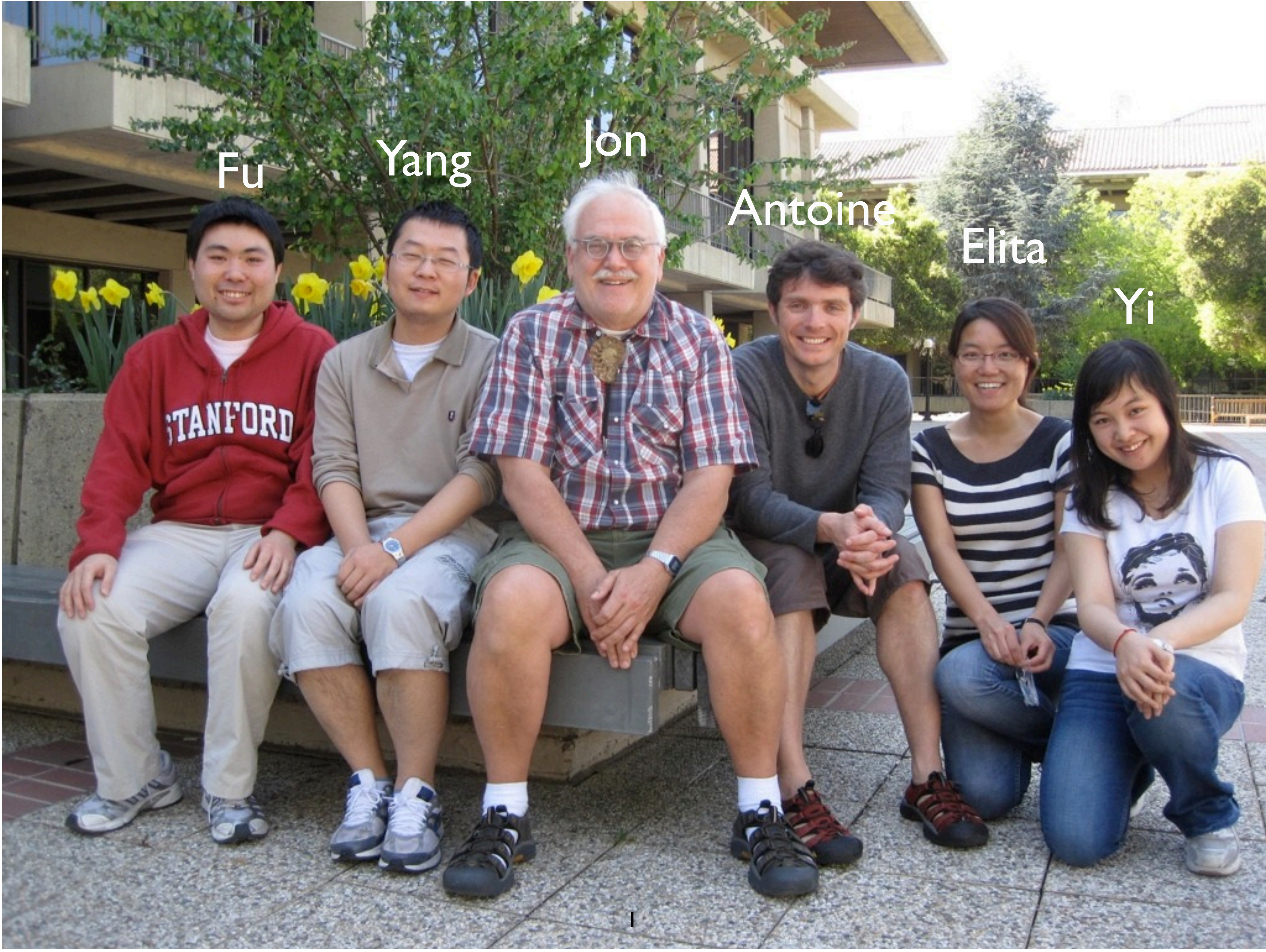
Fu
Yang
Jon
Antoine
Elita
Yi
STANFORD

Sparsity decon in the log domain with variable gain

Jon Claerbout, Antoine Guitton, and Qiang Fu

SEP 2012 spring meeting
Monterey, California

SEP report page 147, page 313

OLD NEWS: We seek sparse deconvolutions by imposing a hyperbolic penalty function.

NEW: Although FT based, we find theory for arbitrary gain(t) and mute(t,x) AFTER decon.

NEW: Results confirm benefit of "gain after decon"

NEW: We have identified a long-needed regularization.

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Sparseness goals

The ℓ_2-norm decon forces a whiteness assumption and forces a "minimum phase" assumption. Both bad.

The sparseness goal should yield a "best" spectrum and (hopefully) the most appropriate phase.

Enhance low frequency only when it aids sparsity.

Seek to integrate reflectivity to obtain log impedance.

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Logarithmic parameterization

$$r_t \;=\; \mathrm{FT}^{-1}\; D(\omega)\;\exp\left(\sum_{\tau\neq 0} u_\tau Z^\tau\right)$$

$D(\omega)$ is the FT of the data.

r_t is reflectivity (and residual)

u_τ are the free parameters.

. $u_0 = 0$ is mean log spectrum.

Lag-log space.

Strange!

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Gain and sparsity

$$q_t = g_t \, r_t$$

where:

r_t is the physical output of the filter

g_t is the given gain function, often t^2

q_t is the gained output, also called the "statistical signal" to be sparsified.

$$q_t = g_t \; r_t$$

$$H(q_t) = \sqrt{q_t^2 + 1} - 1$$

softly clipped residual

$$\frac{dH}{dq} = H'(q) = \frac{q}{\sqrt{q^2+1}} = \text{softclip}(q)$$

r_t is the physical output of the filter
g_t is the given gain function
q_t is the gained output,
$H(q)$ is the hyperbolic penalty function.
Choose g_t so that $q_t \approx 1$. What percentile?
"Sparsity" is $1 \,/\, \sum_t H(q_t)$

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$$r_t = \mathrm{FT}^{-1}\ D(Z)\ e^{\cdots + u_2 Z^2 + u_3 Z^3 + u_4 Z^4 + \cdots}$$

$$\frac{dr_t}{du_\tau} = \mathrm{FT}^{-1}\ D(Z)\ Z^\tau e^{\cdots + u_2 Z^2 + u_3 Z^3 + u_4 Z^4 + \cdots}$$

$$\frac{dr_t}{du_\tau} = r_{t+\tau}$$

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$$\frac{dr_t}{du_\tau} = r_{t+\tau}$$

You think you have seen this before....?

$$r_t = \mathrm{FT}^{-1}\ D(Z)\ e^{\cdots + u_2 Z^2 + u_3 Z^3 + u_4 Z^4 + \cdots}$$

$$\frac{dr_t}{du_\tau} = \mathrm{FT}^{-1}\ D(Z)\ Z^\tau e^{\cdots + u_2 Z^2 + u_3 Z^3 + u_4 Z^4 + \cdots}$$

$$\frac{dr_t}{du_\tau} = r_{t+\tau}$$

No, you likely saw $d_{t+\tau}$.

