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Relative performance of moveout-based multiple-suppression methods for amplitude variation with offset (AVO) analysis and common midpoint (CMP) stacking

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ABSTRACT

We apply three moveout-based methods for multiple suppression to simple synthetic common midpoint (CMP) gathers and compare their performance in terms of level of multiple rejection and, more importantly, in terms of (1) suitability of processed output for use in analysis of amplitude variations with offset (AVO), (2) primary-to-multiple amplitude (p/m) ratio of the data after CMP stacking, and (3) preservation of primary wavelet shape and amplitude on the stack. The three approaches are filtering in the frequency–wavenumber f–k domain, the Hampson method of filtering in the parabolic Radon transform domain, and a hybrid method that improves upon Hampson's approach by using a variation of Harlan's statistical pattern recognition approach to separate primaries from multiples in the parabolic Radon transform domain.

The f–k approach is unsuitable prior to AVO analysis; moreover, it is little better in suppressing the multiples than the CMP stack itself. Hampson's method performs considerably better and is suitable for both purposes, although it can yield distorted AVO response where multiples are relatively strong on input. At about 50% additional computation effort over that of Hampson's method, the hybrid approach has superior treatment of amplitude behavior with offset. For many situations, the hybrid method yields up to twice the primary-to-multiples amplitude ratio on the CMP stack.

INTRODUCTION

Prestack multiple-suppression methods enhance the primary-to-multiples amplitude (P/M) ratio, preserve signal quality on stacked common midpoint (CMP) or common-reflection-point data, and improve data for amplitude versus offset (AVO) analysis by suppressing the multiples while preserving primary signal amplitude. Multiples, the seismic waves that bounced downward at least once before being recorded at the surface of the earth, are generally unwanted because their traveltimes do not fit time–distance relationships used to image the subsurface with primary reflections. Wave-theory–based methods for suppressing free surface-related multiples and internal multiples, where the data are used as prediction operators, hold promise for attacking multiples independent of knowledge of subsurface velocity or structure, provided data-set consistency requirements are met (Verschuur et al., 1992, 1995; Verschuur and Kelamis, 1997; Weglein et al., 1997; Dragoset, 1999). Meeting these conditions, especially for land data, is difficult at best; suppression, particularly of internal multiples, remains problematic in practice.

Traditional moveout-based methods have limitations as well; in particular, the subsurface must be relatively simple so the moveout of primaries and multiples can reasonably be considered hyperbolic and symmetric about zero offset, with relatively mild amplitude variations. Also, the performance of these methods decreases when the differential moveout between primaries and multiples becomes small. Still, moveout-based approaches are the workhorse of multiple suppression with land data.

Of moveout-based approaches, perhaps the most unsung, and yet powerful and universally applied, process for suppressing multiples is the CMP stack and its counterparts, stacks of dip-moveout (DMO) processed or prestack-migrated data. A key drawback of the stack [including stacking with trace weights designed for optimum suppression of multiples (Schoenberger, 1996)], however, is that the multiple suppression is achieved only upon stacking the data, so the process is of no help in suppressing multiples prior to AVO analysis or key data-quality–dependent processes such as velocity estimation, statics estimation, and deconvolution. Moveout-based methods for suppressing multiples on unstacked data can enhance not only the quality of AVO analysis and other prestack

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processing but also the level of multiple suppression on the stack beyond that which could be achieved with CMP stacking alone.

Prestack moveout-based methods consist of three main steps: (1) application of an invertible transformation—frequency–wavenumber (f–k) or parabolic Radon (τ–q)—to map the primaries and multiples to separate regions in those domains, (2) application of a suitably tapered mute to suppress the multiples while ideally preserving the primaries, and (3) inverse transformation to return the multiple-suppressed data to the time domain. The level of multiple suppression with these methods depends on the ability of the chosen transform to separate the primaries and multiples. The quality of this separation in turn depends on the moveout difference between primaries and multiples on the original CMP trace gathers. Because that moveout differential is a function of source–receiver offset, among other things, and because each of the multichannel prestack approaches introduces its particular end effects on the CMP trace gathers in the offset–time (t–x) domain, each has the potential shortcoming that multiple suppression and primary preservation will be offset dependent, thus posing problems for AVO analysis of the processed data.

It is of interest to assess the performance of moveout-based prestack multiple suppression methods in terms of their ability to (1) preserve signal amplitude variation for AVO analysis, (2) enhance P/M ratio on the stack, and (3) preserve primary wavelet phase on the stack. Here, we compare these measures of performance for f–k filtering and for the τ–q approach of Hampson (1986) applied to simple synthetic CMP trace gathers containing moveout-corrected primaries and undercorrected multiples. Moreover, we introduce a third moveout-based approach into the comparison. This hybrid approach combines strengths of the Hampson method and a variation of the method of Harlan et al. (1984) that uses a statistical scheme to further suppress multiples while the data are in the τ–q domain.

We begin by reviewing the three methods, detailing the hybrid method. We then introduce the model data and show the results of multiple reduction with each method on a sample of the model data. The next two sections analyze these results from the perspective of AVO preservation and the quality of the CMP stack. We conclude with a qualitative ranking of each method as they relate to characteristics of the data, in terms of relative P/M ratio and AVO.

OVERVIEW OF MOVEOUT-BASED PRESTACK METHODS

f–*k* filtering

The f–k filtering approach to multiple suppression is a multichannel method that operates on successive CMP gathers. After a CMP gather is NMO corrected and 2D Fourier transformed, primary and multiple events ideally occupy different portions of the f–k domain, so application of some form of muting of the portion containing the multiples, followed by inverse Fourier transformation, should yield a CMP gather with multiples suppressed. A taper of amplitudes between the pass and reject zones reduces the artifacts of Gibb's phenomenon, which can distort the signal obtained after the data are inverse transformed back to the t–x domain.

Unfortunately, because residual multiple events have neither linear moveout nor constant amplitude with offset, it is impossible to simultaneously suppress the energy of the multiples and preserve the energy of the primaries at the short offsets, so offset-dependent distortion in the amplitudes of the processed primaries is unavoidable.

Parabolic *τ*–*q* filtering (Hampson's method)

In Hampson's method, the NMO-corrected CMP gathers are transformed to the τ–q domain using a discrete parabolic Radon transform (Beylkin, 1987). Ideally, this process transforms constant-amplitude events with parabolic moveout,

$$t(x) = \tau + qx^2,$$

in CMP gathers into amplitudes at isolated points in τ–q space. Conversely, points in the τ–q domain are mapped, ideally, into constant-amplitude parabolas in t–x space.

