

INTRODUCTION

A long-standing assumption in applied seismology is that seismic sources are minimum phase. This means the source wavelet and its inverse (the decon filter) are both causal. In this setting, the source wavelet is deduced from (1) the presumption of causality and (2) the observed spectrum. Performing this calculation in four deep-marine regions yields three-lobed wavelets, but the first lobe is always larger than expected while the third lobe is smaller. These results (Figure ??) contradict the Ricker idea based on a surface ghost at the marine gun and another at the hydrophone.

sourceswidth=3in Estimated shot wavelet from four deep-marine regions. The minimum-phase assumption says they should be Ricker wavelets, but they lack that symmetry. Top two from 4ms data, bottom two from 2ms data.

The results in Figure ?? are based on data corrected for divergence. Spectra were averaged over multiple hundreds of inner-offset seismograms. They were not smoothed. Finally, the Kolmogoroff method (see below) deduces the source wavelets shown.

From the discrepancy between the Ricker idea and the minimum-phase idea we here presume the Ricker idea the more correct one. Then we show how phase spectra can be devised that impose Ricker symmetry on the shot waveform estimate.

Seismogram polarity becomes more apparent in seismic data when deconvolution removes the correct source wavelet. Unfortunately, predictive deconvolution does not do a good job defining that wavelet. Predictive decon assumes minimum phase while marine seismology typically exhibits the Ricker wavelet, a wavelet which is both theoretically and practically, marginally or doubtfully minimum phase. The problem is resolved in this paper by shifting the time origin to the middle main lobe of the Ricker wavelet and simultaneously estimating both this (now noncausal) shot waveform and its inverse. A byproduct of this approach is a debubble process giving results of outstanding clarity. See Figure 1.

Parameterizing the logarithm of the spectrum in the time domain lays out parameter choices in a natural way along the “quefreny” axis. We discovered this when our inverse theory project lacked a regularization provided here. Results here are excellent, and computed in $N \log_2 N$ time. We regard them as a final analytical stage before invoking iterative inverse theory.

BASICS OF LAG-LOG SPACE

We describe a wavelet starting from the logarithm of its Fourier transform $F(\omega)$, $\ln F(\omega) = U(\omega)$, so $F(\omega) = e^{U(\omega)}$. Express U as a Z -transform, $U = \sum_{\tau=0}^{2048} u_{\tau} Z^{\tau}$ where $Z = e^{i\omega\Delta t}$, so it's a Fourier transform. The u_{τ} values are our parameterization of the wavelet. The τ axis is called the “quefreny” axis.

The property of exponentials that $e^{A+B+C} = e^A e^B e^C$ has meaning when we exponentiate a Z -transform $\exp(A + B + C) = \exp(\sum_{\tau=1}^{2048} u_{\tau} Z^{\tau})$. The Z -transform sum may be split up into small lags, medium lags, and large lags. This decomposes a filter (or waveform) into a sequence of three filters, each with its own meaning where for example in common marine seismology:

$$\begin{aligned}
 e^{A+B+C} &= e^A e^B e^C & (1) \\
 e^{\sum_{\tau=1}^{2048} u_{\tau} Z^{\tau}} &= (e^{\sum_1^2}) (e^{\sum_3^{15}}) (e^{\sum_{16}^{2048}}) & (2) \\
 (\text{wavelet}) &= (\text{continuity})(\text{Ricker})(\text{bubble}) & (3)
 \end{aligned}$$

Zeroing or suppressing fourier components of a logarithm, such as A , B , and C , is a convenient way of making an operator more unitary. Equation (2) defines the boundaries of the three regions abruptly although in practice we blend them smoothly. Changing the sign of $(A + B + C)$ switches context from an impulse response to its inverse. Both are parameterized by the same A , B , and C . We may specify u_{τ} from prior knowledge, or from knowledge gained from various kinds of data averaging, or from some