Residual orthogonal to fitting function
becomes
Residual orthogonal to itself

$$r_t = \mathrm{FT}^{-1}\ D(Z)\ e^{\cdots + u_2 Z^2 + u_3 Z^3 + u_4 Z^4 + \cdots}$$

$$\frac{dr_t}{du_\tau} = \mathrm{FT}^{-1}\ D(Z)\ Z^\tau e^{\cdots + u_2 Z^2 + u_3 Z^3 + u_4 Z^4 + \cdots}$$

$$\frac{dr_t}{du_\tau} = r_{t+\tau}$$

Physical output gradient
w.r.t. lag-log variable

$$q_t = r_t \quad g_t$$

$$\frac{dq_t}{du_\tau} = \frac{dr_t}{du_\tau}\ g_t = r_{t+\tau}\ g_t$$

Statistical gradient

amazing result coming

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EXPLORATION PROJECT

the step

$$\Delta u_\tau = \sum_t \frac{dH(q_t)}{du_\tau} \qquad \tau \neq 0$$

$$= \sum_t \frac{dq_t}{du_\tau} \frac{dH(q_t)}{dq_t}$$

$$\Delta u_\tau = \sum_t (r_{t+\tau}) \quad (g_t H'(q_t)) \qquad \tau \neq 0$$

A crosscorrelation: Compute it in the Fourier domain.

Jon's favorite theory slide.

STANFORD EXPLORATION PROJECT

the step

$$\Delta u_\tau = \sum_t \frac{dH(q_t)}{du_\tau} \qquad \tau \neq 0$$

$$= \sum_t \frac{dq_t}{du_\tau} \; \frac{dH(q_t)}{dq_t}$$

the softly clipped residual

$$\Delta u_\tau = \sum_t (r_{t+\tau}) \quad (g_t H'(q_t)) \qquad \tau \neq 0$$

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STANFORD EXPLORATION PROJECT

the step

$$\Delta u_\tau \quad = \quad \sum_t \frac{dH(q_t)}{du_\tau} \qquad\qquad \tau \neq 0$$

$$= \quad \sum_t \frac{dq_t}{du_\tau} \; \frac{dH(q_t)}{dq_t}$$

the softly clipped residual

$$\Delta u_\tau \quad = \quad \sum_t (r_{t+\tau}) \quad (g_t H'(q_t)) \qquad\qquad \tau \neq 0$$

A crosscorrelation: Compute it in the Fourier domain.
At convergence this is a delta function.
Special case: stationary L2 then r(t) is white.
Amazing generalization to
(1) non-causal, (2) gain, and (3) sparsity!

Jon's favorite theory slide.

From $\Delta\mathbf{u}$ to $\Delta\mathbf{r}$

Skipping lots of algebra
(including a linearization)
given the gradient step $\Delta\mathbf{u} = (\Delta u_\tau)$
and the residual $\mathbf{r} = (r_t)$,
the residual perturbation is $\Delta\mathbf{r} = \mathbf{r} * \Delta\mathbf{u}$.
("$*$" is convolution)
and the sparsity perturbation is
$\Delta q_t = g_t \, \Delta r_t$.

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Minimizing $H(\mathbf{q} + \alpha \Delta \mathbf{q})$

At each q_t fit hyperbola to parabola (Taylor series).
A sum of parabolas is a parabola. Easy getting α.

$$\alpha \quad = \quad - \quad \frac{\sum_t \Delta q_t H'_t}{\sum_t (\Delta q_t)^2 H''_t}$$

Update the residual $\mathbf{q}$ and unknowns $\mathbf{u}$.
Form new Taylor series and iterate.

Recall stationary ℓ_2: $\quad \alpha = -\ (\Delta \mathbf{r} \cdot \mathbf{r}) / (\Delta \mathbf{r} \cdot \Delta \mathbf{r})$

Newton's method.

Quick peek at the algorithm: math to code key

Lower case letters for variables in time and space like `d` $= d(t, x)$, `dq` $= \Delta q(t, x)$, `u` $= u_\tau$.

Upper case for frequency domain like `R` $= R(\omega, x)$, and `dU` $= \Delta U(\omega)$.

Asterisk $*$ means multiply within an implied loop on t or ω.

The algorithm is brief.

```
D =   FT(d)
U = 0.
Remove the mean from U(omega).
Iteration {
     dU = 0
     For all x
          r = iFT( D * exp(U))
          q = g * r
          dU = dU + conjg(FT(r)) * FT(g*softclip(q))
     Remove the mean from dU(omega)
     For all x
          dR = FT(r) * dU
          dq = g * iFT(dR)
     Newton iteration for finding alfa {
          H'  = softclip( q )
          H'' = 1/(1+q^2)^1.5
          alfa= - Sum( dq * H' ) / Sum( dq^2 * H'')
          q = q + alfa * dq
          U = U + alfa * dU
          }
     }
```


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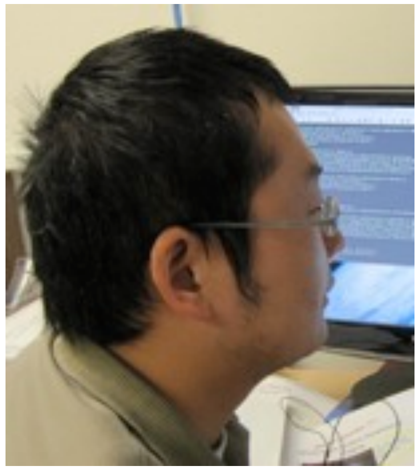

Instability! Yikes!

