and large offset becomes $(t - t_e)t$ where t_e is the first-earth-arrival time. Results show this improvement is well worth while. Present code assumes normal moveout applicable and vertical travel time depth is known. We need code for routinely identifying t_e.

INTRODUCTION

Kjartansson's thesis (I recall) points to the mathematical function t^2 as a general purpose gain function for seismic data. A simple mathematical model suggests this function. This model assumes constant Q absorption along the entire ray path. But there is no absorption on the water path to wide offset or to bottom in deep water. The water-path time t_e is a function of both shot and geophone locations. To find t_e, here we will make simplifying assumptions that vertical water depth is known and normal moveout relates the offsets. The resulting gain function will be $(t - t_e)t$ after t_e and gain equal zero before. Improved results are remarkable, motivating further work to find t_e directly from the data instead of estimating it with the NMO equation.

Review of Kjartansson's t-squared

One power of t arises from spherical spreading. Energy spreads out on a sphere whose area grows as t^2. But we are interested in amplitude, not energy, which implies the square root of t^2 namely t.

Seismograms have their highest frequencies at early times. Later they have been damped by Q leaving only the lower frequencies then. Beginners often believe the way to compensate for absorption is with exponential gain, but that is wrong because exponentials describe sinusoidal waves, not the broad spectral band that our data is.

The most basic absorption law is the *constant* Q model. According to it, energy diminishes in proportion to the number of wavelengths in space, or in proportion to the number of periods in time, the factor of proportionality being $1/Q$. For a downgoing wave the absorption is proportional to the frequency ω and proportional to time in the medium which is the distance z divided by the velocity v. Altogether the spectrum of a wave passing through a thickness z will be changed by the factor $e^{-|\omega|(z/v)/Q}$ where Q is called the Quality factor of the medium.

We may define spectral bandwidth by setting $e^{-|\omega|(z/v)/Q}$ to be some arbitrary cutoff constant, say, e^{-3}. In other words, $|\omega|_{\text{cutoff}} = vQ/(3z)$. Introducing travel time

t as z/v gives $|\omega|_{\text{cutoff}} = Q/3t$. The later you look on a seismogram, the narrower the spectral bandwidth. You compensate for this by gaining your data by another power of t, hence Kjartansson's t^2. Fortuitiously, this result is independent of the unknowns Q and v, and of the cutoff threshold e^{-3}.

Non-zero offset and deep water

For waves in the water path, regardless their direction of propagation we wish to delay the absorption effect until the time the waves first enter earth sediment t_e. My first guess at the gain rule $G(t)$ was this:

$$G(t) \quad = \quad \begin{cases} t & \text{for } t < t_e \\ t^2/t_e & \text{for } t > t_e \end{cases} \tag{1}$$

Notice $G(t)$ is continuous at $t = t_e$ and has appropriate behavior before and after.

Now I believe equation (1) is wrong. It feels like it needs $t - t_e$. Try this instead:

$$G(t) \quad = \quad \begin{cases} 0 \times t & \text{for } t < t_e \\ (t - t_e)t & \text{for } t > t_e \end{cases} \tag{2}$$

Equation (2) is also continuous at $t = t_e$ because it is zero there. The idea is that there is infinite bandwidth in the water path and on the water bottom reflection so they should be multiplied by zero. People might be upset when they can no longer see the water bottom! Head waves either? Well, they'd see both the head waves and the water bottom if they chose their t_e a little early.

Figure 1 shows an early result. This result is delightful. There are strong multiples in the middle, but primaries are loud and clear from top to bottom. Critical-angle events formerly dominated, but are now subdued. Back scatter events at 5s are now more evident. The need for subsequent mute, or AGC, or revised clip is now reduced or eliminated. Subsequent processes such as velocity analysis, migration, and stack should change in subtle, but perhaps interesting ways. Figure 2 shows a near-trace section. Results are similar, but there the new gain is better in a simple way — stronger at early time.

At wide offset in Figure 1 we see events beyond the water asymptote, both head waves and deep reflections. Here is why: To avoid suppressing this good signal, I boosted the velocity parameter to 2500m/s. Formerly we might play with gain via the parameter `tpow`$=\gamma$ in t^γ where we had no physical model for γ. Now we may play with gain where we have two physical parameters: (1) vertical travel time to water bottom `tau`$=\tau$, and (2) `velocity` v, a rough guess of first earth arrival time t_e as a function of offset. We need an along-path first-arrival time finding program. Then the only user parameter should be how much (if any) earlier to set t_e.

