Skewed Pulses

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ABSTRACT

Analogous to bandpass filters, Jon provides a two-parameter definition of a compact pulse with a sharp onset and a slow decay. Its decay/risetime ratio defines the pulse skewness. The immediate application is for modeling and image estimation. Since pulse skewness is occasionally observable on field data we should design a nonstationary process to highlight it in hopes of identifying an interpretive value. Marine sources always have an inherent low cut. This low cut is in addition to the low cut that is coming from a source ghost, possibly also a receiver ghost, and inherent low cut in the transfer functions of hydrophones and geophones. One way to produce such inherent low cut is to take the difference between two skewed wavelets. The two skewed wavelets are normalized so they would have the same area, so their difference has no zero zero frequency ("DC") content.

INTRODUCTION

Wavefield modeling requires a source waveform. It is commonly taken to be a band-limited impulse. The method of band-limitation is rarely given much thought. Most often the resulting signal is simply taken to be time-symmetric. Such a choice is offensive to elder geophysicists like Jon who commonly observe "innovation functions" being non-symmetric. Besides use for modeling, pulse non-symmetry might have diagnostic power on signals which resemble the sound of boiling water, volcanic tremor, signals like fracking generated microseisms, and signals like earthquake codas.

JON SAYS: SKEWED HIGH CUT

Futterman

A compact and causal function (one that vanishes before $t = 0$) is the Futterman function (or Futterman wavelet). Over many decades in frequency, laboratory studies have shown that seismic wave amplitudes generally dissipate strength in proportion to frequency. The presumption that this be strictly true is called the constant-Q media assumption. The wavelet is defined by the two ideas that it should emerge from a constant-Q medium and it should be causal—it should vanish at negative lags. Spectra from more distant reflectors trend to lower frequencies so this function is a propagation phenomenon, not a source waveform. The wavelet depends on the measurement location. Never-the-less, nearby any travel-time distance t_0 the Futterman wavelet remains a suggestion for a local shot waveform.

A notable aspect of the Futterman wavelet is that it has a rapid onset with a slower decay. Low frequencies appear to travel slower than high frequencies simply because it is impossible to stuff low frequencies into a single t_0 location.

After propagating a distance of z in a medium of velocity v and quality Q, high frequencies are diminished in proportion to $\exp(-|\omega|z/(vQ) = \exp(-|\omega|t_0/Q)$. The corresponding time-domain function is called the Futterman wavelet.

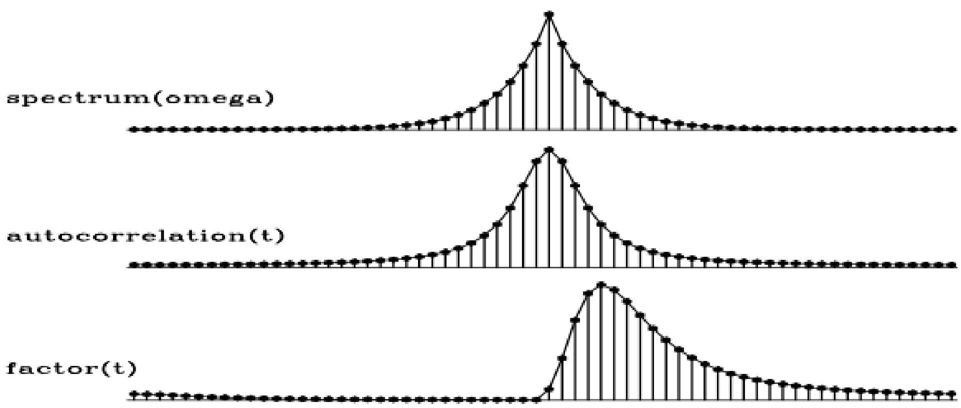

Figure 4.7: Autocorrelate the bottom signal to get the middle, then Fourier transform it to get the top. Spectral factorization works the other way, from top to bottom. hlx/. futterman

Figure 1: Kolmogoroff Spectral factorization. (From GIEE section 4.2) [**NR**]