Sometimes there are time shifts.
Sometimes the polarity is wrong.
I'm going to work on velocity instead.

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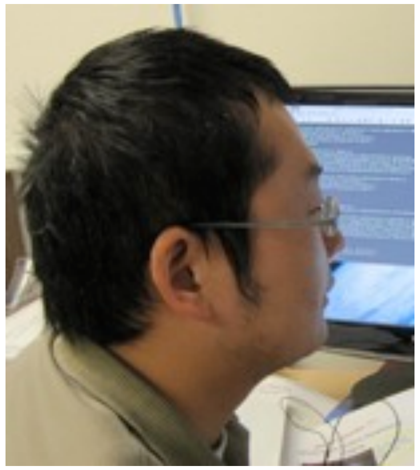
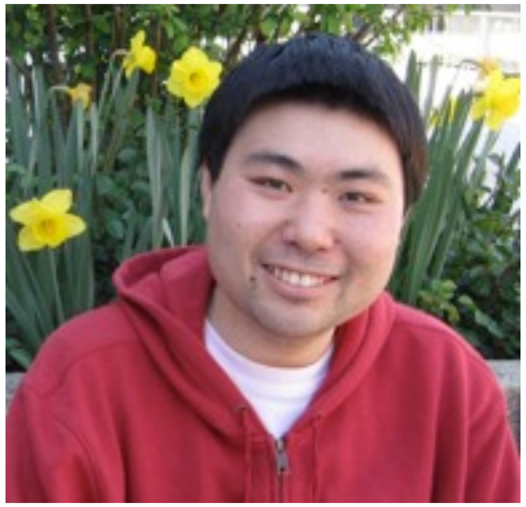
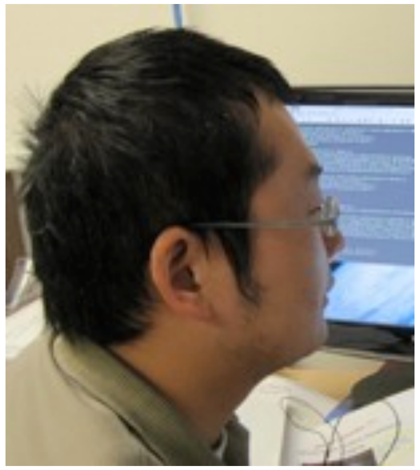


Instability! Yikes!

Try preconditioning.
Try regularization.

I tried them.
I'd rather do Q tomography.

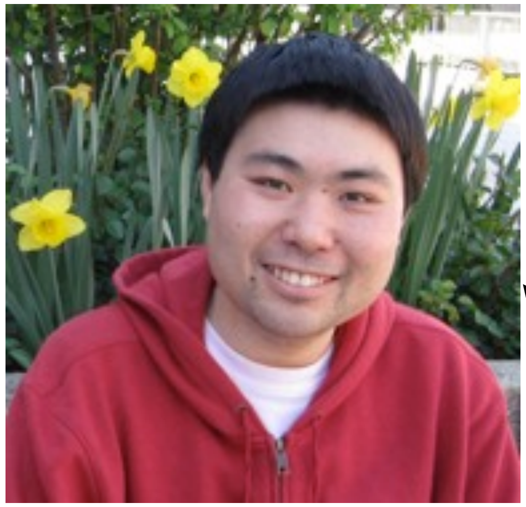


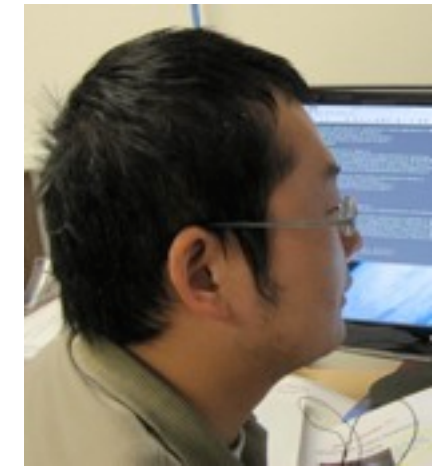


Instability! Yikes!

Masking the gradient fails.
Here are the sample histories
you asked for.

I'm going to Houston.





Instability! Yikes!

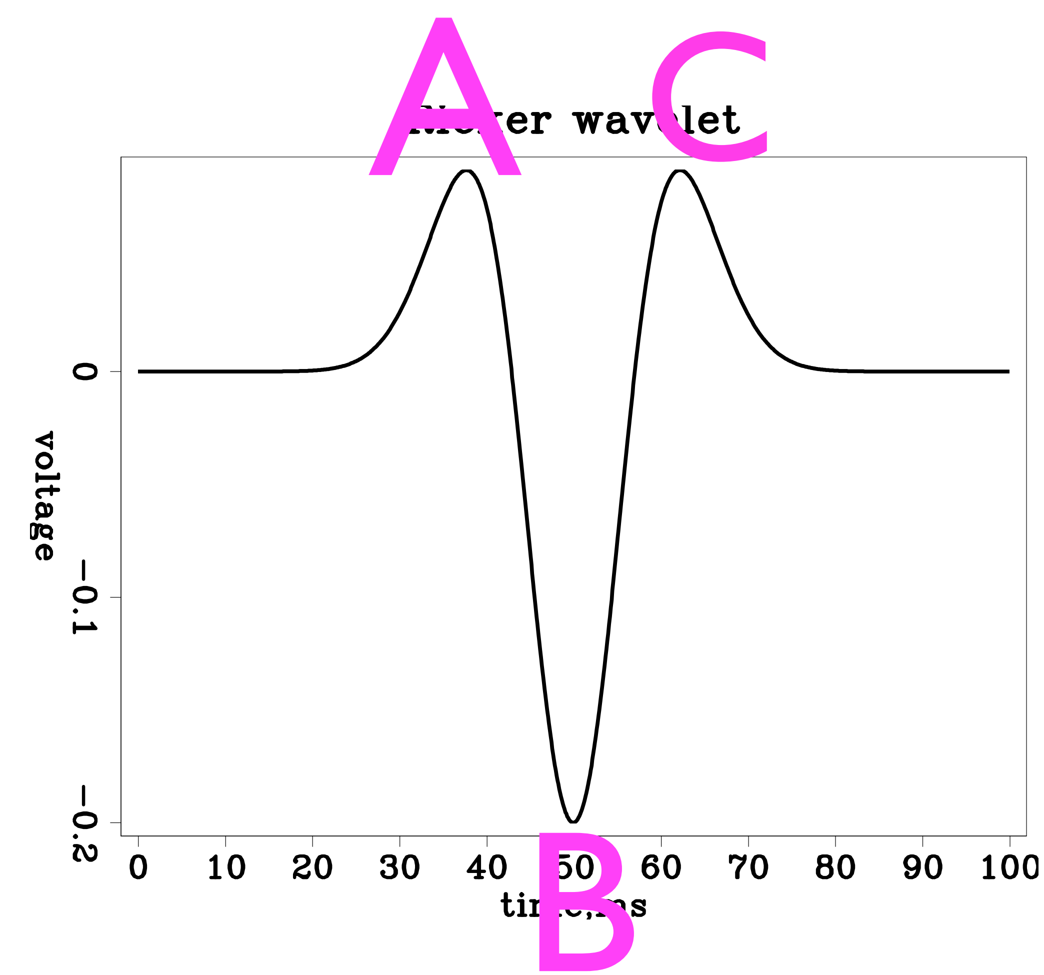
Antoine! Help!