Because they are nearly horizontal after the NMO correction, the primaries in the trace gathers are mapped near the $q = 0$ line, whereas the multiples, which remain undercorrected with a moveout close to parabolic, are mapped away from that line. A suitable attenuation factor (so-called tapered mute or, simply, mute) is applied in the τ–q domain to suppress the energy in the region corresponding to the multiples, and the resulting data (primaries, one hopes) are inverse transformed to the t–x domain.

In practice, to minimize an artificial appearance in the resulting primary data, the method is often implemented in a slightly different way: the primary energy region is suppressed first so the inverse transform actually recovers the multiples. Subtracting the multiples-only data from the original data back in the t–x domain then yields the primaries-only estimate.

Further issues—spatial sampling and aliasing in the transform domain, degree of focusing of primaries and multiples, and truncation artifacts—must be addressed in the practical use of the method (Alvarez, 1995). Aliasing, a problem with the Radon transform as it is with any transform that deals with discrete data, tends to introduce noise in the τ–q domain that introduces artifacts back in the data space. Conditions for avoiding aliasing in either the data space or the model space (parabolic Radon transform) are given in Hugonnet and Canadas (1995). To satisfy these aliasing conditions, the data may have to be high-cut filtered or the offset range restricted. An alternative is to use a q sampling that is inversely proportional to the temporal frequency (Schonewille and Duijndam, 2001). Some irregularity in spatial sampling of the data can be accommodated in Hampson's method if the τ–q transform is computed using Beylkin's f–x discrete Radon transform by defining the forward operator as going from the model to the data space (Beylkin, 1987; Foster and Mosher, 1992).

Provided a perfect NMO correction of primaries, and within the validity of the approximation of the parabolic residual moveout of the multiples, Hampson's method would yield virtually perfect results if the range of offsets in a CMP gather were infinite and the data exhibited no spatial aliasing. For gathers with a finite range of sampled offsets, however, τ–q transformation introduces two distinct problems: (1) truncation artifacts and (2) imperfect focusing of the primaries. The first problem is particularly serious at the near offsets where the moveout discrimination between primaries and multiples is relatively poor. Truncation artifacts appear as horizontal lines in the τ–q domain (Alvarez, 1995; Kabir and Marfurt, 1999) and as noise in the inversely transformed data (Alvarez, 1995). The second problem manifests itself as primary energy leaking into the multiple region and vice versa.

Under ideal conditions, so-called high-resolution parabolic Radon transform approaches are effective in concentrating both the primaries and the multiples to small regions of the τ–q domain. Sacchi and Ulrych (1995) use a nonlinear sparseness algorithm in the frequency domain with a minimum-entropy model constraint in the q-direction and show that fewer q-values are required to represent the data correctly, in addition to increasing resolution of hyperbolic events in the velocity space. Herrmann et al. (2000) propose a noniterative, dealiased, high-resolution Radon transform that they claim can accommodate small spatial aperture, insufficient spatial sampling, and small moveout difference between primaries and multiples. Their method exploits sparseness and the use of local windows in the τ–q domain. The increased focusing of both primaries and multiples in the τ–q domain is attractive because it decreases the contamination of primaries with residual multiples. Problems arise, however, where amplitude, or—worse—the polarity of the primaries varies with offset and residual moveout departs from being parabolic. These realities of field data can compromise the focusing power of any of these transform approaches. Therefore, whether or not use of high-resolution approaches translates into a more faithful amplitude preservation of recovered primaries in field data remains an open question.

A hybrid method

To address the problem of leakage between the primaries and multiples portions of the τ–q domain, we combine Hampson's method with the S/N separation algorithm of Harlan et al. (1984).

Harlan's method uses a statistical approach to separate signal from noise on the basis of their difference in moveout pattern. This difference is exploited by applying to the data (signal plus noise) an invertible linear transformation that focuses or concentrates the signal, but not the noise, in the transformed domain. By focusing, we mean increasing the sparsity of the model representation of a given event. For example, a linear event across many traces in a stacked section in principle requires only two parameters in the slant-stack (τ–p) domain: the slope p of the event and the time intercept τ at some reference midpoint. Therefore, we say the event has been focused by the linear τ–p transform. Nonlinear events cannot be similarly described by only two parameters after applying of a linear τ–p transform; thus, they are not as well focused by that particular transformation. A different transformation (such as a parabolic Radon transform where the event is parabolic in the t–x domain) can accomplish the desired focusing. Then, in the transformed domain the signal and the noise can be identified and separated based on some measure of focusing. The details for specific focusing measures are described at length in Harlan (1986, 1988) and Alvarez (1995).

In Harlan's method, a sample of data d is taken to be the sum of signal s and noise n; that is,

$$d = s + n. \quad (1)$$

Under the assumption that the signal and the noise are statistically independent, random variables, their probability density functions $p_s(x)$ and $p_n(x)$, satisfy (Papoulis, 1965)

$$p_d(x) = p_s(x) * p_n(x), \quad (2)$$

where the asterisk denotes convolution and $p_d(x)$ is the probability density function (pdf) for the data. Given this assumption, by knowing of any two of the probability density functions, we can compute the other. In particular, the signal probability density function can be computed from the data and noise probability density functions. This is achieved with a deconvolution that satisfies the constraints that the samples of $p_s(x)$ are nonnegative and the area under the curve of $p_s(x)$ for all x is unity. Harlan (1988) computes $p_s(x)$ by posing an optimization problem to find the signal probability density function that minimizes the difference between $p_d(x)$ and $p_n(x) * p_s(x)$ in the least-squares sense, subject to the above-mentioned constraints.

The data probability density function $p_d(x)$ can be estimated directly from the data samples, but the noise probability density function $p_n(x)$ cannot. One can obtain a pessimistic estimate by assuming all samples in the model domain [where $p_s(x)$ is to be estimated] consist entirely of noise. This can be done by destroying the spatial coherency of the data in the t–x domain—for instance, by randomly reversing the polarities of the traces (Harlan et al., 1984)—and then taking the parabolic τ–p transform of this altered data set. Assuming that all samples in the model domain are noise is more realistic than it may seem since we choose the transform to focus the signal to small regions of the model domain. Therefore, most of the model-based samples correspond to background noise (events unfocused by the transformation.) The question remains as to when, if ever, the assumption of statistical independence between signal and noise is warranted. This is debatable, but it is a basic assumption that seems to be satisfied by most data sets.