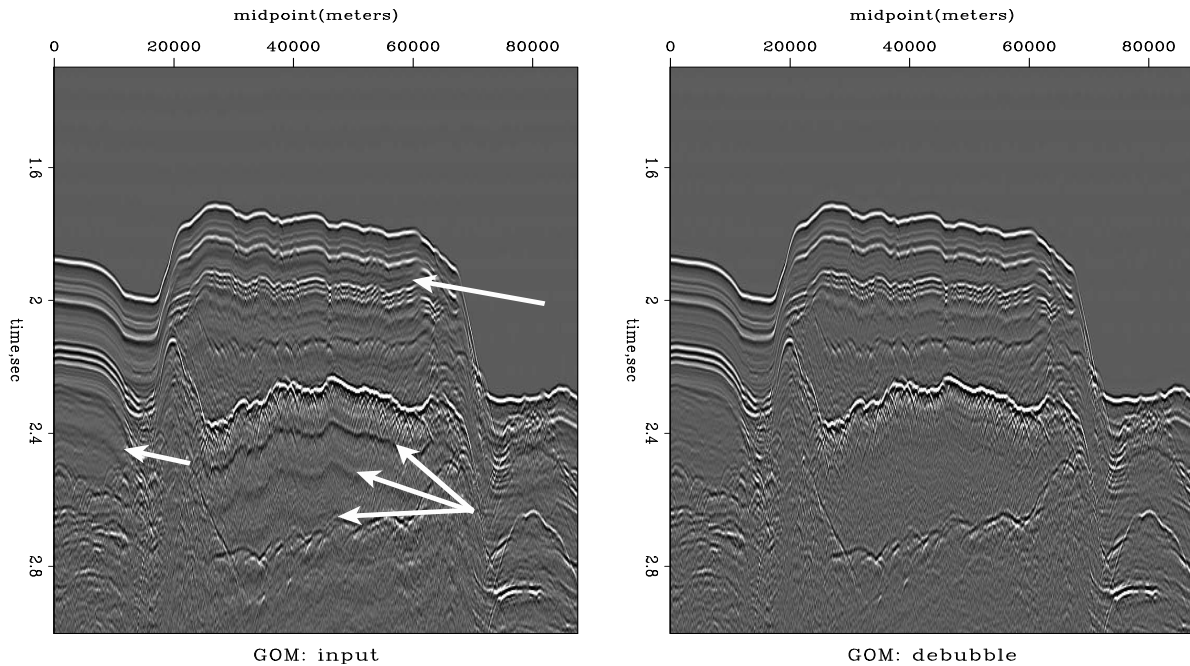


Figure 1 Gulf of Mexico data before and after pseudo-unitary debubble.

mixture of the two. Commonly we begin from Kolmogoroff spectral factorization giving us all the u_τ (see below). We may design a filter e^{A+B+C} by over-riding Kolmogoroff with $A = 0$ and $B = 0$. To see what happens, consider the filter $e^C = 1 + C + C^2/2! + \dots \approx 1 + C$. Examine its leading coefficients. They are $(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, u_{16}, u_{17}, u_{18}, \dots)$. Figure 1 shows the application of such a filter. This operation on the data is called “debubbling”. The 15 interval gap on 4ms data is 60ms, a number half way between the “end of the ghosts” and the “onset of the bubbles”. This result may be described as “textbook quality” (meaning it is the best we have ever produced). Setting all of A , B , and C to zero makes a unitary filter that is simply an impulse. Suppressing components of the logarithm suggests the appellation “pseudo unitary”.

Kolmogoroff spectral factorization

When a time function such as c_t vanishes at all negative time lags it is said to be causal. Its Z transform $C(Z) = c_0 + c_1Z + c_2Z^2 + c_3Z^3 + \dots$, with $Z = e^{i\omega\Delta t}$ is really a Fourier sum. Its square $C(Z)^2$ convolves a causal with itself, so it is causal. Each power of $C(Z)$ is causal, hence $e^C = 1 + C + C^2/2 + \dots$, a sum of causals, is causal. The time-domain coefficients for e^C could be computed putting polynomials into power series or by Fourier transforms. The wavelet e^C has inverse e^{-C} . A causal with a causal inverse is said to be “minimum phase”.