Instability. Yikes!

Antoine: I changed the gain by 10% and the spike jumped from B to C.

Jon: Awful! I thought I had a great starting solution at B

Jon: Make me a movie as a function of iteration.

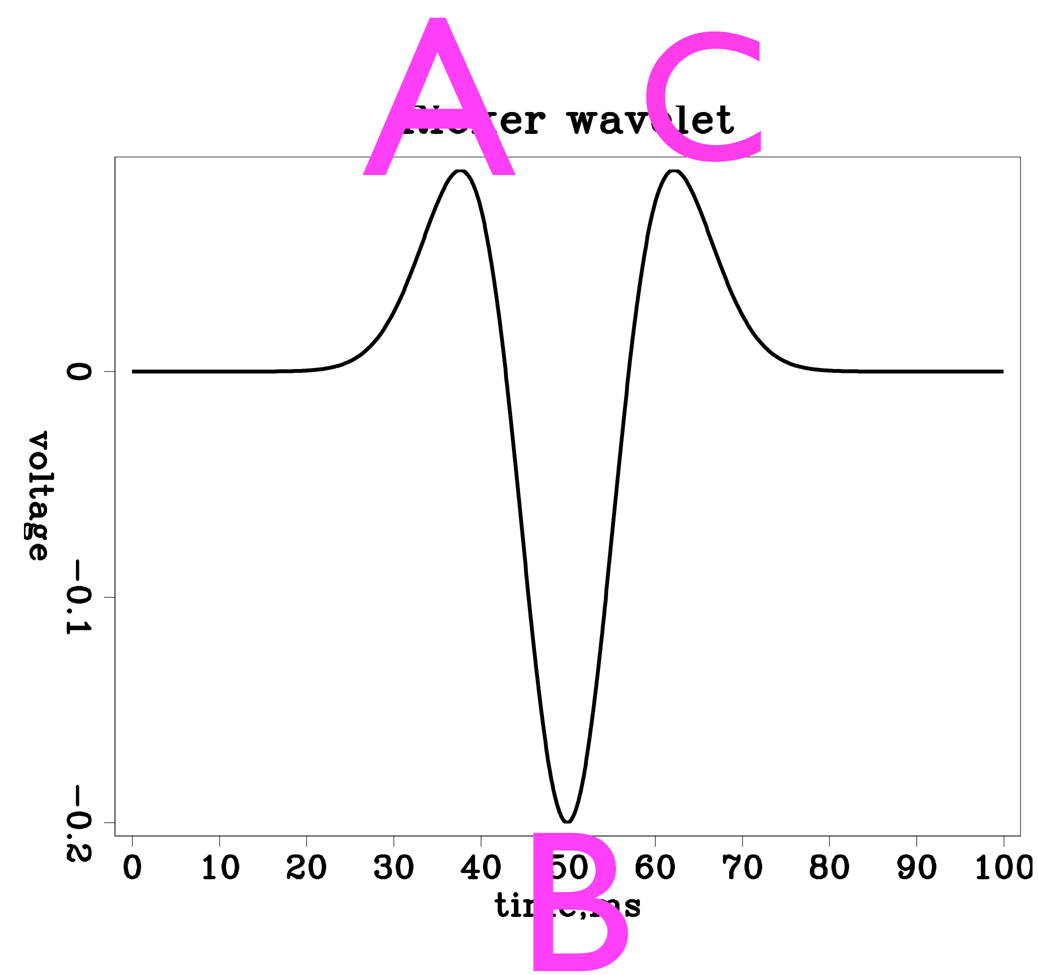


Instability. Yikes!