In applying Harlan's algorithm, we choose the key parameter for identifying the signal component of the data in the transformed domain as the *reliability indicator*, an indicator of the accuracy in the statistical identification of the signal. Specifically, Harlan defines it as the conditional probability that the estimated signal is within a certain percentage of its true (unknown) value:

$$\text{reliability} \equiv P[(-c\hat{s} < s - \hat{s} < c\hat{s})|d] = \frac{\int_{\hat{s}-c\hat{s}}^{\hat{s}+c\hat{s}} p_s(x)p_n(d-x)dx}{\int_{-\infty}^{\infty} p_s(x)p_n(d-x)dx}, \quad (3)$$

where $P[x|d]$ is the conditional probability of x given d, s is the signal amplitude (unknown), $\hat{s}$ is the estimated signal amplitude, d is the data amplitude (signal plus noise), and c governs the uncertainty that can be accepted in estimating the signal [typically $c < 5\%$; see Alvarez (1995)]. For a given value of c, we compute the reliability indicator for every sample in the transformed domain. We then associate high values of the reliability indicator with signal samples and low values with noise samples. The user supplies a reliability threshold such that samples with a computed reliability indicator less than the threshold value are interpreted as noise and attenuated. Thus, through our choice of the reliability threshold value, we trade off noise rejection and signal preservation.

This pattern recognition is based not on some shape in the 2D transformed domain but, rather, on the presumption that focusing the desired feature (signal) causes its amplitude to stand out relative to that of noise. Thus, Harlan's method suppresses background noise (unfocused events), leaving unchanged the amplitudes of coherent (focused) events in the transformed domain. The method differs from conventional noise-filtering algorithms in that the signal and the noise need not be mapped to different regions of the transformed domain; rather, the signal must be concentrated with high amplitude in a relatively small region, whereas the noise is spread—typically with weaker amplitude—over a large region. Here, noise means anything not focused by the transformation.

Harlan's method cannot be used directly to suppress multiples on a CMP gather because both the primary and the multiple reflections generally have approximately hyperbolic moveout; therefore, the parabolic Radon transform that focuses the primary reflections, based on their residual parabolic moveout after NMO correction, will also focus the multiples.

Our hybrid approach combines strengths of both Hampson's τ–q filtering method and Harlan's statistical S/N separation algorithm. For the parabolic Radon transform used here, q, which characterizes the curvature of the parabolic moveout, is measured as the moveout (in milliseconds) at the largest source–receiver offset (2970 m in our tests). The first step is to apply Hampson's method: the data in the CMP gathers are NMO corrected with the velocity of the primaries, such that the primaries become horizontal (or nearly so) across each gather, while the multiples remain under-corrected with a residual moveout close to parabolic (Hampson, 1986). A horizontal event can be considered as a parabola of zero curvature, so a discrete parabolic Radon transform applied to the NMO-corrected CMP gathers will concentrate the energy of the primaries in a small region of the τ–q plane near the line of zero moveout or curvature (i.e., $q = 0$). The multiples, with their nearly parabolic residual moveout, will also be focused, but in a different region of the τ–q domain. In accordance with Hampson's method, we apply a tapered mute to suppress the most identifiable multiple energy (the portion that does not overlap the primary region). Where multiples are considerably stronger than primaries, a portion of the multiple energy (we call it *residual multiple energy*) is mapped to or near the region occupied by the primaries. Because the residual multiple energy is not well focused (although the bulk of the initial multiple energy is), this residual multiple energy that has leaked into the primary region (see Figure 4, below) becomes the target for applying a version of Harlan's statistical S/N separation algorithm. Truncation artifacts are not focused by the transformation and therefore contribute to the noise we wish to suppress. Finally, we apply an inverse τ–q transform to bring the data back to the t–x domain.

The additional forward τ–q transform required to estimate the pdf of the noise and the 1D inversion needed to compute the pdf of the signal increases the total computation time by about 50% over that of Hampson's method. A computer implementation of the method in C language is available as module *suharlan* of the SU package developed at Colorado School of Mines (Cohen and Stockwell, 2002).

MODEL DATA

To compare the performance of the different methods, we create four data models simulating CMP gathers, each containing four primaries and four multiples. NMO correction with the primary velocities results in gathers with primaries perfectly aligned and multiples undercorrected. The four models differ in the relative amplitudes of primaries and multiples, and in the offset dependence of these amplitudes. In model 1 (Figure 1), the amplitudes of all primary events are the same and do not vary with offset. The P/M amplitude ratio for all traces in this model is small—1:4—and, as in all four models, two of the multiples are time coincident with primaries at zero offset and the other two are not. The time-coincident events simulate contamination of primaries by residual multiples, and the noncoincident ones are included to assess distortion of uncontaminated primaries that arises after attempts at multiple suppression. At the largest source–receiver offset (2970 m), the moveout difference between primaries and multiples is, from early to late events, 160, 120, 90, and 80 ms.

Model 2 is the same as model 1 except that the P/M ratio increases to 1:1. Comparing the relative performance of the various methods for these two models gives an idea of the robustness of these methods in preserving primaries while reducing multiples.

Model 3 differs from model 1 in that amplitude decreases linearly with offset for both the primaries and the multiples, such that the amplitudes on the far trace are only half those on the near trace. The purpose of this model is to test the ability of the various methods to suppress multiples where the data have been imperfectly corrected for offset-dependent variation in amplitude, such as in imperfect correction for geometrical spreading or absorption.

Model 4 introduces an even stronger decrease of amplitude with offset, such that the amplitudes on the far trace are −0.5 times those on the first trace; that is, the longer-offset traces are polarity reversed. Phase changes for primaries are likely to

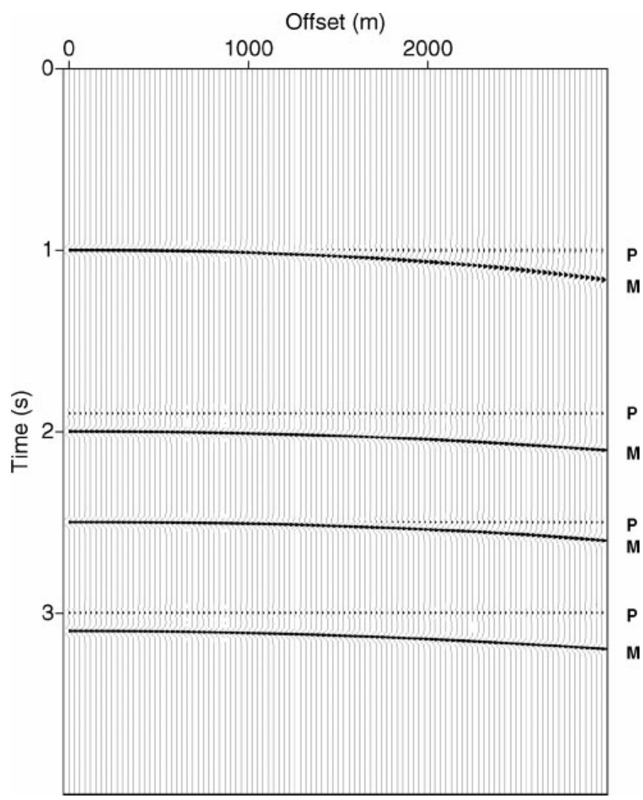

FIG. 1. Simulated NMO-corrected CMP gather for model 1. The peak amplitude of the multiples (M) is four times that of the primaries (P). Neither the primaries nor the multiples have any offset-dependent amplitude variation.

occur in practice when the elastic parameters exhibit a strong change across an interface, although the situation in model 4 is more extreme than might be encountered in practice in that the multiples vary similarly with offset. This model data set is intended to compare the performance of the methods under particularly adverse conditions. Table 1 summarizes the characteristics of the four model data sets.