Relate c_τ to amplitude $r = r(\omega)$, phase $\phi = \phi(\omega)$.

$$|r|e^{i\phi} = e^{\ln|r|+i\phi} = e^{\sum_\tau c_\tau Z^\tau} = \exp\left(\sum_\tau c_\tau Z^\tau\right) \quad (4)$$

Given a spectrum $r(\omega)$ we can construct a minimum-phase filter with that spectrum. Since $r(\omega)$ is a real even function of ω , the same may be said of its logarithm. Let the inverse Fourier transform of $\ln|r(\omega)|$ be u_τ , where u_τ is a real even function of time. Imagine a real odd function of time v_τ .

$$|r|e^{i\phi} = e^{\ln|r|+i\phi} = e^{\sum_\tau (u_\tau+v_\tau)Z^\tau} \quad (5)$$

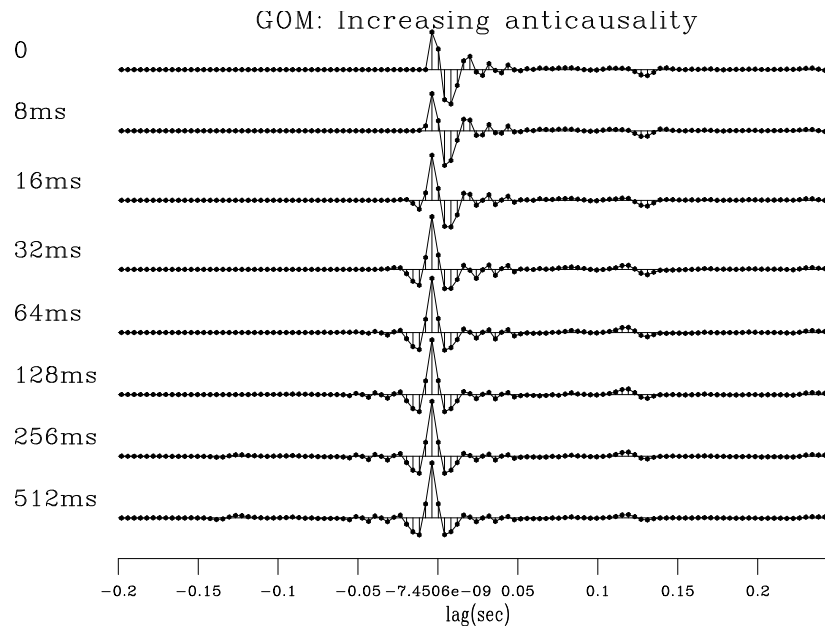


Figure 2 Gulf of Mexico $\Delta t = 4\text{ms}$: Increasing the anticausality in Ricker compliant decon. Too much causes half the bubble to appear before the shot. Numerical values of anticausality refer to the lag τ at which the sine-squared weight upon u_{τ}^{anti} reaches unity where weighting ceases.

The phase $\phi(\omega)$ transforms to v_{τ} . We can assert causality by choosing v_{τ} so that $u_{\tau} + v_{\tau} = 0$ for all negative τ . This defines v_{τ} at negative τ . Since v_{τ} is odd, we also know its values at positive lags. This causal exponent $c_{\tau} = u_{\tau} + v_{\tau}$ creates a causal minimum-phase filter with the specified spectrum. This process is called “Kolmogoroff spectral factorization” because in applications one begins with an energy spectrum $|r|^2$ and factors it into an $re^{i\phi}$ times its conjugate.

Ricker compliant decon

Start with the c_{τ} resulting from a Kolmogoroff factorization. (Optionally you might weight down portions making it more unitary.) Split it into even and odd parts, $v_{\tau} = u_{\tau}^{\text{odd}} = (c_{\tau} - c_{-\tau})/2$ and $u_{\tau}^{\text{even}} = (c_{\tau} + c_{-\tau})/2$ whose sum is c_{τ} . The even part Fourier transforms to the logarithm of the amplitude spectrum. The odd part Fourier transforms to the phase spectrum. Here we monkey with the phase while not touching the amplitude. We simply taper u_{τ}^{odd} towards zero for small lags. Figure 2 examines the consequences of various numerical choices of “small”. As we increase the weight width, the wavelet increases in symmetry near $t = 0$. This is new to us. No longer causal are $c_{\tau} = u_{\tau}^{\text{even}} + u_{\tau}^{\text{odd}}$ and the wavelet of e^C . At the origin is a centered Ricker wavelet. The regularization we needed is $c_{\tau} - c_{-\tau} \approx 0$ for small τ .

