Removal of linear events with combined radon transforms

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SEP120 pages 395-405
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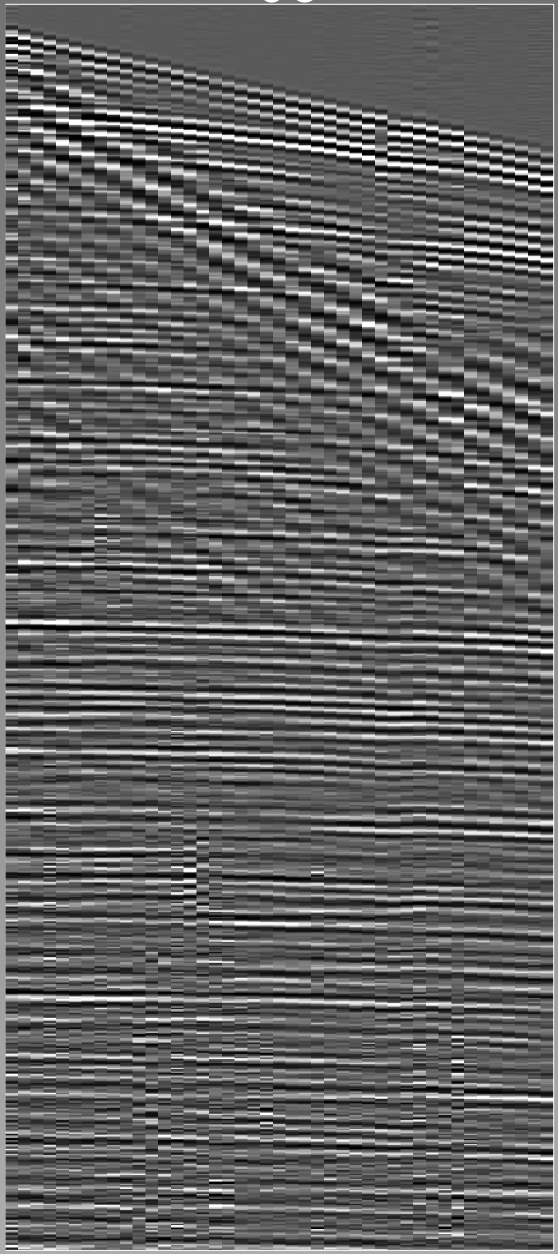
Goals

- Bound-constrained inversion application
- Dual operator inversion
- Inversion comparison
- Analysis vs. synthesis

Land data kinematics

08



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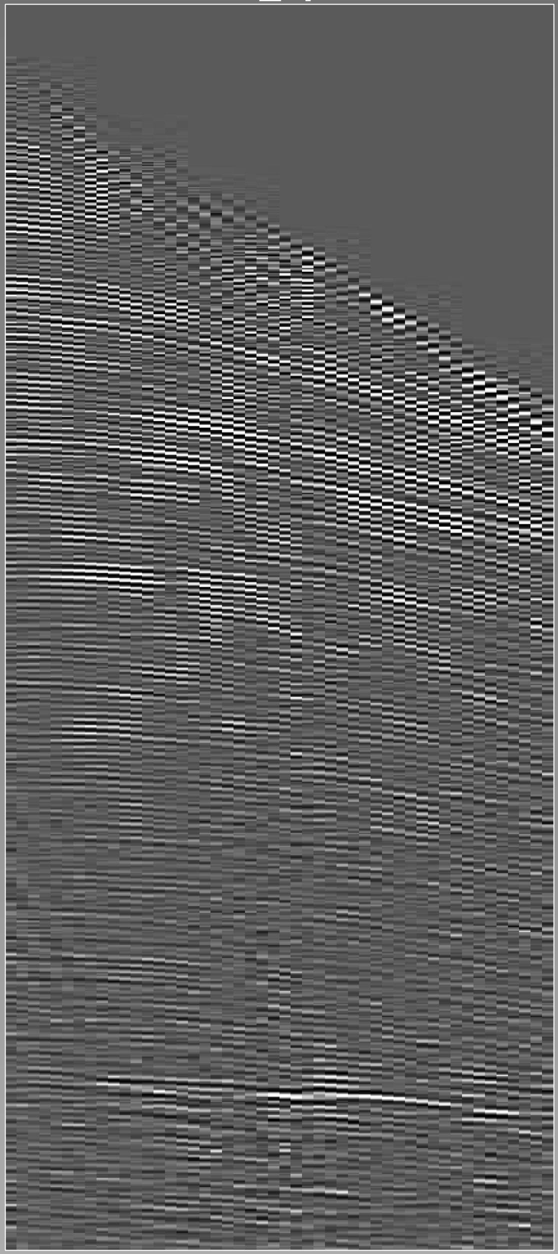

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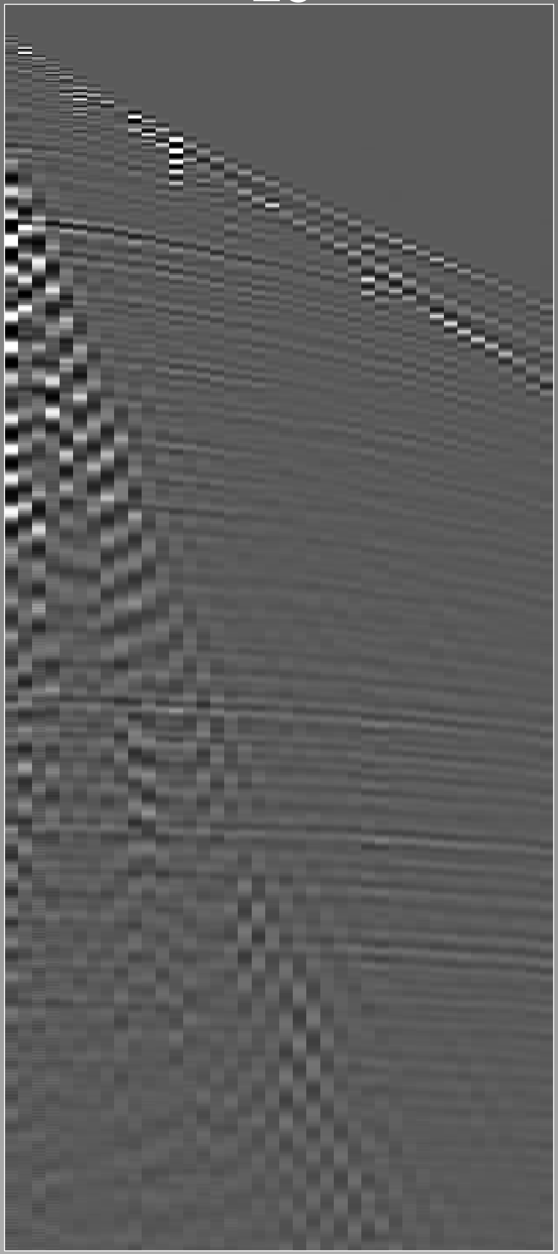