SAMPLE RESULTS OF MULTIPLE SUPPRESSION

We applied each of the three methods—f–k, τ–q, and the proposed hybrid method—to the unstacked data for each of the four models. Here, we detail the results for model 1 and only summarize the results for the other data sets.

f–*k* filtering

Figure 2 shows the output from the f–k filtering method for model 1 (P/M ratio of 1:4, with the same primary amplitude on all traces). Primaries are largely unchanged, but substantial energy from the multiples remains. As expected, at short offsets the multiples are poorly suppressed because the difference in moveout between the multiples and the primaries approaches zero for those offsets. Although multiple suppression in the f–k domain can be applied in any of several different ways and with different choices of parameters that govern action in the pass and reject portions of the f–k space, the performance seen in Figure 2 is representative of results that can be expected with f–k filtering approaches.

Table 1. Characteristics of the four data sets used to compare the performance of the multiple suppression algorithms.

Model	P/M	AVO	Polarity change
1	1/4	No	No
2	1/1	No	No
3	1/4	Linear decrease	No
4	1/4	Linear decrease	Yes

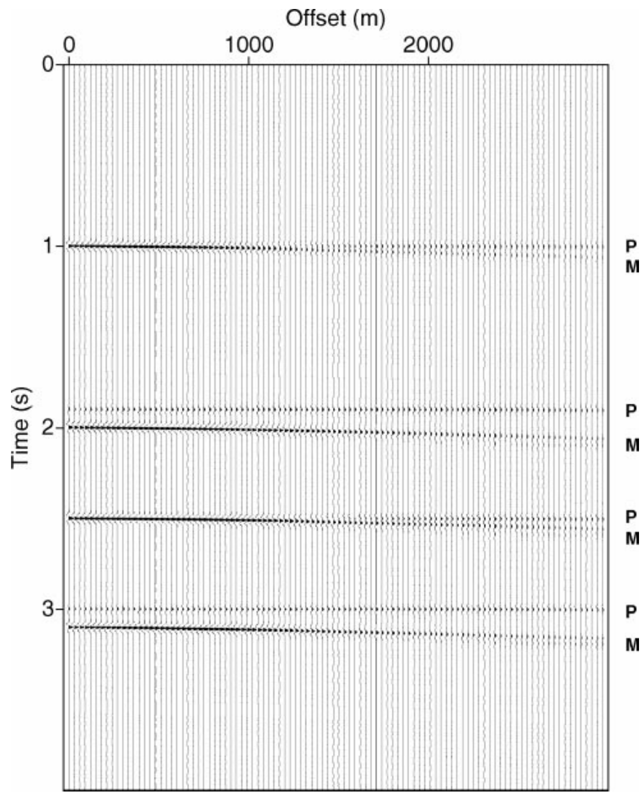

FIG. 2. The f–k multiple suppression of the data in model 1 leaves substantial residual multiple energy, particularly on the shorter offset traces. Where the multiple (M) overlaps a primary (P), one might incorrectly infer that the amplitude of the primary changes with offset.

Parabolic Radon transform method

Multiple suppression by Hampson's parabolic Radon transform approach (Figure 3) is significantly more effective than that by the f–k method.

The earliest multiple in Figure 3 is well suppressed because this multiple and the primary exhibit enough differential moveout to allow a clear separation between the two (see Figure 1). Because their moveouts are closer to those of nearby primaries, causing overlap of amplitudes in τ–q space, the multiples at later zero-offset times are less completely suppressed. The presence of the residual multiple at zero-offset time 2.5 s appears as an increase in apparent amplitude of the primary for the short-offset traces. Figure 4 shows the τ–q transformed data; note in particular the tails of multiples that overlap the primary region in τ–q space and the horizontal artifacts caused by truncation of the near-offset portion of the data. These tails,

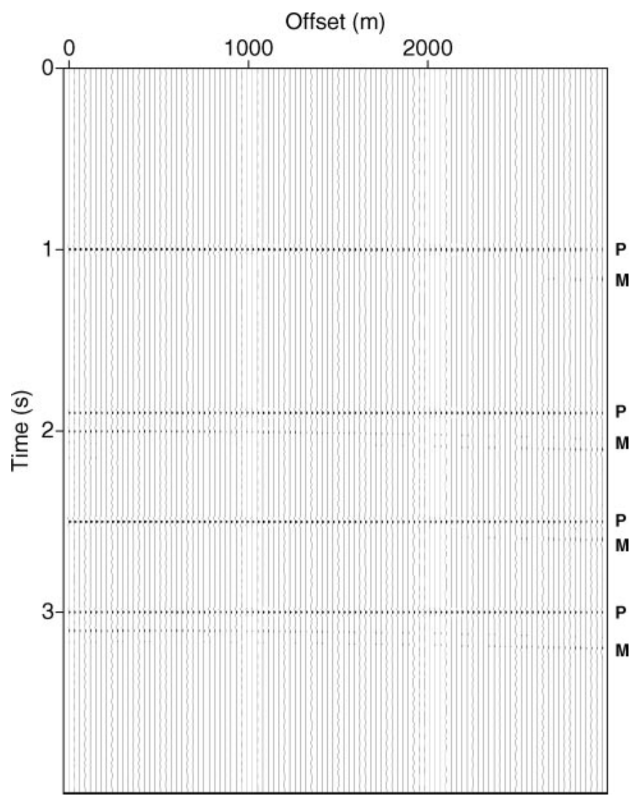

FIG. 3. Result of multiple suppression with Hampson's approach applied to the data of model 1. The residual multiple energy evident on the edges of the section may cause one to misconstrue the amplitudes of primaries as changing with offset, but not so severely as when the f–k method is used for multiple suppression.

which are attributable to the small moveouts on the short-offset traces, cannot be successfully muted in Hampson's method since they overlap the primary region.