Figures 3 and 4 are based on the average spectrum of all the traces shown. This average spectrum creates a single decon filter. Our default choice of 60ms is larger than the Ricker width, about 20ms, but not as large as the bubble delay, about 150ms, a very easy choice.

WHAT IS LEFT TO DO?

This paper starts from a given spectrum, here an average of many traces. In reality, the spectrum might vary from trace to trace. The spectrum will vary from one offset to the next. These basic aspects are not addressed here. Inverse theory incorporates other real-world complications beyond the scope of the present study. Our ongoing work with it shows it reveals polarity even more clearly. Perhaps so because it uses ℓ_1 -like statistics. Also it correctly handles gain and filtering as non commuting operators. The method of this paper is the winner, however, when it comes to clear and simple parameter choice.

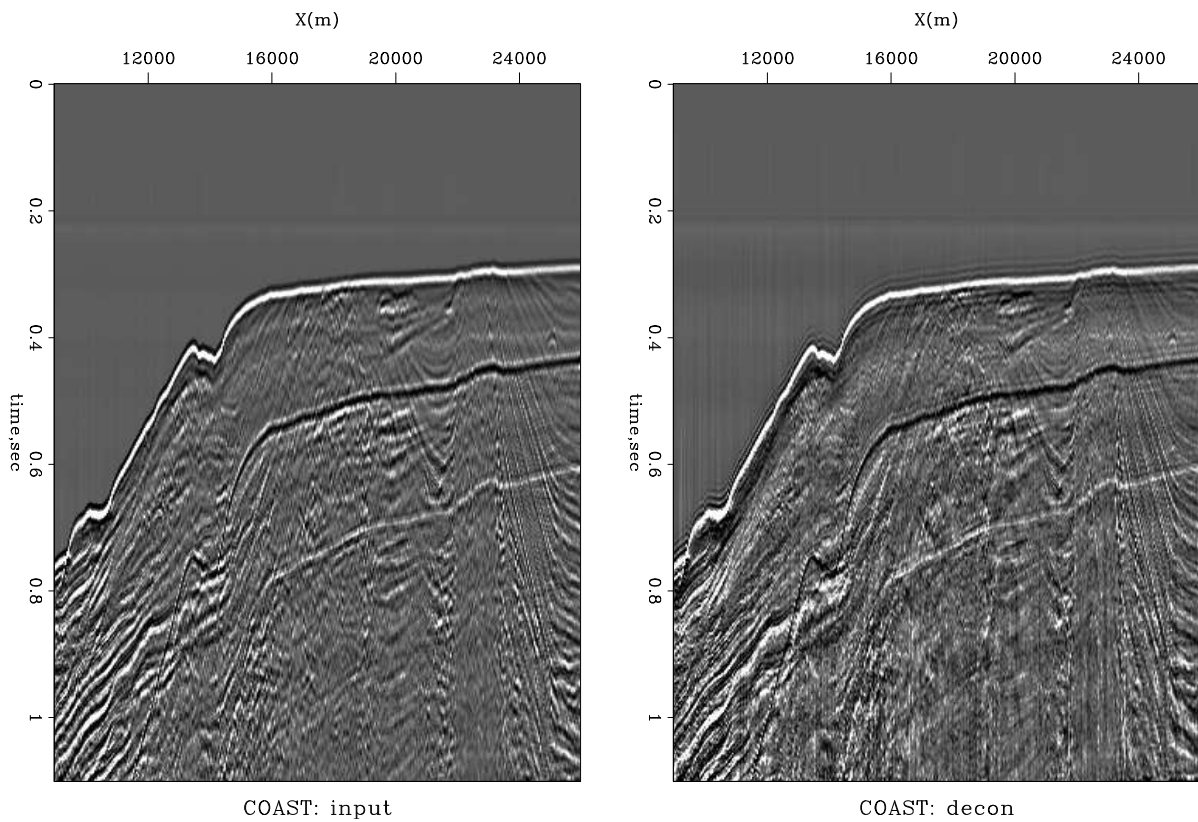


Figure 3 Holbrooke data. Ricker compliant decon changes water bottom wavelet from black-white-black to white. Alternating polarity on multiples is more clear.