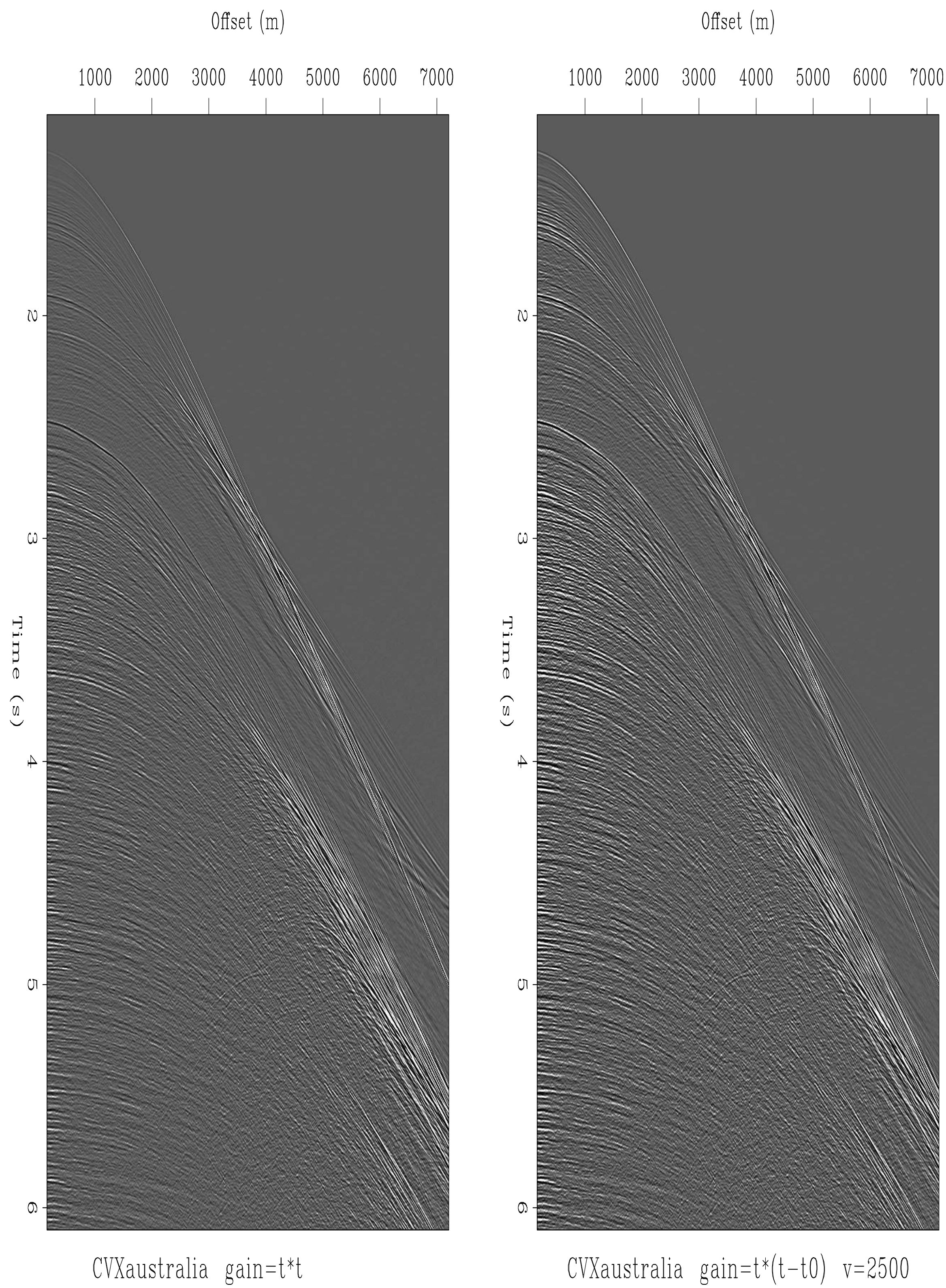

Figure 1: Chevron Australia shot gather gained by t^2 (left) and by $t(t - \sqrt{1.2^2 + x^2/v^2})$ (right). These plots use our default clip percentile, 99%.