with Antoine and Qiang Fu

10 iterations:
good spike at B,
A&C small

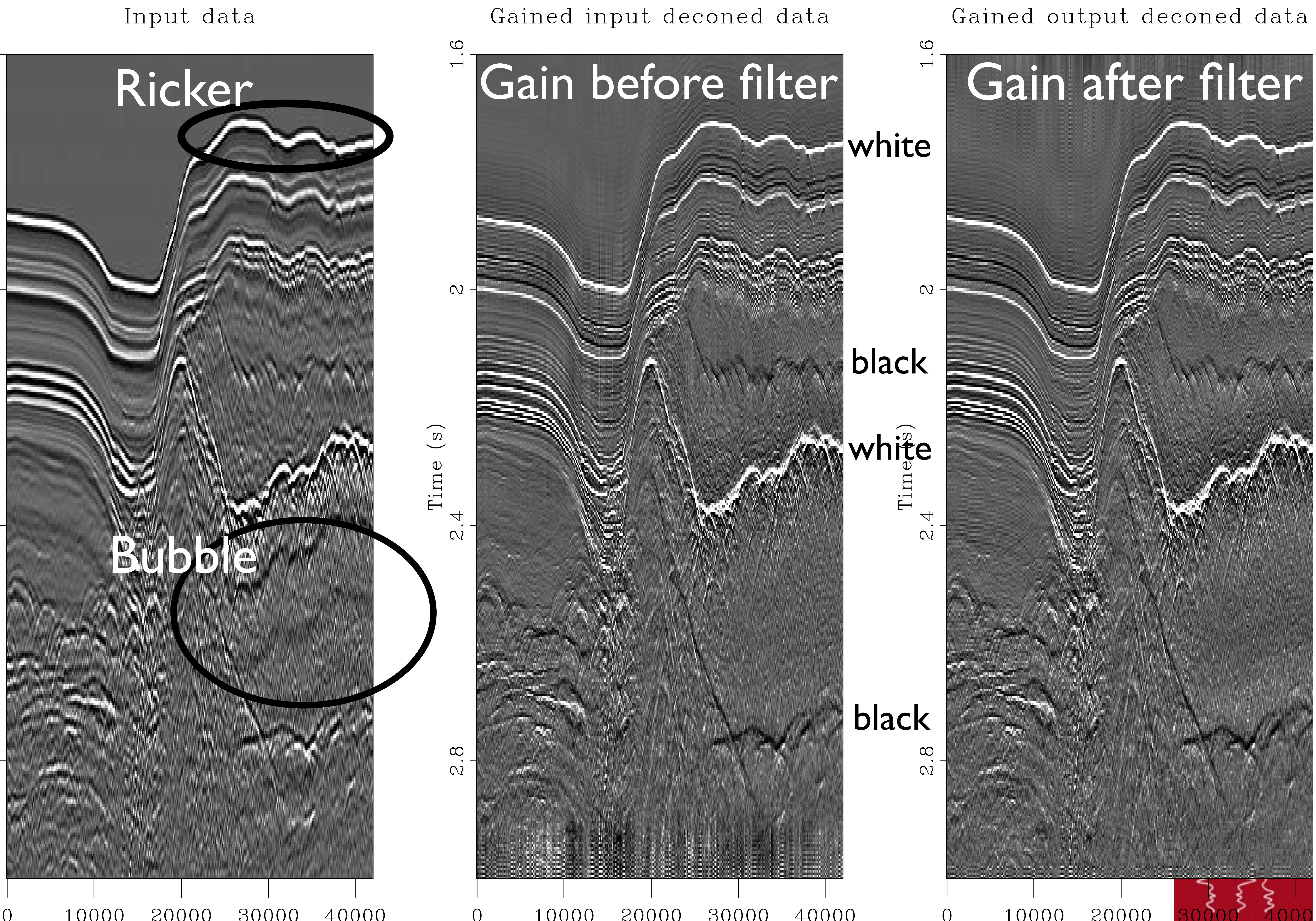
200 iterations:
maybe spikes at A
maybe spikes at B
maybe spikes at C
others small



Thursday, May 24, 2012

"But when it's good, it's really good! Let's look at some of the results."

We'll return to the stability problem later.



