

Constant Q medium

From the absorption law of a material, spectral factorization yields its impulse response. The most basic absorption law is the *constant Q* model. According to it, for a downgoing wave the absorption is proportional to the frequency ω , proportional to time in the medium z/v , and inversely proportional to the “quality” Q of the medium. Altogether the spectrum of a wave passing through a thickness z will be changed by the factor $e^{-|\omega|\tau} = e^{-|\omega|(z/v)/Q}$. This frequency function is plotted in the top line of Figure 1.

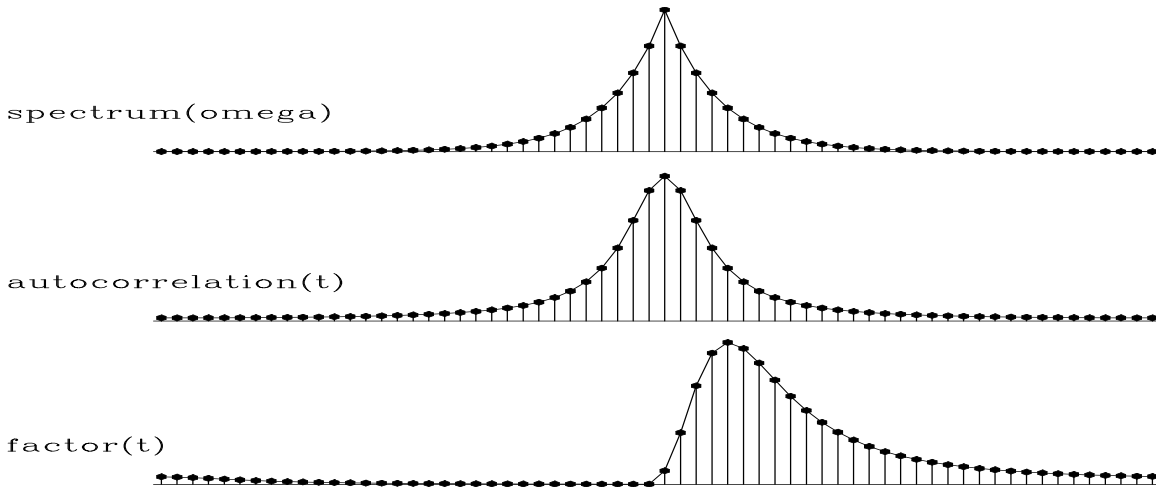


Figure 1: Autocorrelate the bottom signal to get the middle whose FT is the top. Spectral factorization works the other way, from top to bottom.

The middle function in Figure 1 is the autocorrelation giving the spectrum (top). The third is the factorization. An impulse entering the medium comes out with this shape. It is causal. It begins off with a fairly sharp corner and ends with a broad sweep. A low frequency cannot be packed in a small space, so we may say it is spread throughout the wave. The high frequencies are near the sharp corner.

The time-like variable $\tau = (z/v)/Q$ is the variable we get from inverse transforming $|\omega|$. It is not the total travel time. It is the extra delay low frequencies experience from the absorption. In Figure 1 its origin is in the middle of the axis. The one independent parameter is z/Q , not both z and Q .

There is no physics in this analysis, only mathematics. A physical system could cause the factored wave to be more spread out (effectively by an additional all-pass filter), but physics cannot make it more compact because the low frequency cannot be compacted into a smaller space. Everything is as early as it can be starting at $t = 0$.