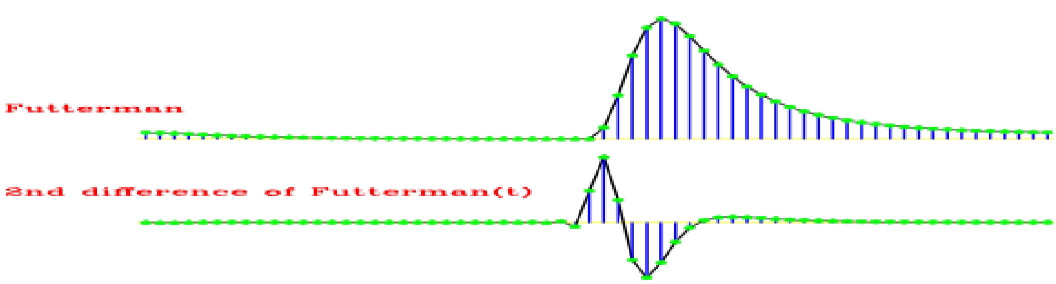

Figure 1.2: The causal constant Q response and its second finite difference. The first two lobes are approximately the same height, but the middle lobe has more area. That third lobe is really small. Its smallness explains why the water bottom could seem a Ricker wavelet (second derivative of a Gaussian) while the top of salt would seem a doublet. (Claerbout) signal/. futter

Figure 2: The Futterman function and its second derivative (From NSDF chapter 1.) For practical purposes, the second derivative is a *two*-lobed wavelet. [**NR**]

The computation of the Futterman wavelet is 20th century mathematics. It is abstract, compact, efficient, and subtle. Complete code for it is found in my free book GIEE in the section named Kolmogoroff Spectral Factorization. (Unfortunately, this wavelet seems impossible to create using differential equations in t.)

Hubbert's Pimple

As an alternative to the Futterman wavelet, we begin from a function known in Geophysics as Hubbert's Pimple. M. King Hubbert devised this function to predict the ultimate decline in worldwide petroleum production. Hubbert's pimple is the specialization of Equation (1) to $\alpha = \beta$. However, of interest here is the skewed case when $\alpha < \beta$.

$$s(t) \quad = \quad \frac{4}{(e^{(t_0-t)/\alpha} \;+ e^{(t-t_0)/\beta})^2} \tag{1}$$

This paper advocates Equation (1) as an appealing impulse that should be widely used to represent source functions. At large $|t - t^0|$ the function damps exponentially both before and after $t = t_0$. For $\alpha < \beta$ the function rises more rapidly in time and then decays more slowly. (At large positive $t - t_0$, we may ignore α. At large negative $t - t_0$ we may ignore β. At large positive $t - t_0$, $s(t)$ tends to $2/(e^{(t-t_0)/\beta})^2 = 2/e^{2(t-t_0)/\beta} = 2e^{-2(t-t_0)/\beta}$.)

In this paper we are concerned with the meaning of α and β and choosing (or estimating) numerical values for them.

Hubbert attacked the important problem of predicting the decline of petroleum production. Sadly, his predictions are fairly worthless because he did not recognize his unwarranted presumption that future decay β numerically matches that of past growth α. Francis Muir and I meticulously derived Hubbert's pimple in its four different forms related to petroleum production data fitting. That development is found in SEP report 136.

Defining skewness

The *rise time* is specified by α while the *decay time* is specified by β. In the frequency domain we might deal with a high-frequency cutoff and a low-frequency cutoff. Here in the time domain α resembles the (inverse) high-frequency cutoff (short wavelength) while β resembles the low frequency (long wavelength) cutoff.

In music, one octave represents a range of a factor of two in frequency, likewise a factor of two in wavelength. The more octaves, the more spectral bandwidth.

$$\text{skewness} \quad = \quad \sigma \quad = \quad \beta/\alpha \quad = \quad 2^{\text{octaves}} \tag{2}$$

Bandwidth is a difference of two frequencies. *Skewness* is a ratio of two frequencies. Natural science doesn't seem to offer units for measurement of skewness, however in mechanical gear shifting for bicycles, I have seen both *percent* and *degrees* as units for measuring gear ratios. I favor *percentage*. It gives the road-distance ratio of a high gear to a low gear. I'm admiring a bicycle (Gazelle c380 HMB) in which the highest gear goes nearly four times as far as the lowest one, 380% to be exact.