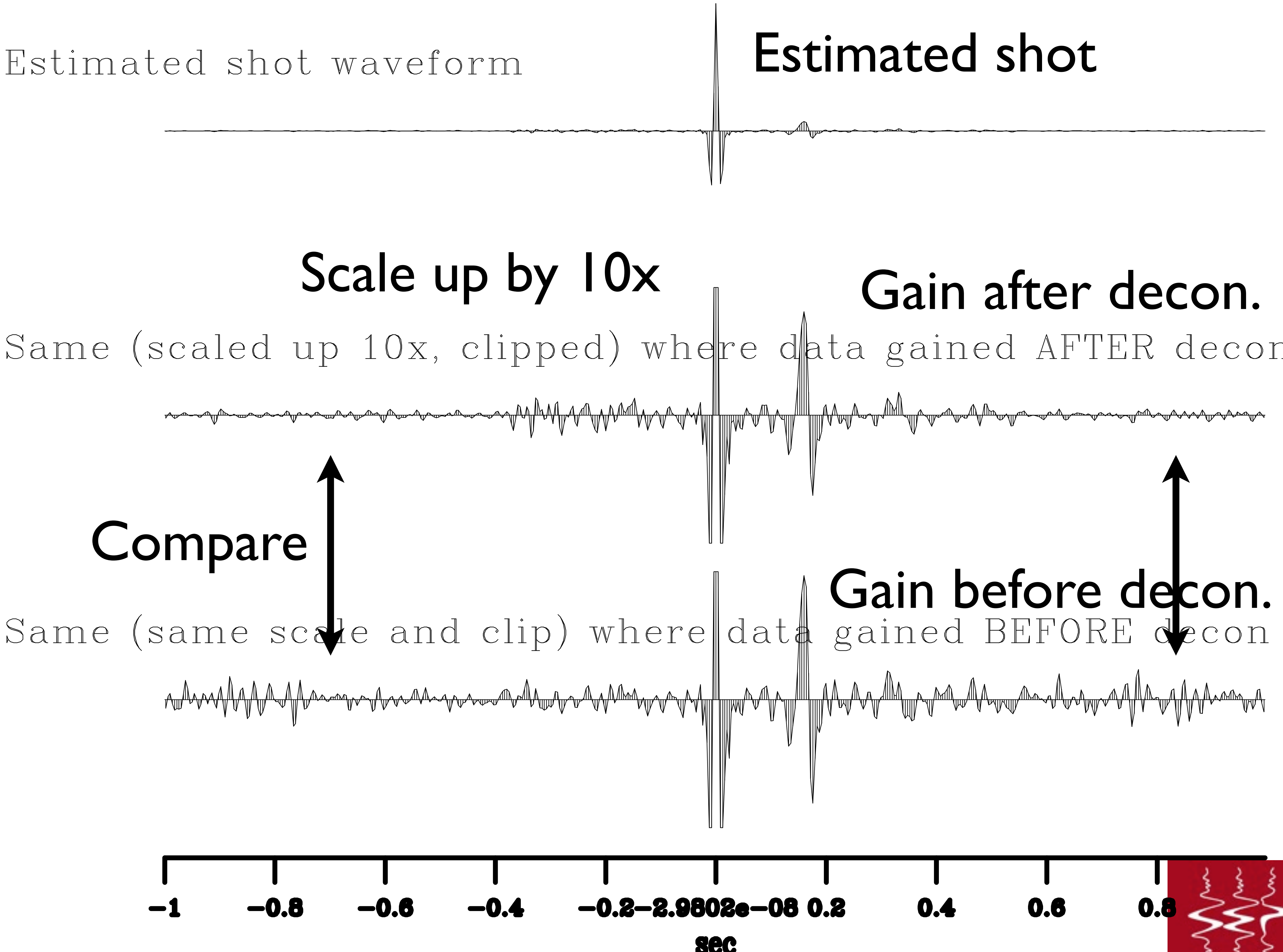
first seen by Yang Zhang

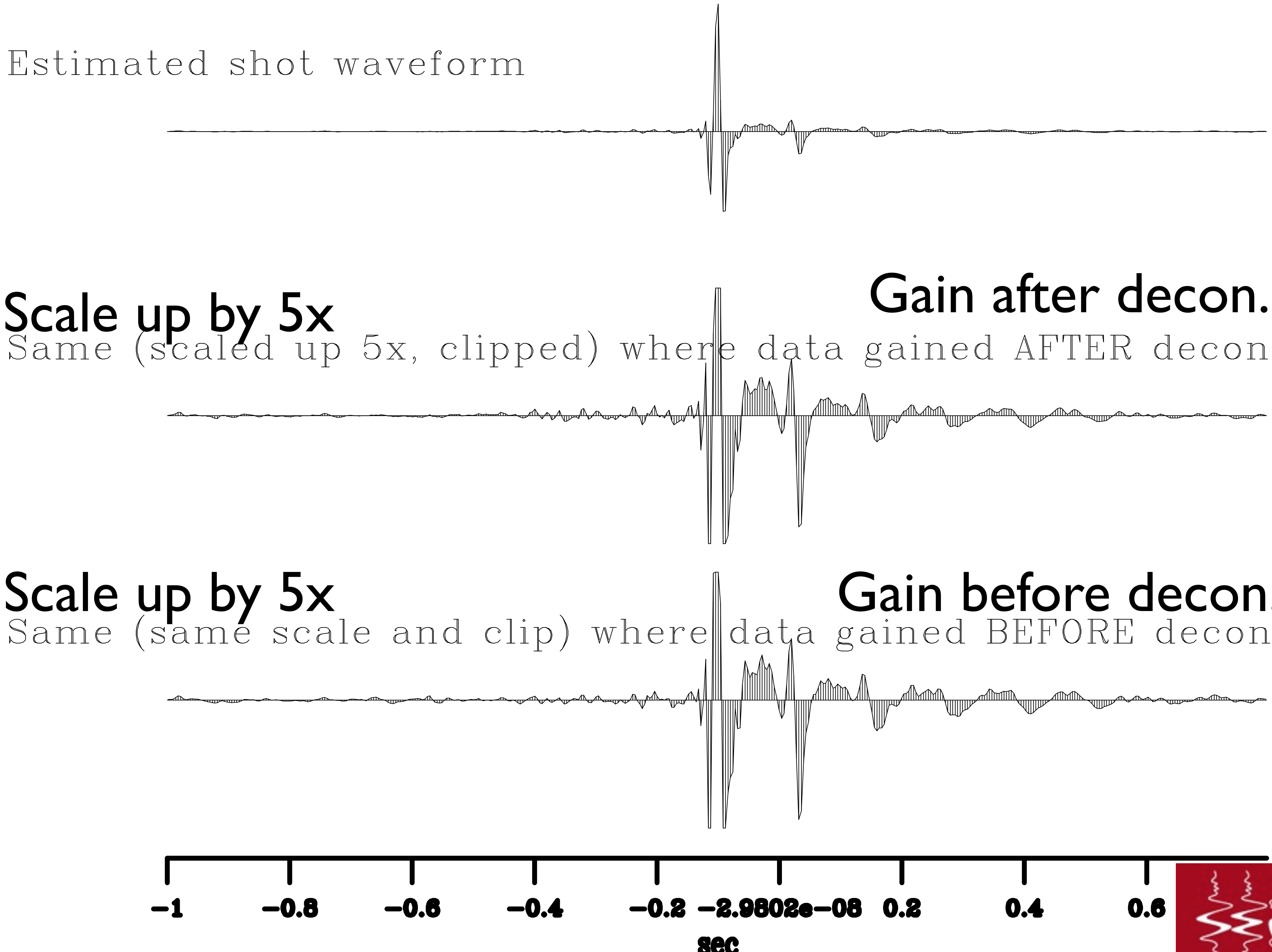
Prepare to compare gain before with gain after