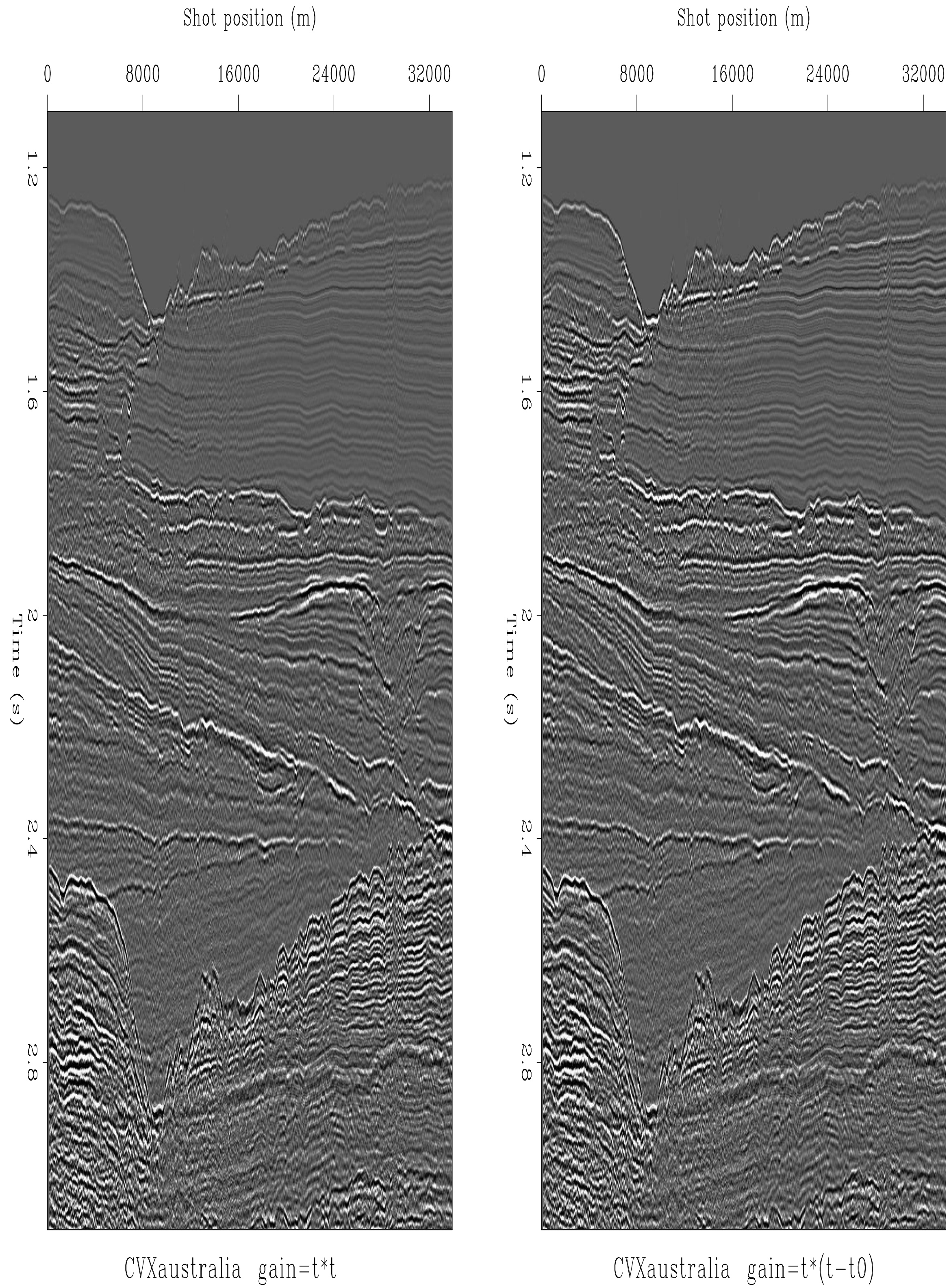

Figure 2: Chevron Australia near trace section gained by t^2 (left) and by $t(t - \sqrt{1.2^2 + x^2/v^2})$ (right). The new gain is better at early time.

An expeditious approximation

Figure 3 shows the result of the expeditious approximation that the water path is only its horizontal distance ignoring the actual slanted path. The first earth reflection time t_e is not difficult to obtain, but it's not effortless either in view of the many complications arising in volume production. Since the offset is always known, this expeditious approximation $t_e = |x|/v$ is nearly effortless to install and use by default. Serendipituously, Figure 3 shows results even better than the more accurate calculation in Figure 1!

Where do good ideas come from? In my experience they arise when a period of intense concentration is interrupted by enforced idleness such as getting on a bicycle, walking home, or (at my age) awaking for no good reason in the middle of the night. It suggests not to fill such idle time with frivolous distractions.