Outline

- Two Radon operators as one
- Crosstalk in a synthetic example
- Model space
- Data space residuals
- Data space signal
- Conclusion

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Dual operator inversion

$$\mathbf{Lm} = \mathbf{d}$$

where

$$\mathbf{L} = [\mathbf{L}_1 \ \mathbf{L}_2] \ ,$$
$$\mathbf{m} = \begin{bmatrix} \mathbf{m}_1 \\ \hline \mathbf{m}_2 \end{bmatrix} \ .$$

S. Chen, D. Donoho, & M. Saunders, 1999

Atomic Decomposition by Basis Pursuit,

SIAM Journal on Scientific Computing

Hyperbolic & Linear Radon Transforms

$$\begin{aligned} \mathbf{L}_h \mathbf{m}_h + \mathbf{L}_l \mathbf{m}_l &= \mathbf{d} \\ \epsilon^2 \mathbf{I} \mathbf{m}_h + \epsilon^2 \mathbf{I} \mathbf{m}_l &= 0 \end{aligned}$$

Hyperbolic & Linear Radon Transforms

$$
\begin{aligned}
\mathbf{L}_h \mathbf{m}_h + \mathbf{L}_l \mathbf{m}_l &= \mathbf{d} \\
\epsilon^2 \mathbf{I} \mathbf{m}_h + \epsilon^2 \mathbf{I} \mathbf{m}_l &= 0
\end{aligned}
$$

Quatro Inversiones

- Bound constrained
- l^1 norm
- Cauchy norm
- l^2 Least-squares

Quatro Inversiones

- Bound constrained

$$\text{min } ||f(\mathbf{m})||^2 \text{ subject to } \mathbf{m} \in \{\Re^N < 0\} \ +$$
$$\text{min } ||f(\mathbf{m})||^2 \text{ subject to } \mathbf{m} \in \{\Re^N > 0\}$$

A. Guitton, 2004, SEP117 pages 51-63, *Bound constrained optimization: Application to the dip estimation problem*

Quatro Inversiones

- Bound constrained
- l^1 norm

$$\min |f(\mathbf{m})|^1$$

Quatro Inversiones

- Bound constrained
- l^1 norm
- Cauchy norm

$$\text{min } ||f(\mathbf{m})||^2 + \epsilon^2 \sum_i^n \ln(b + m_i^2)$$

Quatro Inversiones

- Bound constrained
- l^1 norm
- Cauchy norm
- l^2 Least-squares

$$\min \, ||f(\mathbf{m})||^2$$

Signal extraction

$$\mathbf{s}_h = \mathbf{d} - [\mathbf{L}_h \mathbf{L}_l] \left[\frac{0}{\hat{\mathbf{m}}_l} \right]$$

or

$$\mathbf{s}_h = \mathbf{d} - \mathbf{L}_l \hat{\mathbf{m}}_l$$

Signal extraction

$$\mathbf{s}_h = \mathbf{d} - [\mathbf{L}_h \mathbf{L}_l] \left[\frac{0}{\hat{\mathbf{m}}_l} \right]$$