Hybrid method

The hybrid approach (Figure 5) reduces the residual multiple energy that leaked into the primary region in the τ–q domain. As a result, the multiple suppression is excellent on all offsets and for all events. The effectiveness of the noise-separation step in attenuating the residual multiple energy can be assessed by referring to Figure 6, which shows the τ–q transform of the data in model 1 after the multiple suppression. The first step, the mute, removes most of the multiple energy (with moveouts larger than those governed by the slanted dashed line in Figure 4). Then, Harlan's noise-separation algorithm attenuates the residual multiple energy in the tails that leak into the primary region. In this case (model 1), the suppression process results in a slight decrease in signal amplitude. Since in this model the input multiples are four times stronger than the primaries, the residual multiple energy is strong enough to compete in amplitude with portions of the primary energy, especially the reflection for the deepest reflector (see Figure 4), for which differential moveout between primaries and multiples is smallest. The noise-separation step therefore has limited capability for distinguishing between residual multiple energy and primary energy on the basis of amplitude. Here, our choice of the reliability threshold favors reducing the residual multiple at the expense of some loss of signal amplitude.

As noted below, the small decrease in primary amplitude is likely acceptable for AVO analysis because the reduction is uniform across offset. If this small decrease of primary amplitude is considered excessive, the reliability threshold can be adjusted to allow more samples to be identified as signal.

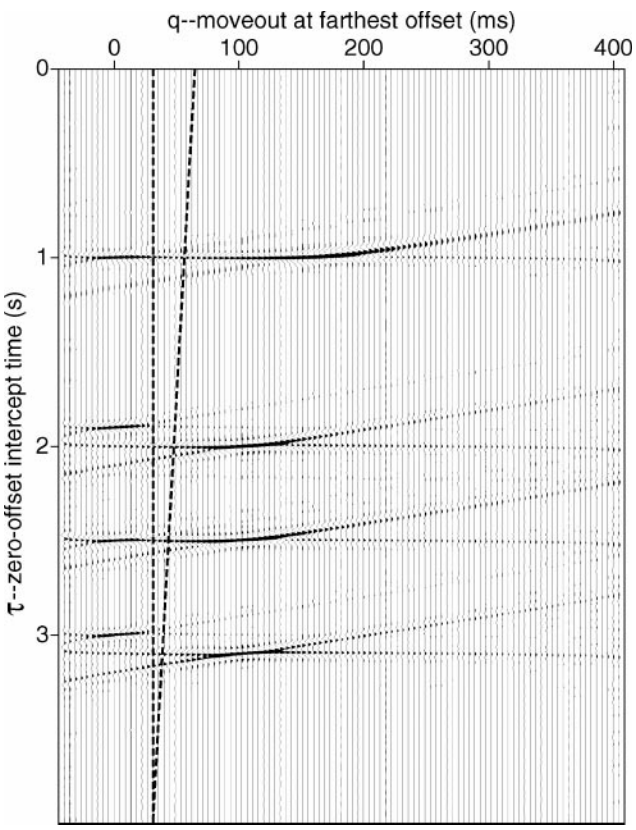

FIG. 4. The τ–q transform of NMO-corrected data in model 1. To the right of the slanted dash line is the multiple-energy region; to the left of the vertical dashed line is the primary region. The region between the dashed lines is the taper zone. Because of the large relative strength of the multiples, substantial residual energy from the multiples (the tails of the main energy) is mapped to the q-region of the primaries. Here, q is expressed as moveout on the trace with offset 2970 m.

IMPLICATIONS FOR AVO

To compare quantitatively the performance of the three algorithms from the standpoint of their value for AVO studies, we plot the amplitude of the primary reflections, after multiples have been suppressed, as a function of offset for each model and each multiple-suppression method. The amplitudes are measured as the peak of the wavelet at the two-way traveltime corresponding to each input primary, which, at times, may be contaminated by residual multiple. In Figure 7, a solid black line denotes the amplitude of a primary in the absence of multiples in the input data for model 1 (Figure 1). Any difference between this curve and one of the curves of apparent primary amplitude after multiple suppression indicates erroneous AVO. That variation could be a combination of the

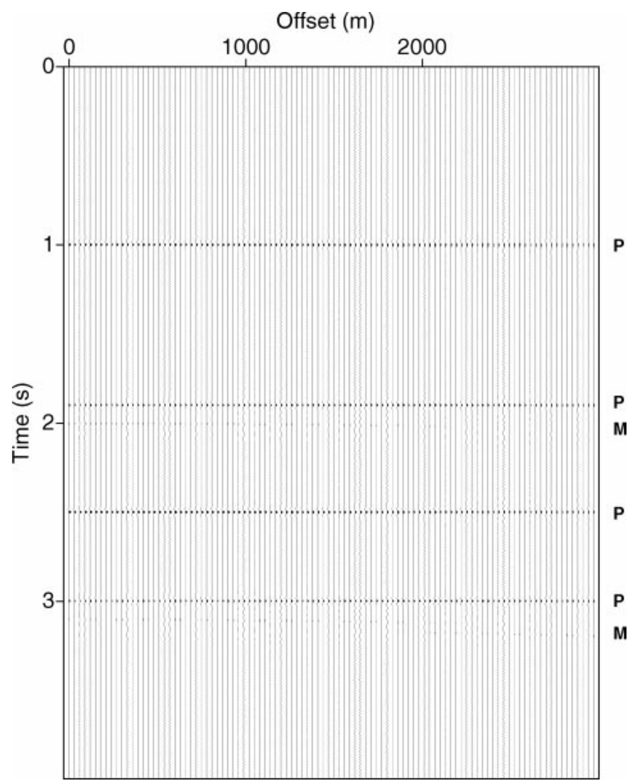

FIG. 5. Model 1 after the multiple energy has been attenuated by the hybrid method. The bulk of the multiple energy (i.e., to the right of the slanted dashed line in Figure 4) was suppressed by the mute, and the residual multiple energy that leaked into the primary region (to the left of the vertical dashed line in Figure 4) was attenuated by the S/N separation algorithm.

amplitude variation of the output primary and contamination of the primary by residual multiples. A thin dashed line in Figure 7 indicates the apparent amplitude of the primary in the input data contaminated by overlapping multiples. For primaries not coincident with multiples at zero-offset times, this curve coincides with the solid black line. The caption describes the other curves in the figure.

Where primaries are contaminated by multiples (Figures 7a and 7c), the contamination is so severe that AVO analysis using the input data would be meaningless. The f–k filtering (dashed black line) has failed to suppress the strong multiples sufficiently on the smaller offset traces for those primaries; therefore, those f–k filtered events are also useless for AVO study. For those primaries that are not coincident with multiples (Figures 7b and 7d), the f–k filtered primary amplitudes are closer to the ideal behavior but are still distorted at the short and long offsets because of edge effects of the f–k filter process. In any case, the f–k algorithm is unsatisfactory when the goal is to analyze AVO behavior.