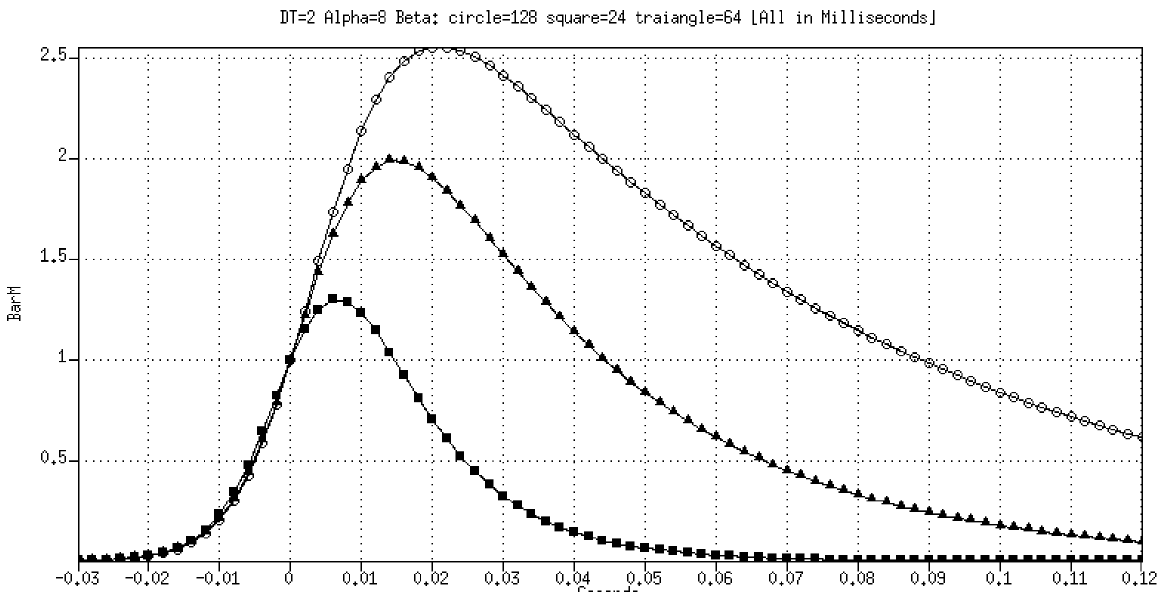

Figure 3: Asymmetric pulses of common short wavelength risetime with three different long-wavelength cutoffs. At $t = 0$ all plots take the numerical value 1.0. Signals for $\Delta t = .002\, ms$, $\alpha = 8\, ms$, $\beta = (24, 64, 128)\, ms$ for skewnesses of $\sigma = (3, 8, 16)$. [**NR**]

In reflection seismology, one might say that we have a frequency range going from 8 to 128 Hz. We can write this as $(8 : 128) = 8 \times (1 : 16) = 8 \times (1 : 2 \times 2 \times 2 \times 2)$. Since each 2 corresponds to an octave, such reflection data has a bandwidth of 16, namely 4 octaves. While that might characterize marine data, land data has more like 3 octaves. Land data has a skewness (or inverse frequency range) of 800%. Inversion code testers struggling with 3-D space might work with two octaves, 400%.

The second time derivative

Imagine a sharp Gaussian impulsive source slightly below the ocean surface and a receiver nearby. Reflections from the surface effectively form a second derivative of the Gaussian creating a downgoing wavelet called the Ricker wavelet which would be a three-lobed wavelet of high frequency. As soon as that wave enters the sea floor sediment, the earth's Q causes a severe reduction in the high-frequency content and the wavelet becomes the convolution of the sharp Gaussian, the second difference, and the Futterman. Practically speaking, that's a two-lobed wavelet.

Theoreticians and modelers like to take as a source the Ricker wavelet, a three-lobed wavelet. Observationally, we do expect the three-lobed Ricker wavelet from