or

$$\mathbf{s}_h = \mathbf{d} - \mathbf{L}_l \hat{\mathbf{m}}_l$$

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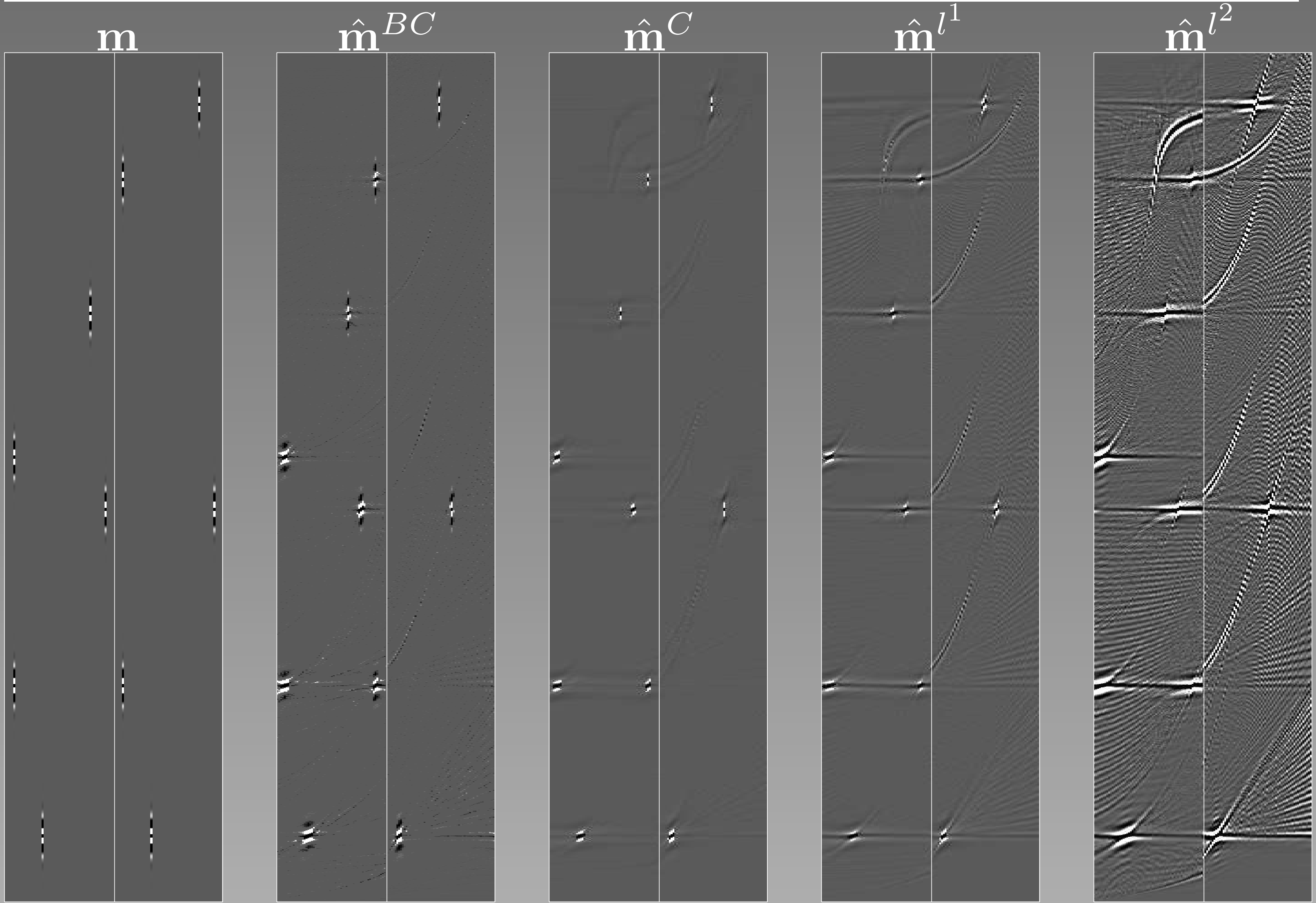
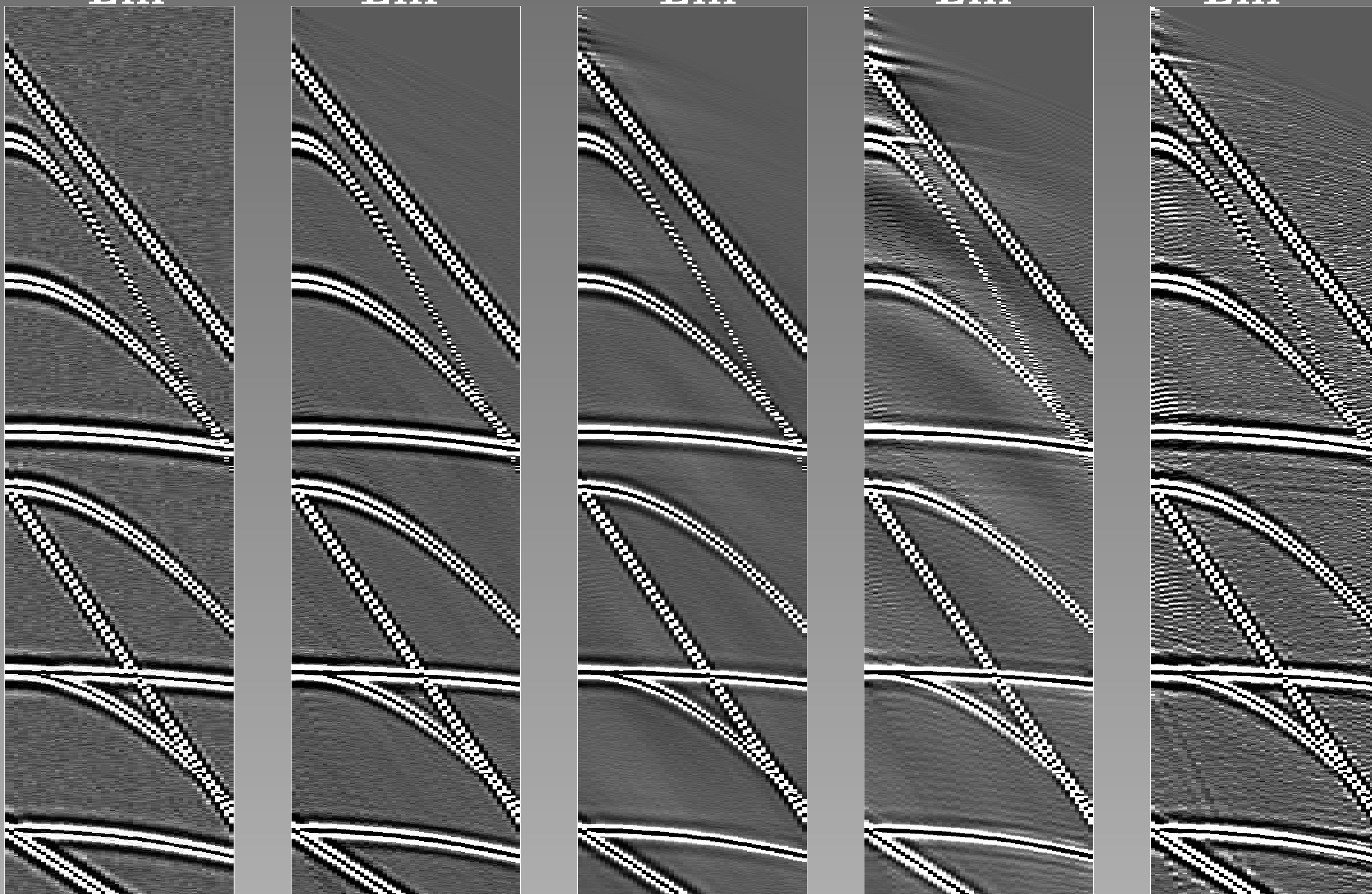
Inverted model space