Hampson's approach (solid gray line) performs considerably better for all primary events, but amplitudes depart from the true amplitude for long offsets and, more importantly, short offsets. Although not as severe as the departures when the f–k filter approach is used, the amplitude variations at short offset can nevertheless distort AVO analysis. For the third primary (Figure 7c), for example, the departure from the true amplitude at zero offset is almost 50%. More importantly, a strong, roughly linear AVO is present for offsets between about 800 to 1600 m. The results in Figures 7b and 7d indicate that the primary amplitudes at near offset were reduced in the process of suppressing the multiples, likely from rejection of tails of the primaries at high q-values in the τ–q transform. The increase in amplitude for short offsets for the other two reflections is not because of a change in actual primary amplitude but rather because of the contamination by the residual multiples. The variations are relatively small, so these results are likely acceptable for AVO analysis over a large range of shorter offsets.

Because the level of residual multiples present is much smaller than that for the other two methods, the hybrid approach performs the best of the three methods where primaries are contaminated by the multiples. For those primaries not coincident with the multiples, use of the algorithm resulted in a general reduction in amplitude, which, however, is more or less uniform for all offsets, much as happened with Hampson's method; that is, the curves from these two methods are roughly parallel. Since AVO analysis depends on the relative variation of AVO rather than on the absolute amplitude values, the performance of the hybrid method for AVO analysis is at least comparable to that of Hampson's. Neither of these algorithms did well for the far offsets, for which the departure from the true amplitudes varied rapidly to values as high as 50% in Figure 7c—probably the result of an edge effect of the parabolic τ–q transform.

Although not shown, similar results were obtained for the other three data sets (Alvarez, 1995), indicating that the f–k filtering approach, despite its conceptual simplicity and low computational cost, is clearly unacceptable for multiple suppression where AVO analysis is subsequently to be performed. Hampson's and the hybrid approach perform about the same, giving good results, except when the variation of AVO in the data is so severe that a polarity inversion occurs. The polarity inversion severely decreases the focusing power of the τ–q transform to a point that renders the subsequent application of the noise-separation algorithm useless, since it depends on focusing the signal (primaries) while leaving the noise (residual multiples) unfocused (see Figure 8). In that situation, perhaps some other method, such as that of Lumley et al. (1995), which claims to be tailored specifically to AVO preservation, might prove effective. Local transforms (Hermann et al., 2000; Hugonnet et al., 2001) may also prove helpful in that case, as may wave theory-based approaches if the change in amplitude and phase with offset on the input data is consistent with wave theory.

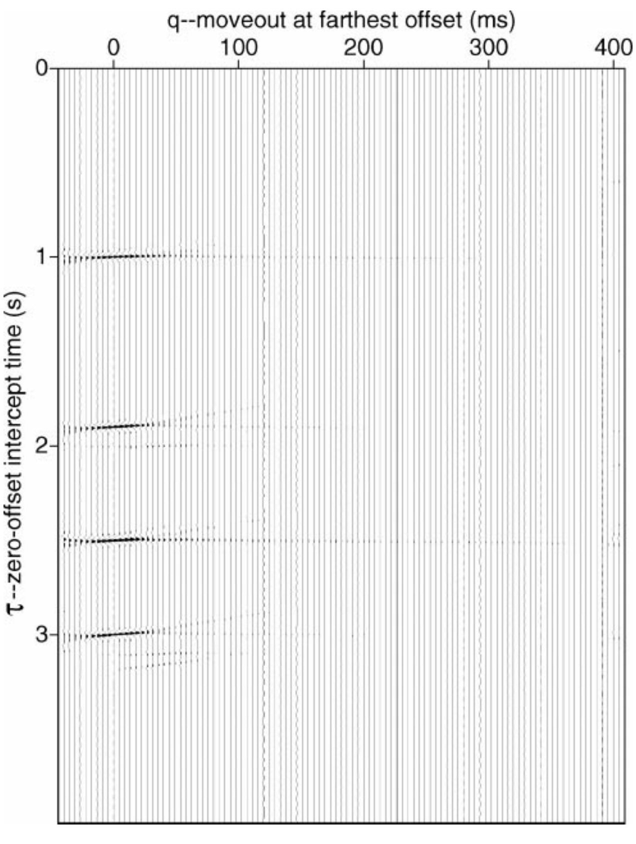

FIG. 6. The τ–q transform of data in model 1 after the multiple energy has been attenuated by the hybrid method. The bulk of the multiple energy (i.e., to the right of the slanted dashed line in Figure 5) was suppressed by the mute, and the residual multiple energy that had leaked into the primary region (to the left of the vertical dashed line in Figure 5) was attenuated by the S/N separation step.

QUALITY OF THE CMP STACK

We now compare the ability of the three multiple-suppression methods to improve the P/M ratio on stacked data, apart from any distortion in shape of the stacked primary events. In Figure 9, stacked traces of NMO-corrected data are plotted in the following order: trace 1, stack of the input data; trace 2, stacked primary-only input data (ideal); and traces 3–5, stacked multiple-suppressed output from the f–k filtering method, Hampson's τ–q method, and hybrid method, respectively.

The stacked traces in Figure 9 are results for model 1. From this figure and similar ones for the other models, we measure the P/M ratio for the stacked traces after applying each of the multiple-suppression methods. The P/M ratio is computed as the ratio of the peak-to-trough amplitude of the output primary to that of the residual multiple. For primaries coincident with multiples, the amplitude of the residual multiple is estimated as the difference of the amplitude of the primary (which has a contribution from the multiple) and the amplitude of an adjacent primary with no contribution from multiples.

The computed P/M ratios are collected in Table 2. The CMP stack itself yields an improvement factor of up to four in P/M ratio for all models with the exception of model 4 (polarity reversal), for which the improvement is small.

The f–k filtering approach yields only marginal improvement in P/M ratio over and above that which the CMP stack itself provides. (Recall that the f–k filter approach is best at suppressing multiples on the longer offset traces—the same ones for which CMP stacking is most effective.)

Hampson's τ–q method yields an improvement in P/M ratio over that of the CMP stack, which ranges from about 3:1 for model 4 to more than 5:1 for model 1. For all of the data sets tested, the improvement in P/M ratio was significant.