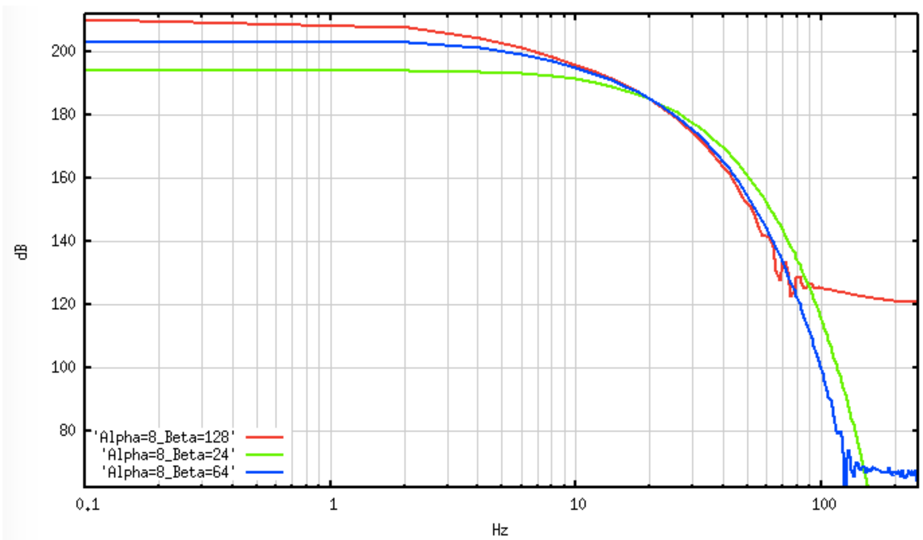

Figure 4: Spectra of $S(\alpha, \beta)$ all with the same α and varying β. SHUKI: At low frequency there is actually no cutoff because the wavelets all contain zero frequency ("DC"). They all do have a high-cut. It depends mainly on the smaller between α and β so there little difference between $\beta = 64$ and $\beta = 128$. The Nyquist frequency for 2 millisecond data is 250 Hz. At the Nyquist frequency, $\beta = 128$ is 45 dB stronger than $\beta = 64$ because the time length of the wavelet was 400 milliseconds. JON: At low frequency $\beta = 128$, *red*, the time domain curve has more area so in frequency the red curve is higher. At high frequency $\beta = 24$, *green*, there is a little more high frequency because of the sharper curvature at the top of the time function. [**NR**]

the water bottom but we expect the two-lobed Futterman 2nd derivative from the basement, from the top of salt, and from each one of the distinguishable layers.

Figure 5 tells us that for 2nd derivatives, the skewed Hubbert wavelet resembles the Futterman wavelet except for the fact that the $t = t_0$ location on the 2nd-derivative of the Futterman is at the wave onset while for the skewed Hubbert, the $t = t_0$ location lies between the two lobes near 0.5 ms.

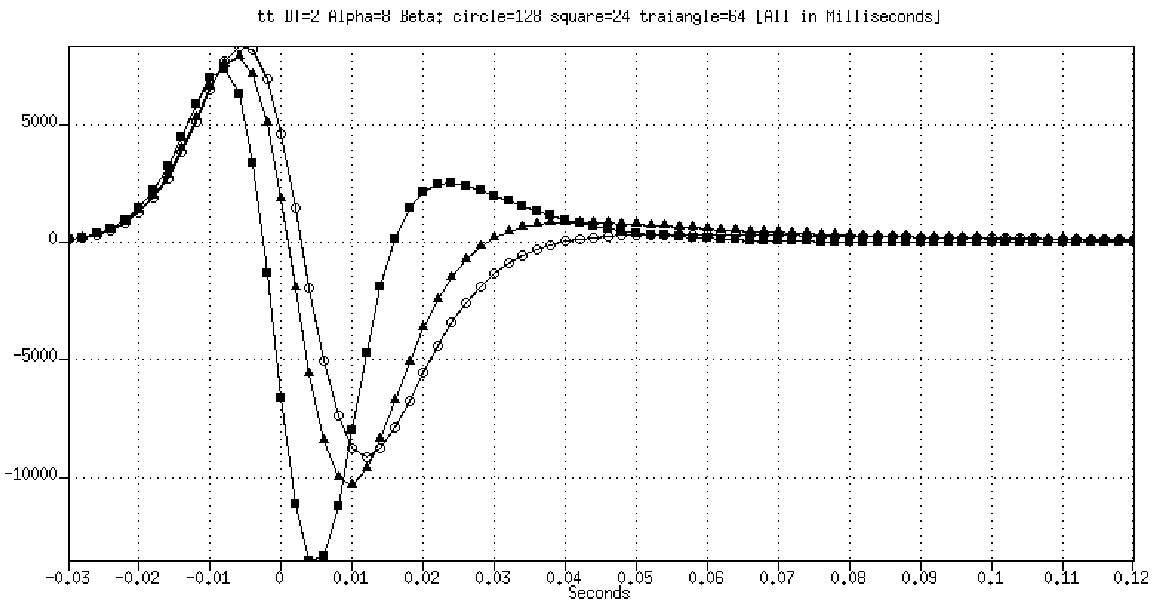

Figure 5: The second time derivative of skewed Hubbert pulses. Fixed short wavelength risetime for three long-wavelength cutoffs. Curves range from the somewhat unskewed (narrowband) triplet toward the two strongly skewed (broadband) doublets. $\Delta t = .002$, $\alpha = 8$ ms, $\beta = (24, 64, 128)$ ms for skewnesses of $\sigma = (3, 8, 16)$. [**NR**]

Estimating skewness

Is skewness σ something that can be observed in field data? It should be easy whenever the pulses are not too close together but it must be more difficult where pulses overlap. Might this skewness be measureable in the sound of boiling water, volcanic tremor, signals like fracking-generated microseisms, and signals like earthquake codas? We need to try.