Data space

$\mathbf{Lm}$ $\mathbf{L\hat{m}}^{BC}$ $\mathbf{L\hat{m}}^{C}$ $\mathbf{L\hat{m}}^{l^1}$ $\mathbf{L\hat{m}}^{l^2}$



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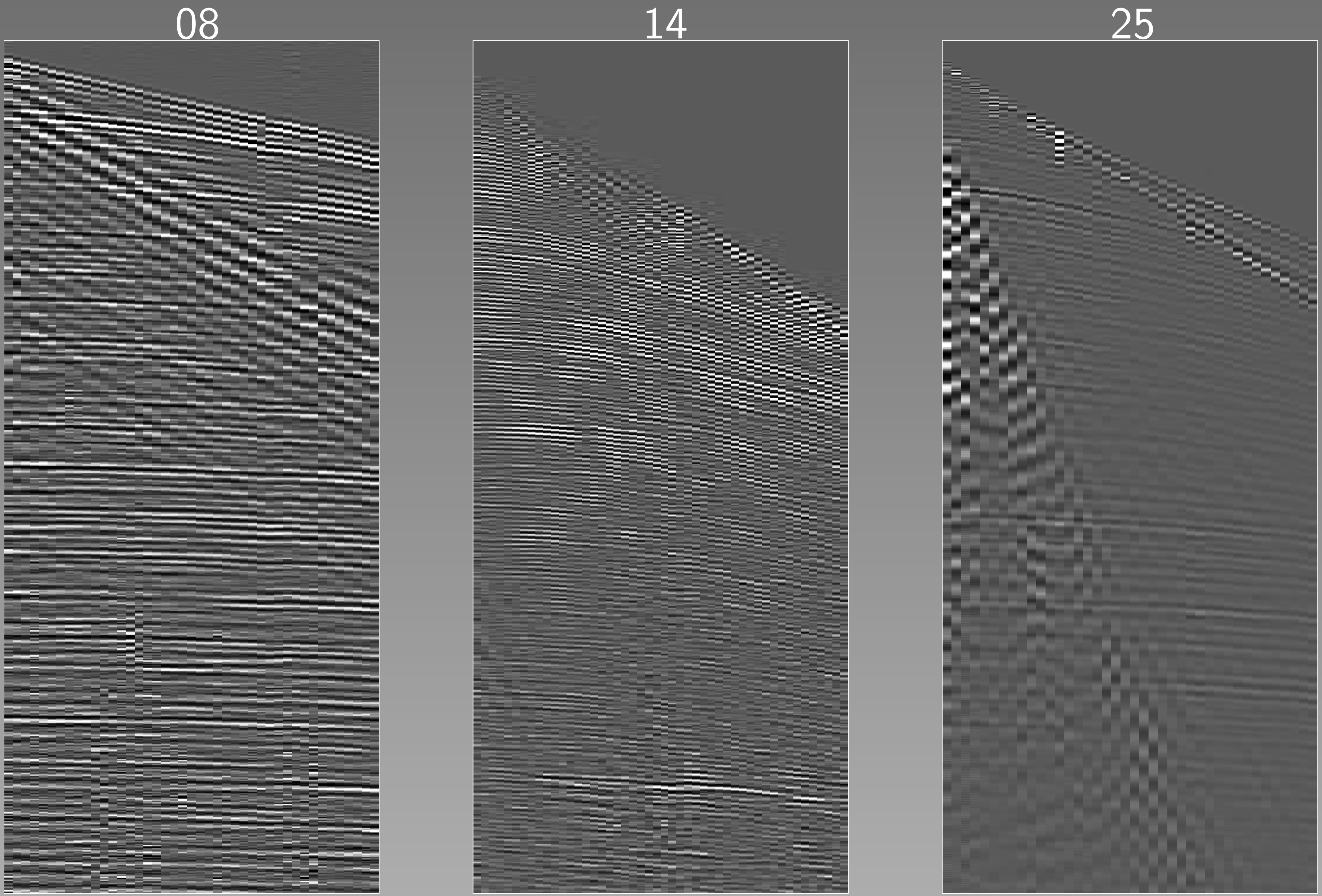
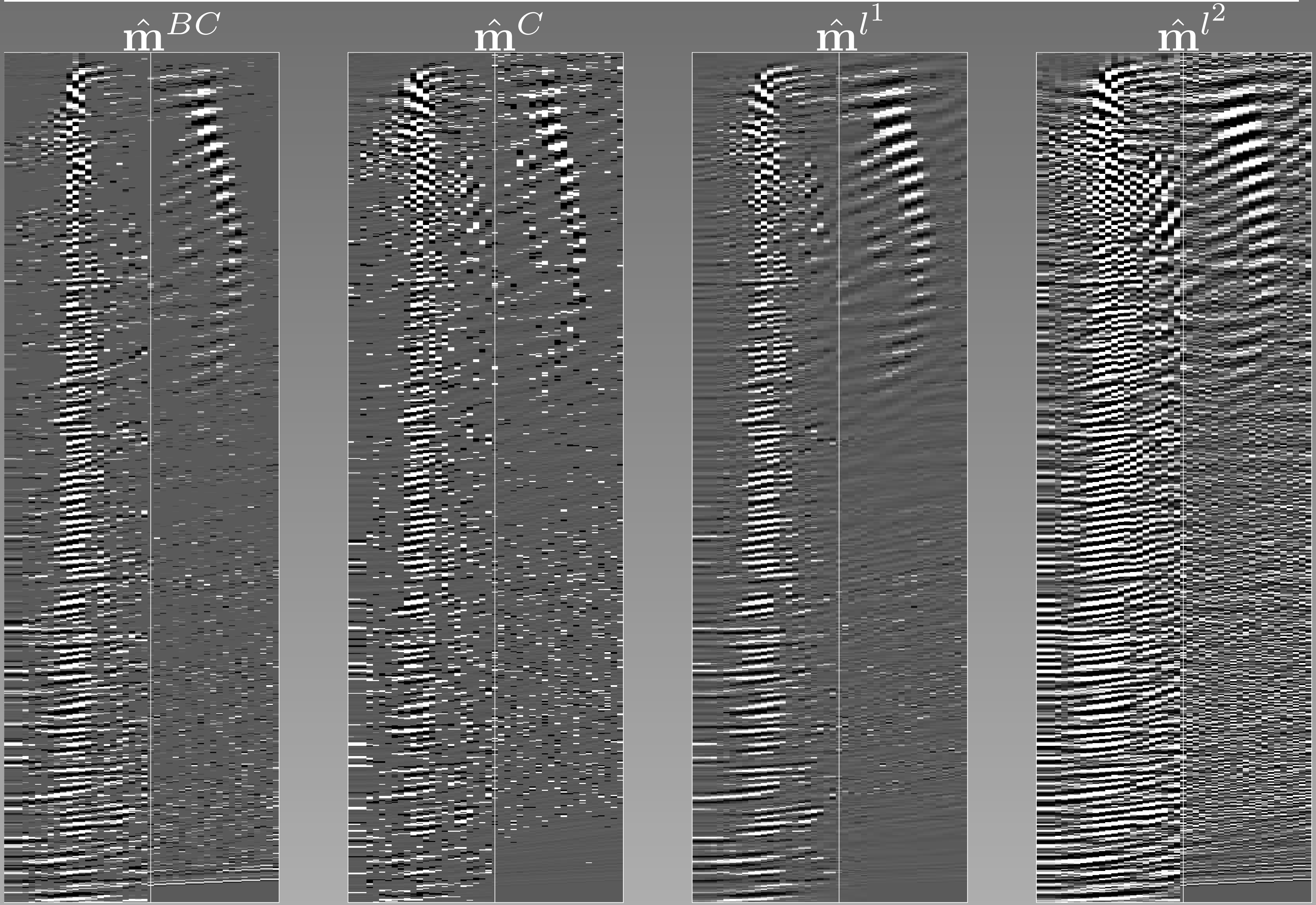
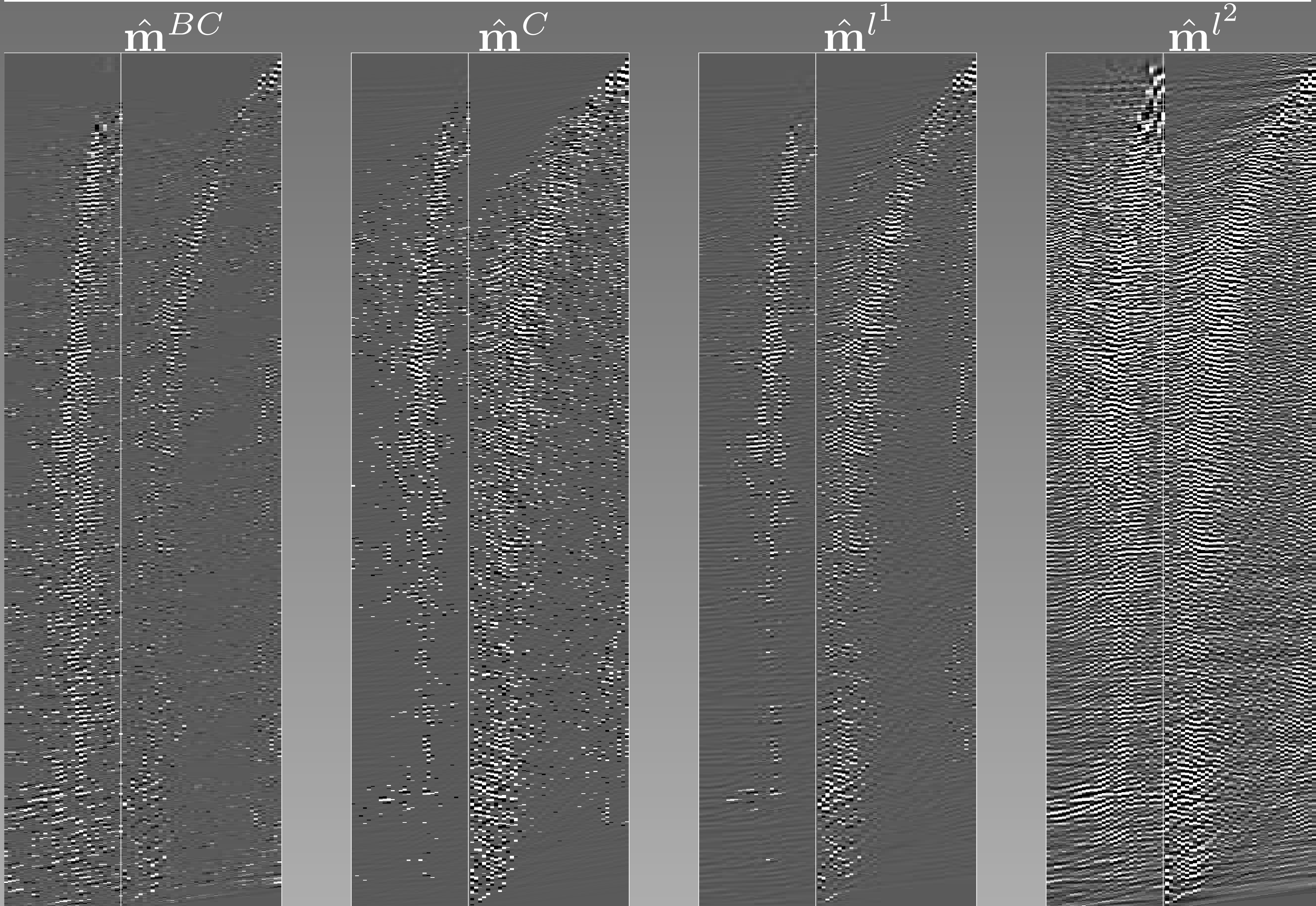
Land data