The hybrid approach further improves P/M ratio for the first and second data sets (those for which the trace-to-trace amplitudes were constant) because the focusing power of the transform is high for these models. The linear decrease of amplitude with offset in model 3 diminishes the focusing power of the

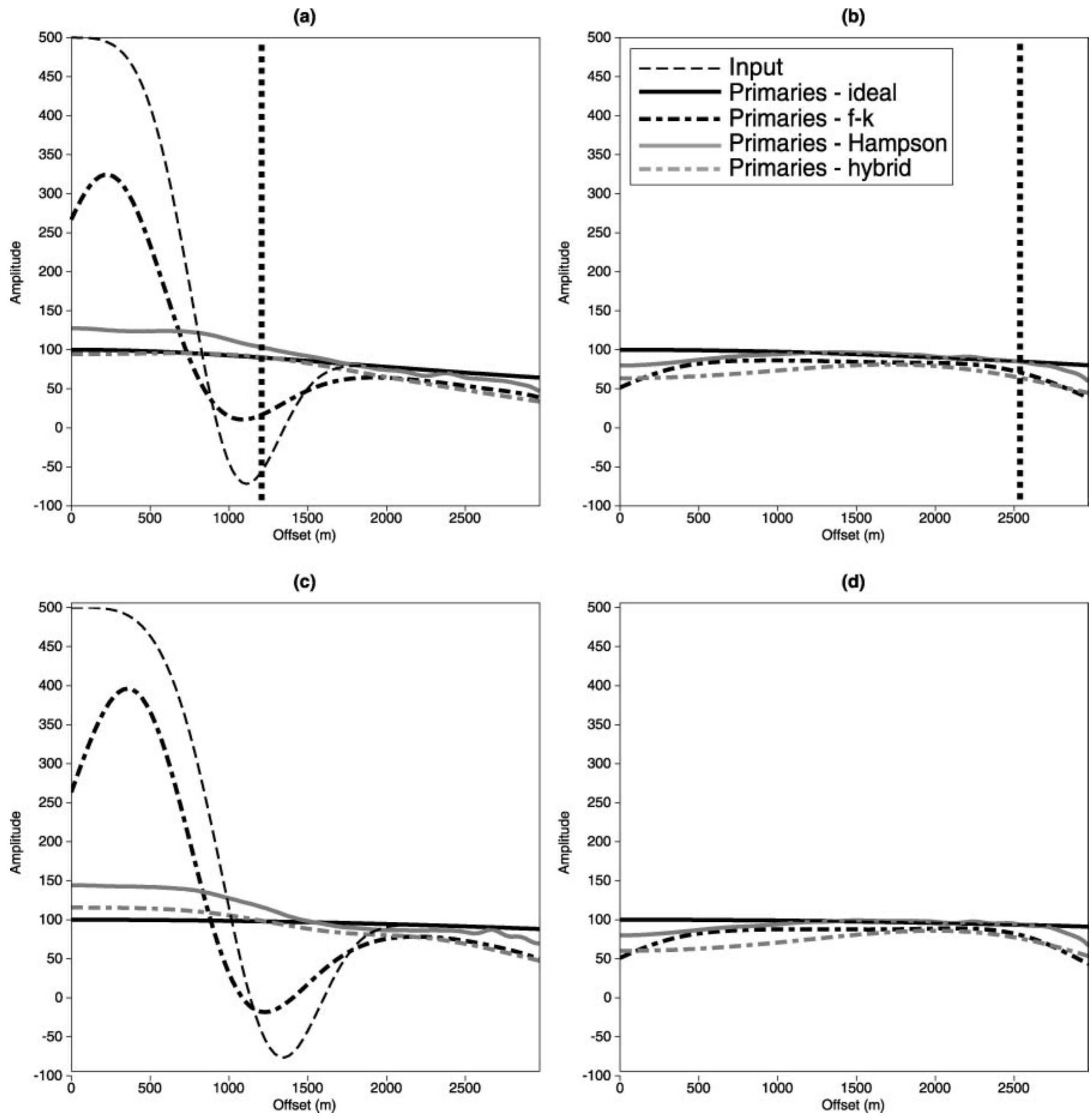

FIG. 7. AVO for primaries in model 1. (a)–(d) results for the primaries from the shallowest to the deepest events. Amplitudes are measured as the peak of the wavelet at the two-way traveltime of each input primary. The solid black line is the amplitude of the input primary in the absence of multiples; the thin dashed line is the apparent amplitude of the input primary in the presence of multiples; and the dashed black, solid gray, and dashed gray lines are the primary amplitude after multiple suppression with the f–k, Hampson, and hybrid approaches, respectively. The dashed vertical line shows where offset equals depth of the reflector. Offsets larger than this value are normally ignored in AVO analysis.

transform. Consequently, the result obtained with the hybrid approach is only marginally better than that of Hampson's. For model 4, the polarity reversal reduces the focus of the primaries even further. The increased smearing of both the primary and multiple energy in the τ–q domain reaches a point where the algorithm cannot discriminate between the primaries and the residual multiples; as a result, the performance of the hybrid method is actually poorer than that of Hampson's method. As mentioned previously, it remains to be seen whether methods such as those of Lumley (1995) and Herrmann et al. (2000), or a wave-theory-based method, might be more successful in this situation.

DISCUSSION

Qualitative measures of the relative performance of the different multiple-suppression methods, in terms of AVO behavior and CMP stack quality, are summarized in Table 3. In the table, we offer qualitative grades of very good, good, fair, and poor based on analyses in the previous two sections. Recall that the first and third primaries in each data set are each contaminated by a multiple, whereas the second and fourth are not.

The results indicate that for model 1 (1:4 P/M ratio) the hybrid algorithm is preferred unless computation cost is the overriding consideration.

From the results of the previous two sections, the coincidence of primaries and multiples at zero-offset time, or lack of it, might seem to govern the choice of multiple-suppression approach to use prior to AVO analysis. In field data, however, from one primary reflection to another, the zero-offset time and polarity of nearby multiples will differ from those of primaries in a generally random fashion. This circumstance tends to favor use of the hybrid approach.

Despite its 50% greater cost, the hybrid approach is favored over Hampson's method wherever the goal is maximum degree of multiple rejection in the stacked data, particularly where the primaries are severely contaminated by multiples and the primary wavelet does not reverse polarity across the range of offsets. In contrast, when the overriding concern is AVO and some residual multiple energy can be tolerated in the stack, then the less costly Hampson approach is a competitive choice.

Table 2. P/M ratio in CMP-stacked traces for the ideal primaries-only data, input multiple-contaminated data, and results of the three multiple-suppression methods. The numbers show the ratio of the peak-to-trough amplitude for primaries to that for multiples.