Sometimes we recognize signal skewness in some areas of shot gathers and sections. Skewness may be widely present but not dominantly so. We should cook up a nonstationary estimator of skewness σ as a function of time and space.

How *might* we pose a nonstationary estimator? How *should* we pose it?

Or perhaps these days we are supposed to forget about statistical-estimation theory and simply produce a huge volume of synthetic data for machine learning? Haha.

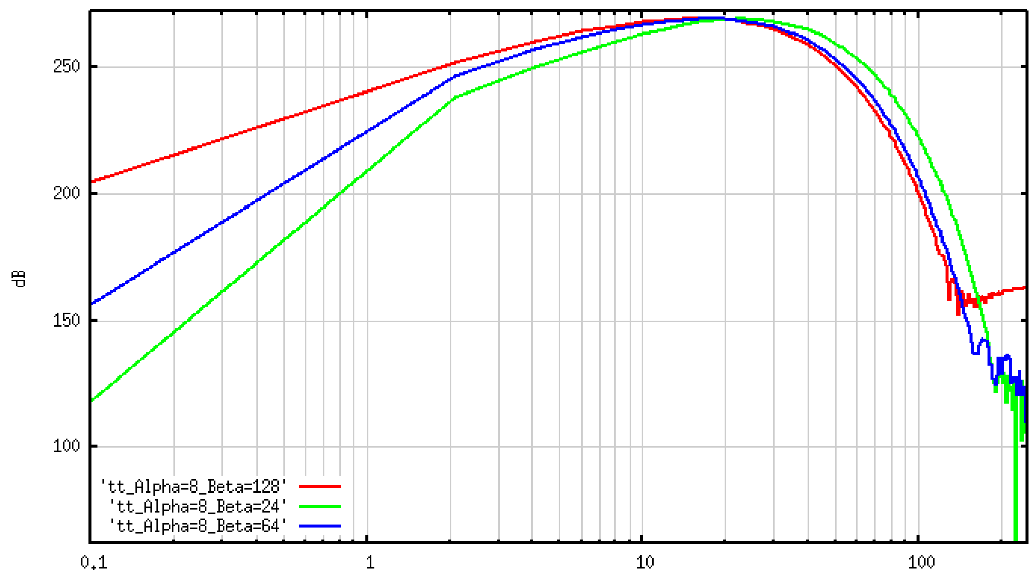

Figure 6: Spectra of the second derivative of $S(\alpha, \beta)$ all with the same α and varying β. The second derivative introduces a low-cut. Interestingly, there is more difference between $\beta = 64$ and $\beta = 128$ in the low-cut than in the high-cut. However, in this example, it is all happening in the sub-Hz range. [**NR**]

Bandpass specification: in f? in t?

Suppose we want a bandpass filter but we are less interested in sharp frequency cutoffs than we are in having a compact filter. Instead of cutoff frequencies, we might specify cutoff wavelengths. Given a wavelength cutoff filter, what would it look like compared to its corresponding frequency domain box? Show both filters in both domains. We might be disappointed and find a clear reason to drop this idea.

SHUKI SAYS: WATCH THE LOW CUT

Seismic wavelets are inherently band limited. Marine Seismic sources are machines that displace water. Chemical explosions create a bubble of CO_2 and steam. Airguns create bubbles of air. Marine Vibroseis usually displace water with a moving piston. The acoustic wave that is radiated from the displaced water is proportional to the second time derivative of the volume of the displaced water. The second derivative is an inherent analog- or physical- low-cut of 12 dB/octave. These include chemical explosives, airguns, and vibes. They can never emit zero frequency. In addition to this inherent 12 dB/octave low-cut, the ghost of the source is approximately a first time derivative that adds 6 dB/octave more to the physical low-cut. If the receiver is on a streamer, then the receiver ghost adds 6 dB/octave more. In addition to the above, hydrophones have a transfer function that includes 6 dB/octave more. All the above physical low cut filters total 24 dB/octave for ocean bottom nodes hydrophone data and 30 dB/octave for streamers. The second derivative in the frequency domain is multiplication by the square of the frequency which in the language of engineers is 12 dB per octave because an octave and 6dB mean the same thing—a factor of 2. They are best modeled by wave-equation modeling and not via the wavelet used in the modeling. When writing modeling code, it is important to include the inherent low-cut. The effects of the ghosts should be added during the wave equation modeling. The following figure shows a modification to the asymmetric inverse cosh wavelet that generates band limited wavelets. The ghosts are angle dependent.

```
double JFCW(double t, double beta,double alpha)
{
double a = exp(-t/beta) + exp(t/alpha); // Asymmetric COSH
return 4.0/a/a;
}
```

