Target-oriented wave-equation least-squares migration/inversion with phase-encoded Hessian

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Running head: **Inversion with phase-encoded Hessian**

ABSTRACT

Prestack depth migration produces blurred images due to limited acquisition apertures, complexities in the velocity model and bandlimited characteristics of seismic waves. This distortion can be partially corrected using the model-space least-squares imaging/inversion approach, where a target-oriented wave-equation Hessian operator is first explicitly computed, and then inverse filtering is iteratively applied to deblur or invert for the reflectivity. However, one difficulty is the cost for computing the explicit Hessian operator, which requires storing a large number of Green's functions, making it challenging for large-scale applications. As a remedy to this difficulty, I present a new method for computing the Hessian operator for the wave-equation-based least-squares migration/inversion problem. The proposed method modifies the original explicit Hessian formula, enabling efficient computation of this operator. A particular advantage of this method is that it eliminates on-disk storage of Green's functions. The modifications, however, also introduce undesired crosstalk artifacts. I examine two different phase-encoding schemes, namely, plane-wave-phase encoding and random-phase encoding, to suppress the crosstalk. I apply the randomly phase-encoded Hessian operator to the Sigsbee2A synthetic data set, where an improved subsalt image with more balanced amplitudes is obtained.

INTRODUCTION

Migration is an important tool for imaging subsurface structures using reflection seismic data. The classic imaging principle for shot-based migration states that reflectors are located where the forward-propagated source wavefield correlates with the backward-propagated receiver wavefield (Claerbout, 1971). However, this imaging principle is only the adjoint of the forward Born-modeling operator (Lailly, 1983), which provides reliable structural information of the subsurface, but blurs the image because of the non-unitary nature of the Born modeling operator. To deblur the migrated image and correct the effects of limited acquisition geometry, complex overburden and bandlimited wavefields, the imaging problem can be posed as an inverse problem based on the minimization of a least-squares functional. The inverse problem can be formulated either in the data space (Lailly, 1983; Tarantola, 1984; Nemeth et al., 1999; Kuhl and Sacchi, 2003; Clapp, 2005) or in the model space (Beylkin, 1985; Chavent and Plessix, 1999; Rickett, 2003; Sjoeberg et al., 2003; Guitton, 2004; Plessix and Mulder, 2004; Valenciano et al., 2006; Yu et al., 2006; Symes, 2008). The data-space approach can be solved iteratively using the gradient-based method (Nemeth et al., 1999; Kuhl and Sacchi, 2003; Clapp, 2005) without explicit construction of the Hessian, the matrix of the second derivatives of the error functional with respect to the model parameters. The iterative solving, however, is relatively costly and converges slowly without proper preconditioning.

On the other hand, the model-space approach requires explicitly constructing the Hessian and applying its pseudo-inverse to the migrated image. The full Hessian of the least-squares functional is too big and expensive to be computed in practical applications; hence some authors (Chavent and Plessix, 1999; Rickett, 2003; Plessix and Mulder, 2004; Symes,

2008) approximate it by a diagonal matrix. In the case of high-frequency asymptotics, and with an infinite aperture, the Hessian is diagonal in most cases (Beylkin, 1985). For a finite range of frequencies and limited acquisition geometry, however, the Hessian is no longer diagonal and not even diagonally dominant (Pratt et al., 1998; Chavent and Plessix, 1999; Plessix and Mulder, 2004; Valenciano et al., 2006). It has been shown by Albertin et al. (2004) and Valenciano (2008) that, in areas of poor illumination, e.g., subsalt regions, the Hessian's main diagonal energy is smeared along its off-diagonals. Therefore, a diagonal matrix has limited effect in deblurring the migrated image, especially in poorly illuminated areas. That is why several authors, e.g., Albertin et al. (2004) and Valenciano (2008), suggest computing a limited number of the Hessian off-diagonals to compensate for poor illumination and improve the inversion/deblurring result.

Since the exact Hessian off-diagonals are expensive to compute, some attempts have been made to reduce the cost by computing the non-diagonal Hessian in an approximate sense. Yu et al. (2006) introduce a lateral invariant non-diagonal Hessian by assuming a 1-D layered medium; Guitton (2004) uses a bank of non-stationary filters to approximate a non-diagonal inverse of the Hessian; with local plane-wave assumptions, Lecomte and Gelius (1998), Gelius et al. (2002) and Lecomte (2008) compute the Hessian in the local phase domain using a ray-based approach. They demonstrate that, since ray tracing conveniently gives local propagation angles of both source and receiver rays, the local scattering wavenumber for image points in the subsurface can be efficiently constructed. However, the high-frequency asymptotic approximation and the caustics inherent in ray theory may prevent the ray-based approach accurately handle complex geologies (Hoffmann, 2001). To better honor the bandlimited nature of the seismic waves, Xie et al. (2006) use an one-way wave-equation-based approach to compute the phase-domain Hessian through local plane-wave decomposition. The above theories of computing the local phase-domain Hessian operator assume the velocity model is locally homogeneous. Although this is a reasonable assumption for most cases, when there are sharp velocity contrasts as in the vicinities of salt boundaries, this assumption becomes less reliable.

Another way of computing the wave-equation non-stationary Hessian operator is through crosscorrelation of the source and receiver Green's functions in the space domain (Plessix and Mulder, 2004; Valenciano et al., 2006). This approach does not introduce any assumptions to the velocity model. However, computing even a limited number of the Hessian off-diagonals in the space domain, by directly implementing the explicit Hessian formula, is very cumbersome. A huge number of Green's functions (easily several hundred terabytes for a typical 3-D survey with a reasonable frequency band) must be pre-computed and stored and then read from the disk to generate the Hessian. Such operations not only require high-volume storage, but also high-speed I/O and network communication. Though computer speed continues to improve rapidly, computing the Hessian in such a way still presents a challenge.

To make the space-domain Hessian more affordable, I describe a method based on the phase-encoding technique. In this method, the original explicit Hessian formula is slightly modified to enable efficient computation of this operator. The proposed method makes the Hessian computation similar to the shot-profile migration, but with slightly modified imaging and boundary conditions for the wavefields. The new method eliminates the need to store Green's functions, but it also introduces crosstalk artifacts. I examine two phase-encoding schemes, plane-wave-phase encoding (Whitmore, 1995; Zhang et al., 2005; Liu et al., 2006) and random-phase encoding (Romero et al., 2000), to attenuate the crosstalk.

This paper is organized as follows. First, I briefly review the theory of formulating the inverse problem in the model space. Next, I discuss how the explicit Hessian can be efficiently computed using phase encoding. Finally, I apply the phase-encoded Hessian to deblur the migrated image for the Sigsbee2A model.

I examine two different phase-encoding schemes to attenuate the crosstalk, namely, plane-wave-phase encoding and random-phase encoding.

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