data ⟶ t-squared gain ⟶ decon

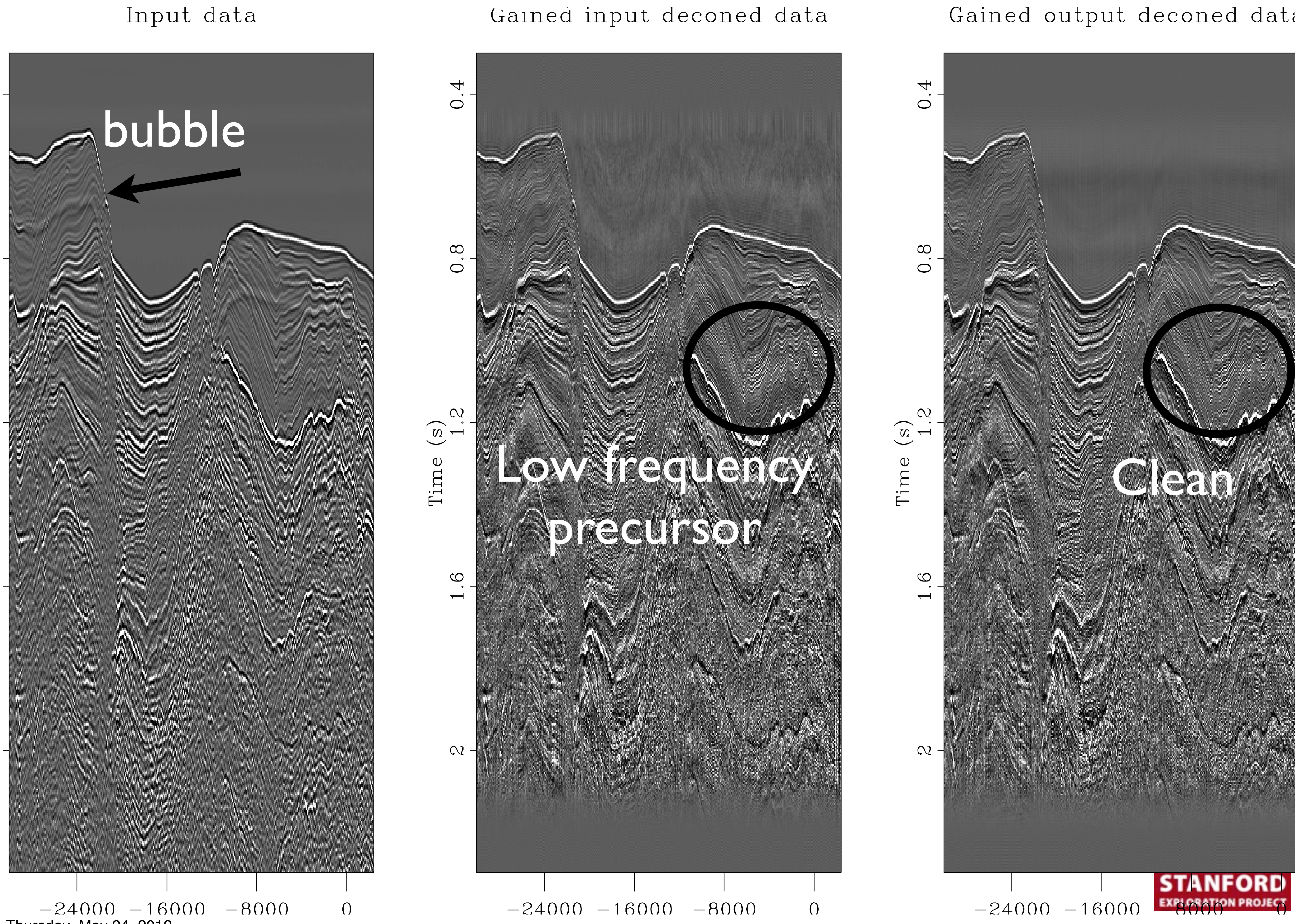
data ⟶ new decon ⟶ t-squared gain

r_t q_t


Estimated shot waveform
Estimated shot
Scale up by 10x
Gain after decon.
Same (scaled up 10x, clipped) where data gained AFTER decon
Compare
Gain before decon.
Same (same scale and clip) where data gained BEFORE decon
−1
−0.8
−0.6
−0.4
−0.2−2.9802e−08 0.2
0.4
0.6
0.8
sec


Estimated shot waveform
Scale up by 5x
Gain after decon.
Same (scaled up 5x, clipped) where data gained AFTER decon
Scale up by 5x
Gain before decon.
Same (same scale and clip) where data gained BEFORE decon
−1
−0.8
−0.6
−0.4
−0.2
−2.9802e−08
0.2
0.4
0.6
sec

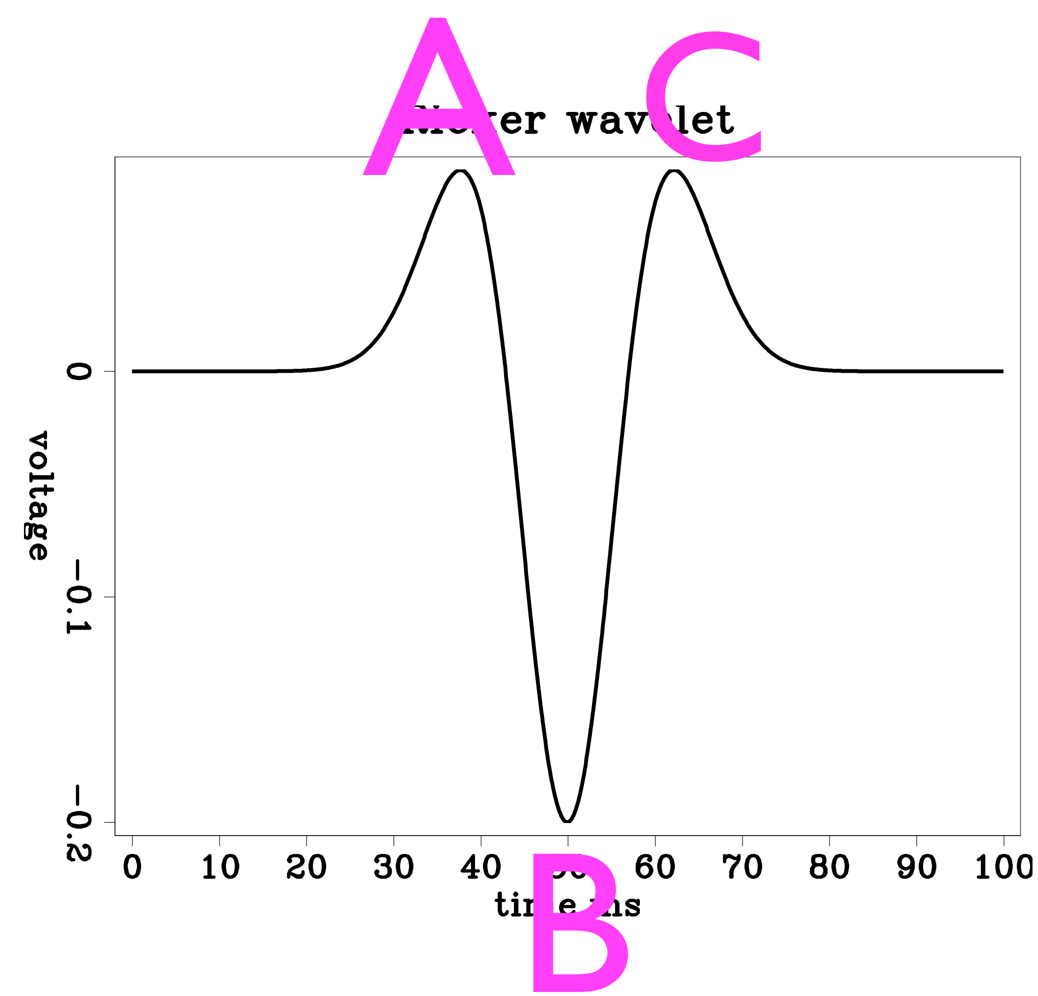
Produced by Antoine



Instability. Yikes!

with Antoine and Qiang Fu

10 iterations,
spikes at B,
A&C small



200 iterations,
maybe spikes at A
maybe spikes at B
maybe spikes at C
others small

Nonlinearity?