Model	Input	Stack	f–k	Hampson	Hybrid
1	0.25	1.0	1.0	5.5	10.2
2	1.0	4.0	4.3	18	40
3	0.25	0.8	0.85	3.9	4.4
4	0.25	0.31	0.33	0.9	0.87

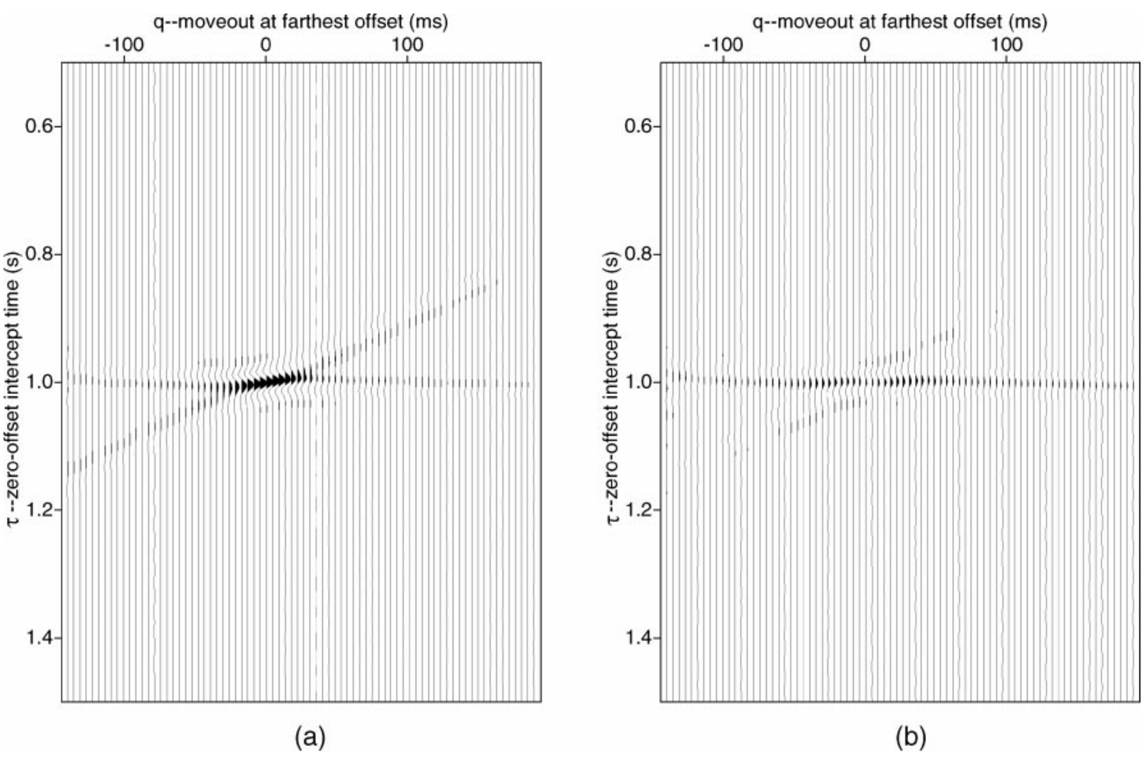

FIG. 8. Detailed comparison of the τ–q transform for the shallowest primary in (a) model 1 (without polarity reversal) and (b) model 4 (with polarity reversal). The polarity reversal severely decreases the focusing power of the transform, rendering subsequent application of the noise-separation algorithm ineffective.

Both the Hampson and hybrid methods yield some variation of apparent primary amplitude, but the processed data are far superior to both the unprocessed data and the f–k filtered output for AVO analysis.

Admittedly, the model data used for this study are simplistic and idealized. Strong NMO stretch for shallow, low-velocity reflections; trace-dependent variations in amplitude of both primaries and multiples, such as those associated with variable source and receiver coupling into the earth; and departures of primary moveout from being hyperbolic are complications that should be investigated. These complications compromise, to various extents, the focusing ability of the parabolic Radon transform, possibly limiting the additional value of the hybrid method. However the deterioration in focusing seen for model 4, with its polarity change across the CMP gather (and for model 3 as well), is likely much greater than the degradation that can be expected from variations in coupling of sources and receivers.

Table 3. Performance comparison for the three methods of multiple suppression. Models 1 and 2 have no variation of primary amplitude with offset, model 3 has linear decrease of amplitudes with offset with no polarity reversal, and model 4 has linear AVO and polarity reversal.

Models	Method	AVO Preservation	Quality of CMP Stack
	f–k filtering	Poor	Poor
1, 2	Hampson	Good	Good
	Hybrid	Good	Very Good
	f–k filtering	Poor	Poor
3	Hampson	Fair	Good
	Hybrid	Fair	Good
	f–k filtering	Poor	Poor
4	Hampson	Poor	Fair
	Hybrid	Poor	Fair

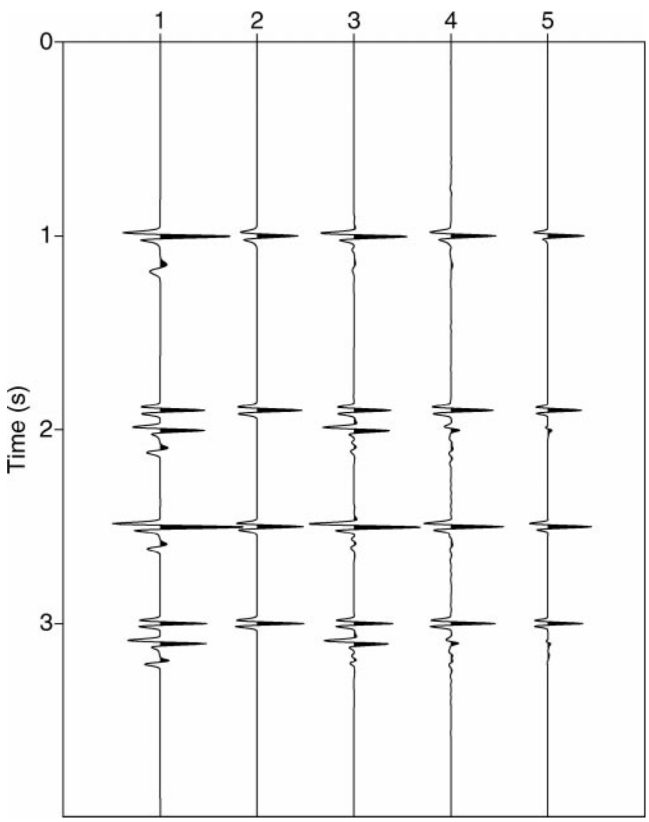

FIG. 9. CMP stacked traces for model 1. The first trace is the stack of the NMO-corrected input data; the second is the stack of the NMO-corrected, primaries-only input data; and the last three traces are stacks of the multiple-suppressed results of f–k filtering, Hampson's method, and hybrid method, respectively.

Practitioners in seismic data processing are aware of the limitations of moveout-based approaches in suppressing multiples where the subsurface is complex and moveout is thus not hyperbolic and where differential moveout between primaries and multiples is relatively small. Likely, the smaller the different moveout, the more promising the hybrid approach, which exploits characteristics of data in the Radon-transformed domain that are not dependent solely on differential moveout. Where differential moveout between primaries and multiples is relatively small, the high-resolution approaches of Herrmann et al. (2000), Cary (1998), and Sacchi and Ulrych (1995) and our hybrid method offer alternative approaches to addressing leakage in the τ–q domain. The high-resolution methods, however, are considerably more costly. Moreover, their ability to reduce the leakage problem is compromised where residual moveout is nonparabolic and amplitude varies with offset. This suggests potential added value in building a hybrid approach that combines a high-resolution technique with the Harlan pattern-recognition method.

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