I don't know why the expeditious result is providing the more uniform distribution of amplitudes in Figure 3 than in Figure 1, whether it is geologically fortuitous, or whether there is a geometrical reason. Making the brash assumption that it represents the prevailing situation suggests implementing it in our routine plot programs (rather than as an independent process). One essential plot parameter would be knowledge of `offsetAxis`, a way to know in a hypercube which axis is the offset. In principal some velocity is needed, call it `mutevelocity`, and perhaps that velocity should be something like an RMS velocity, but in practice, we would mostly use a velocity a little above the water velocity to allow for deep reflections that poke their way out from the water velocity cone.

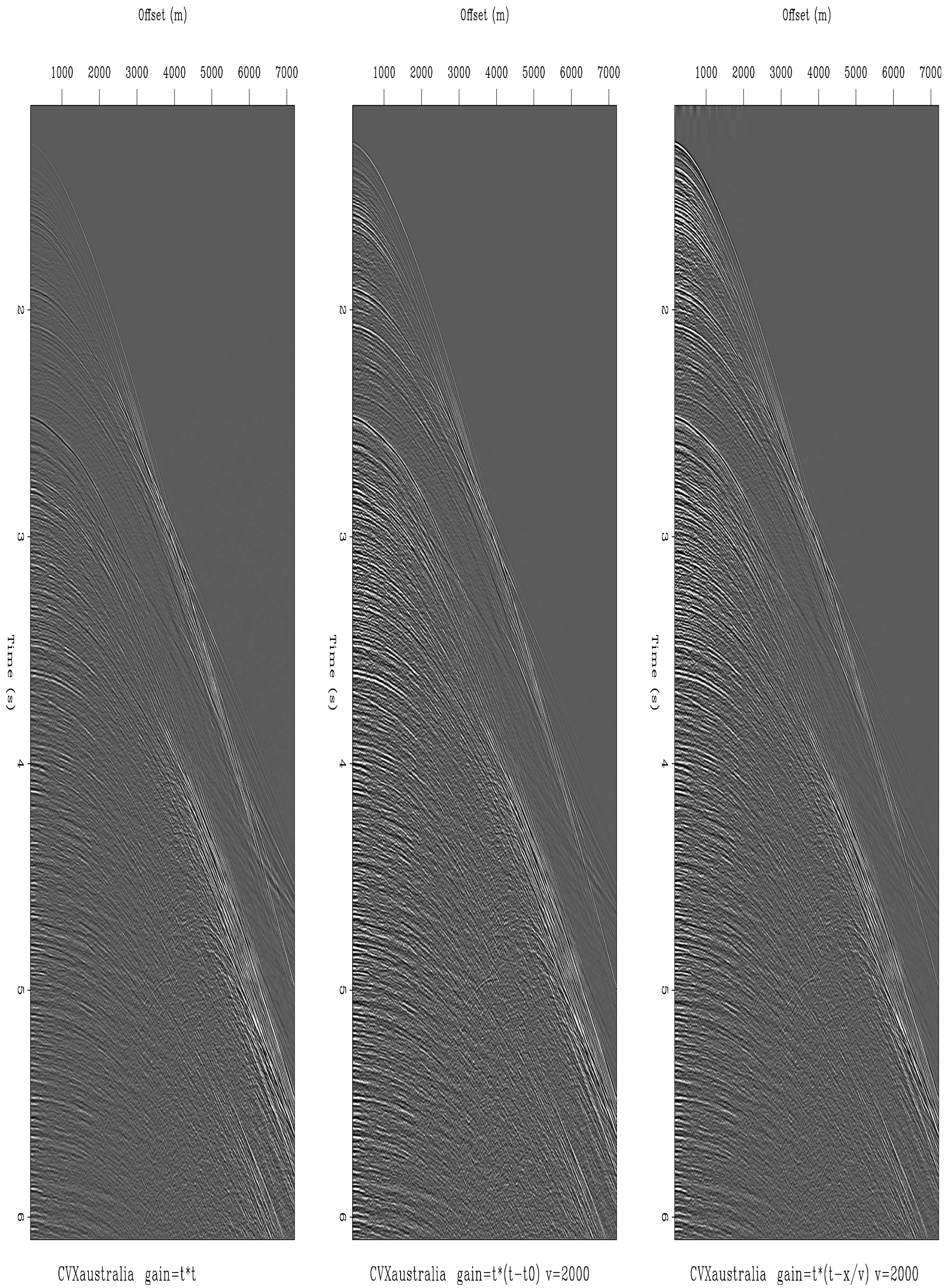

Figure 3: Chevron Australia shot gather gained by t^2 (left), by $t(t - \sqrt{1.2^2 + x^2/v^2})$ (center) and by $t(t - |x|/v)$ (right).