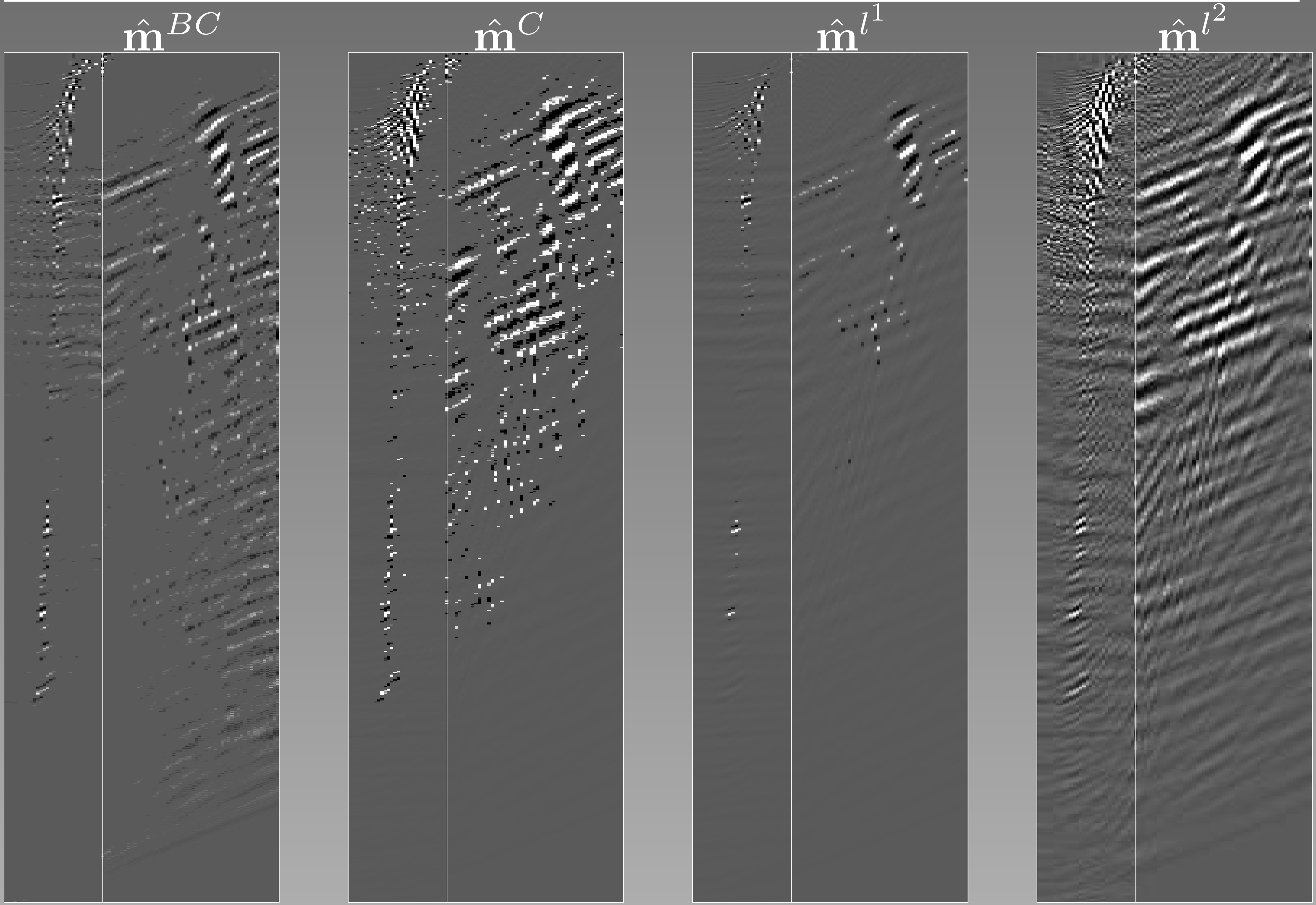
Model space 08



Model space 14



Model space 25



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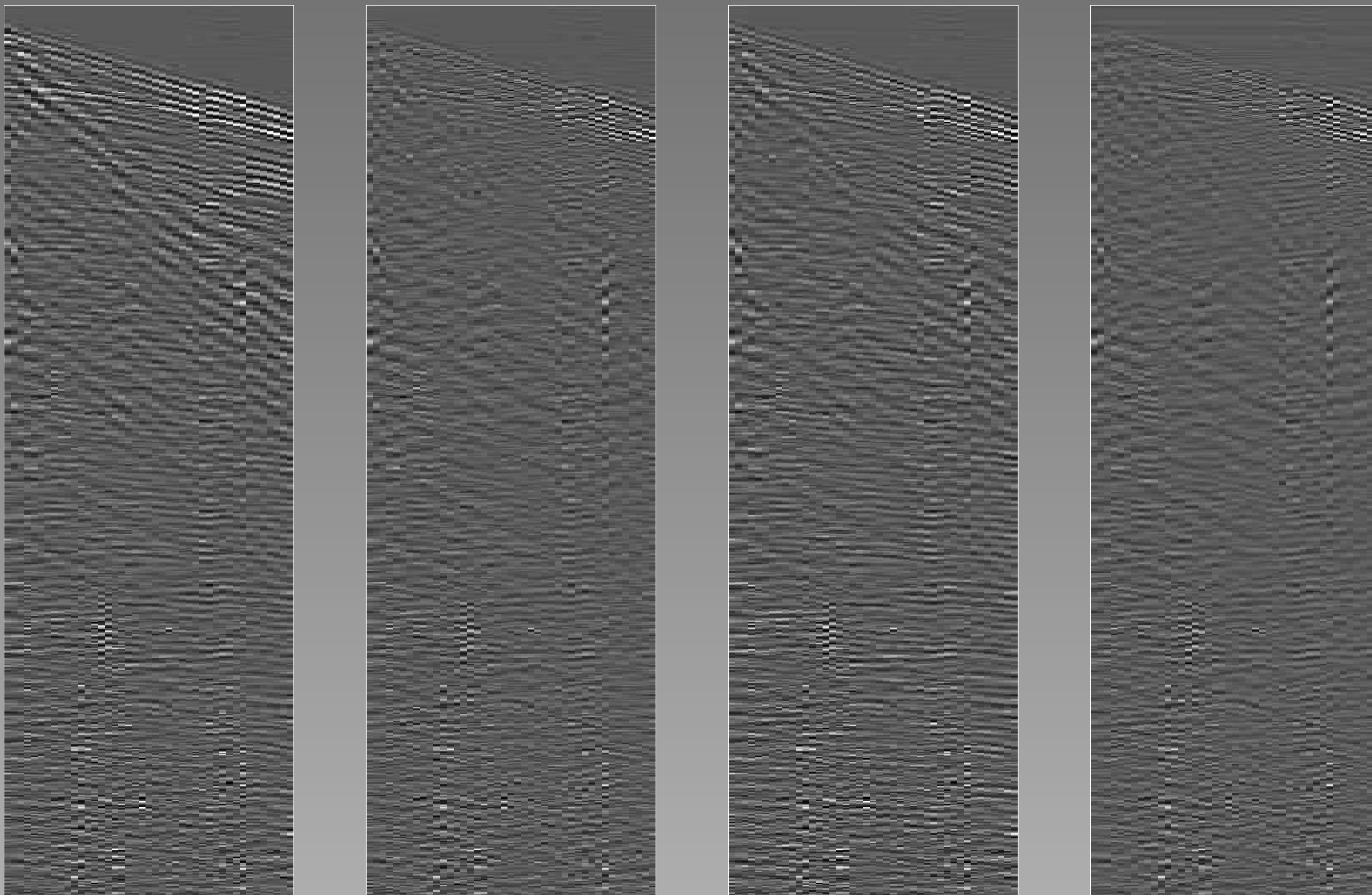
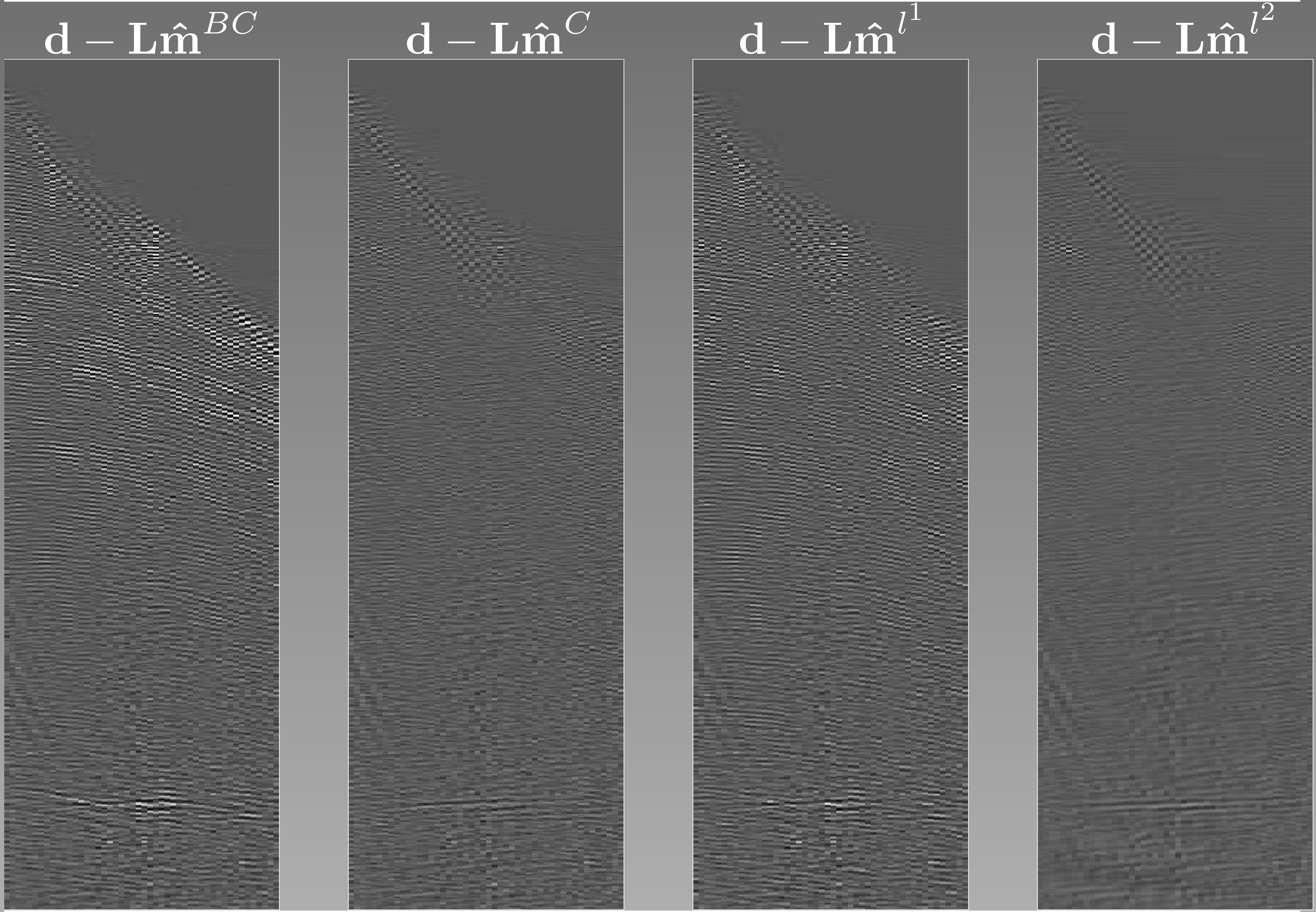
Data space residual 08

$\mathbf{d} - \mathbf{L}\hat{\mathbf{m}}^{BC}$ $\mathbf{d} - \mathbf{L}\hat{\mathbf{m}}^{C}$ $\mathbf{d} - \mathbf{L}\hat{\mathbf{m}}^{l^1}$ $\mathbf{d} - \mathbf{L}\hat{\mathbf{m}}^{l^2}$