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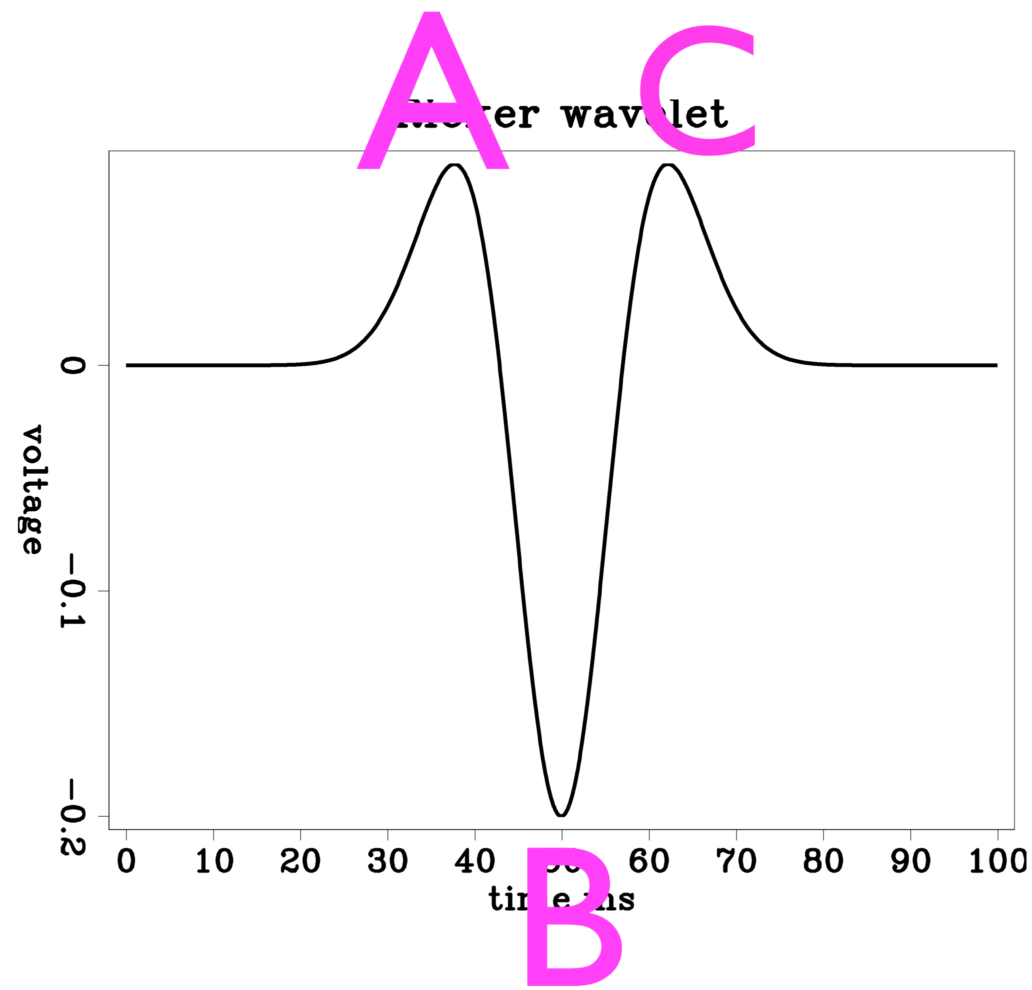
EXPLORATION PROJECT

Instability. Yikes!

with Antoine and Qiang Fu



10 iterations,
spikes at B,
A&C small

200 iterations,
maybe spikes at A
maybe spikes at B
maybe spikes at C
others small

Nonlinearity?

Null space!!

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EXPLORATION PROJECT

Nobody has proven it is a null space problem.

But I think it is,

so I must come up with a regularization.

Basic Regularization

$$0 \approx w_\tau(u_\tau - \bar{u}_\tau)$$

weights

prior model

But how to choose them?

Fancier Regularization

$$0 \quad \approx \quad \sum_t \sum_k w_{k,\tau}(u_\tau - \bar{u}_\tau)$$

$$0 \quad \approx \quad \mathbf{W}(\mathbf{u} - \bar{\mathbf{u}})$$

but what to choose for $\mathbf{W}$ and $\bar{\mathbf{u}}$?

unknown matrix

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Intuitive Regularization

$$0 \;\approx\; w_\tau (u_\tau - u_{-\tau})$$

$$\text{Choose big } w_\tau \text{ where } |\tau \approx 0|.$$

Reduces the phase near t=0,
more like Ricker there.

the happy discovery slide

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Regularization

FFT notation in matrix,
Fortran notation in vectors.

$$\mathbf{0} \approx \begin{bmatrix} r_m(1) \\ r_m(2) \\ r_m(3) \\ r_m(4) \\ r_m(5) \\ r_m(6) \end{bmatrix} = \mathbf{W} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 & 0 & -1 \\ 0 & 0 & +1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & +1 & 0 \\ 0 & -1 & 0 & 0 & 0 & +1 \end{bmatrix} \begin{bmatrix} u(1) \\ u(2) \\ u(3) \\ u(4) \\ u(5) \\ u(6) \end{bmatrix} = \mathbf{WJu}$$

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Report deadline

Only Antoine has seen the results

(if he hasn't been too busy at work).

Any student had too much synthetic data?

Theory innovations

- Two-sided filters escape minimum phase.
- Use sparsity goal instead of whiteness.
- Apply gain and mute AFTER filtering.

Conclusions from testing

- Value of gain theory confirmed by two examples.
- Sparsity is not powerful enough to ensure a "best" phase. Regularization is needed.
- A long-needed regularization is identified.


I'd like to thank the team.
Fu
Yang
Antoine
Elita
Yi
STANFORD

ACKNOWLEDGEMENT

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We'd like to thank Yang Zhang for continued interest.

The end