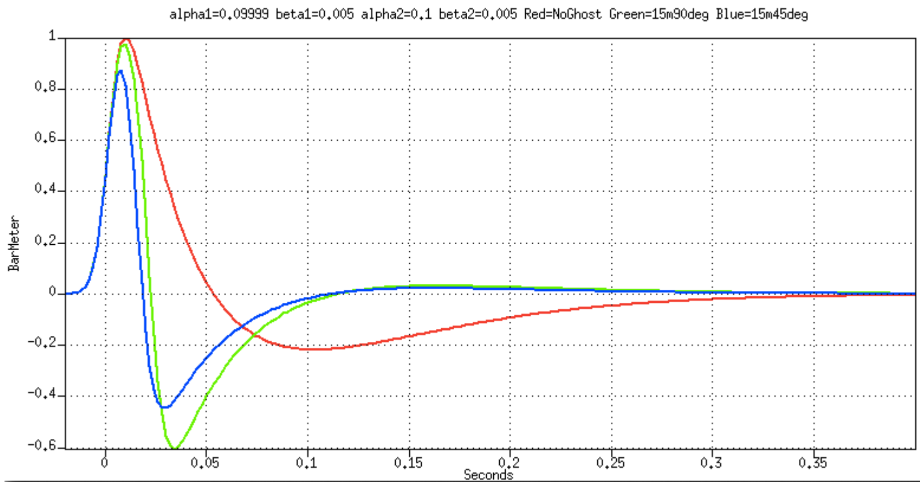

Figure 7: Band Limited JFCW (t,alpha1,alpha2,beta1,beta2) = Normalized(JFCW(t,beta1,alpha2)) - Normalized(JFCW(t,beta2,alpha2)) [**NR**]

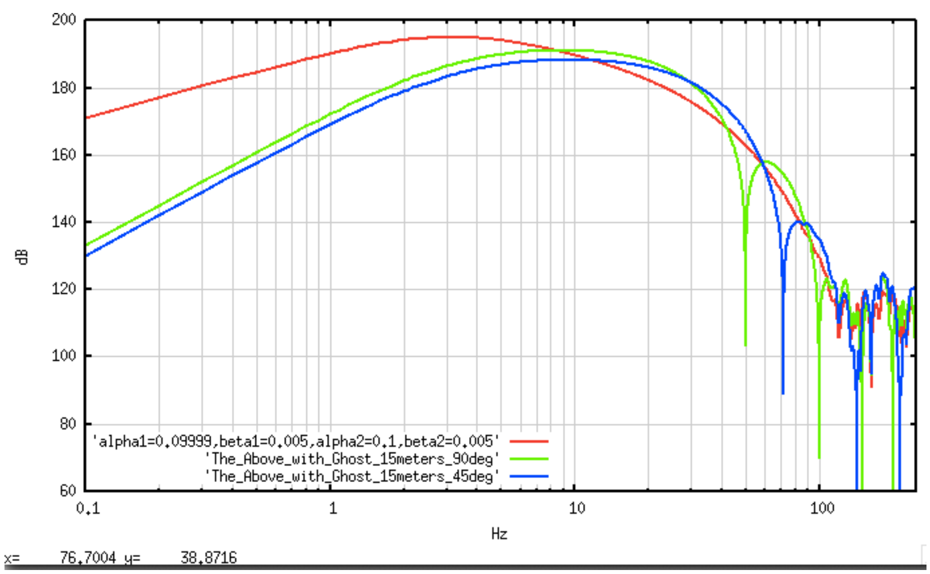

Figure 8: The band limited wavelet in the frequency domain. Red is without ghost. Green is with a ghost for a source deployed at a depth of 15 meters and at an angle of 90 degree down. Note the ghost notches at 0 Hz, 50 Hz and 100 Hz. Blue is with a ghost for a source deployed at a depth of 15 meters and at an angle of 45 degree down. Note the ghost notches at 0 Hz, 71 Hz and 142 Hz. [**NR**]