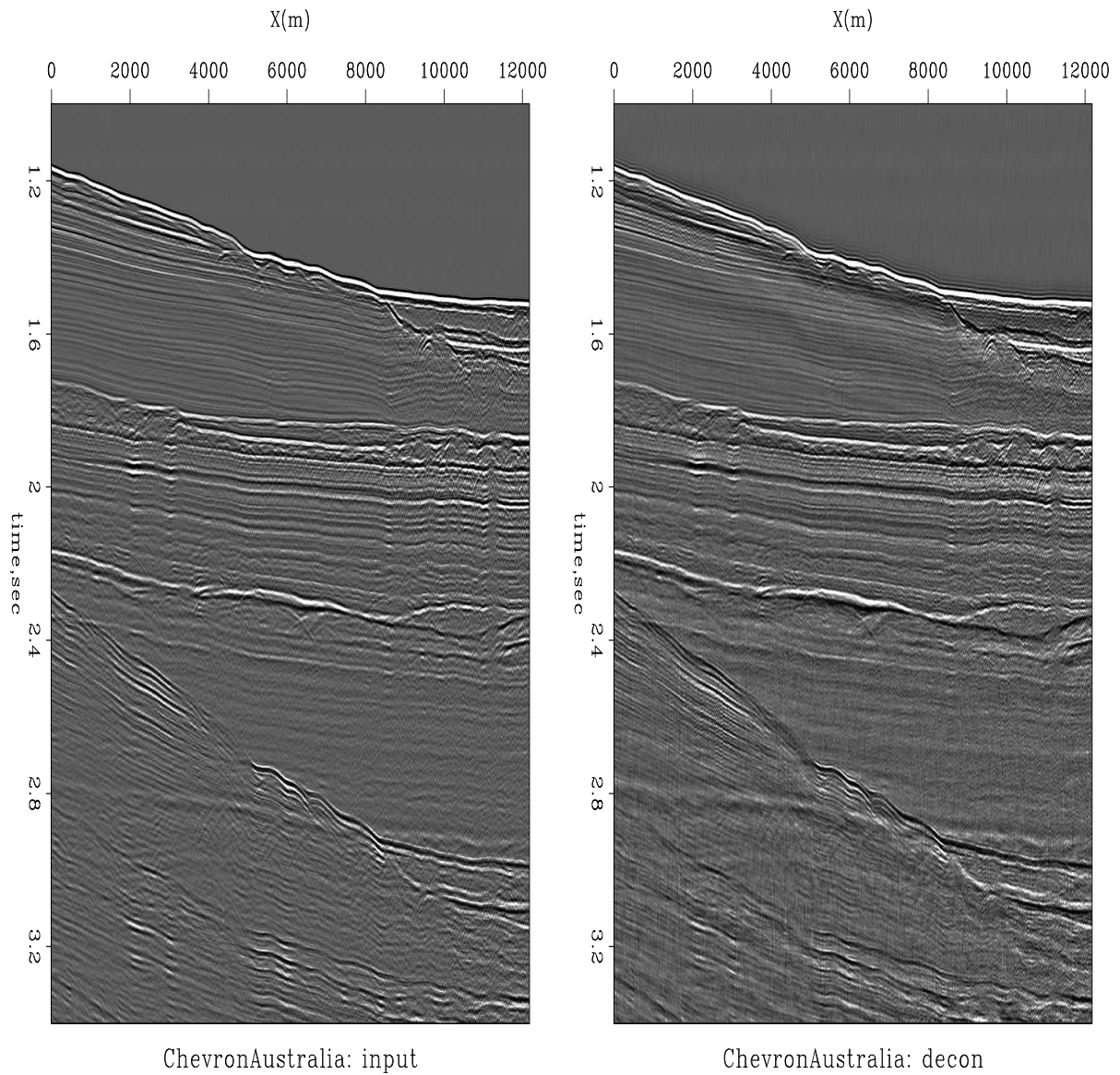


Figure 4 Australia data from Chevron, a near-trace section (7m cable depth). Features are similar to figure 3 but with more interesting events.