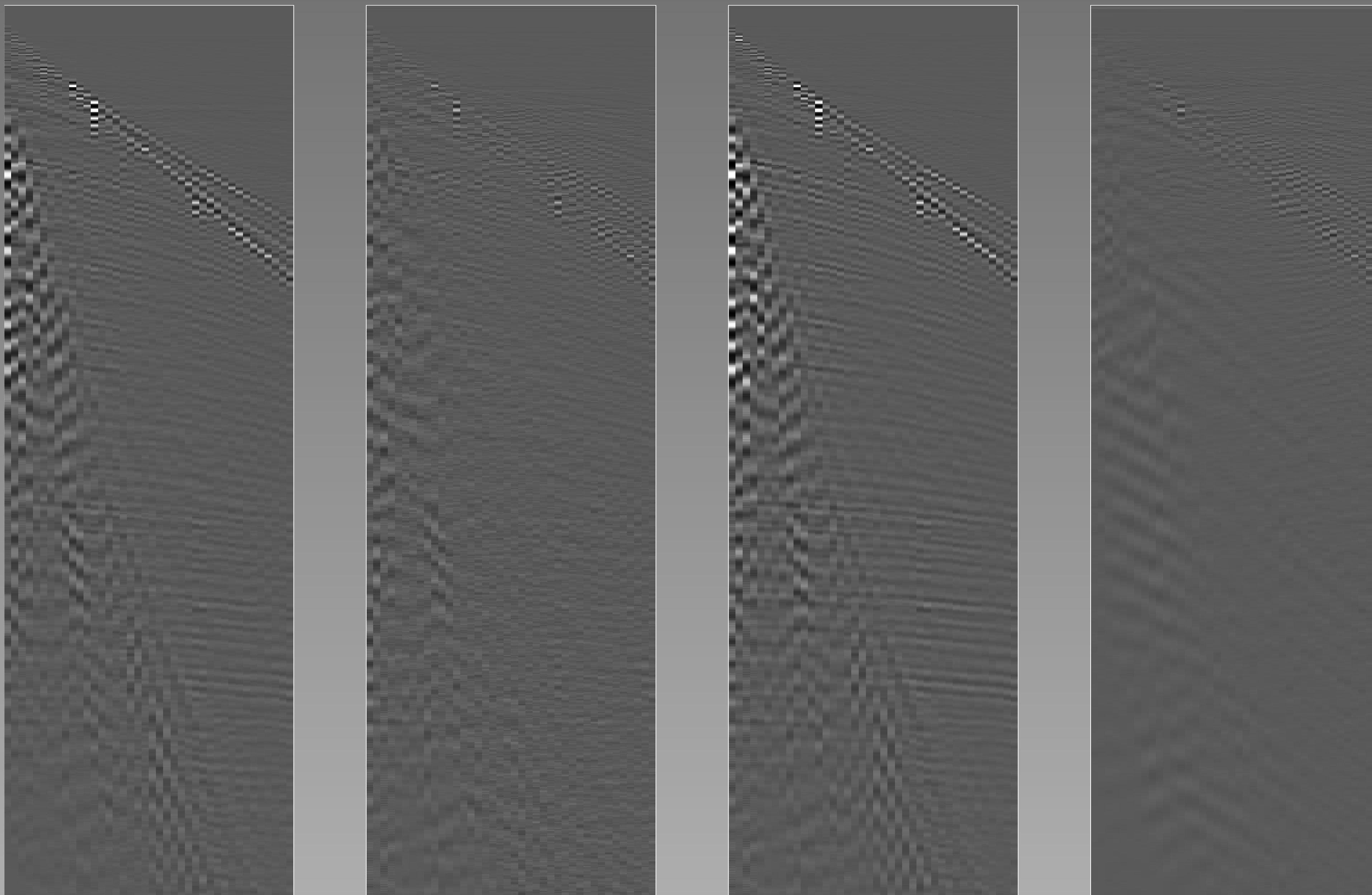


Data space residual 14



Data space residual 25

$\mathbf{d}-\mathbf{L}\hat{\mathbf{m}}^{BC}$ $\mathbf{d}-\mathbf{L}\hat{\mathbf{m}}^{C}$ $\mathbf{d}-\mathbf{L}\hat{\mathbf{m}}^{l^1}$ $\mathbf{d}-\mathbf{L}\hat{\mathbf{m}}^{l^2}$



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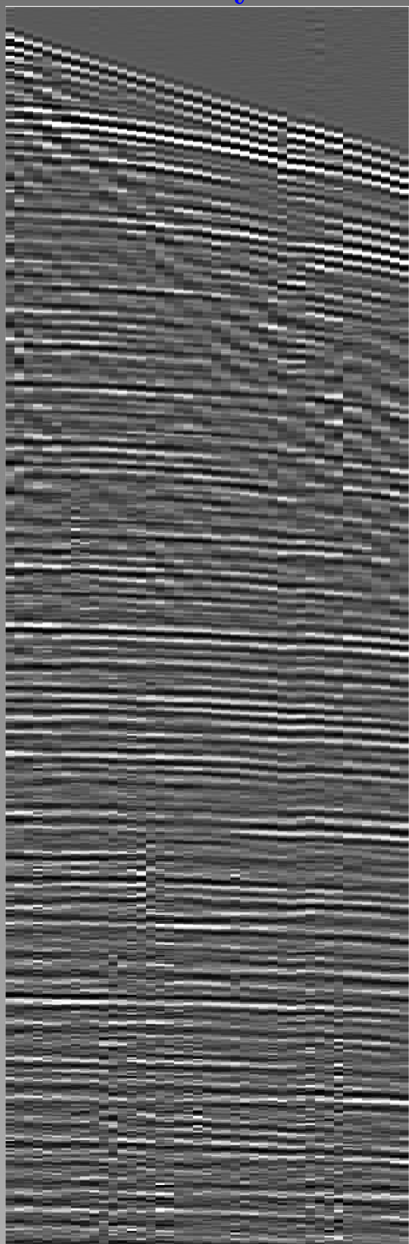
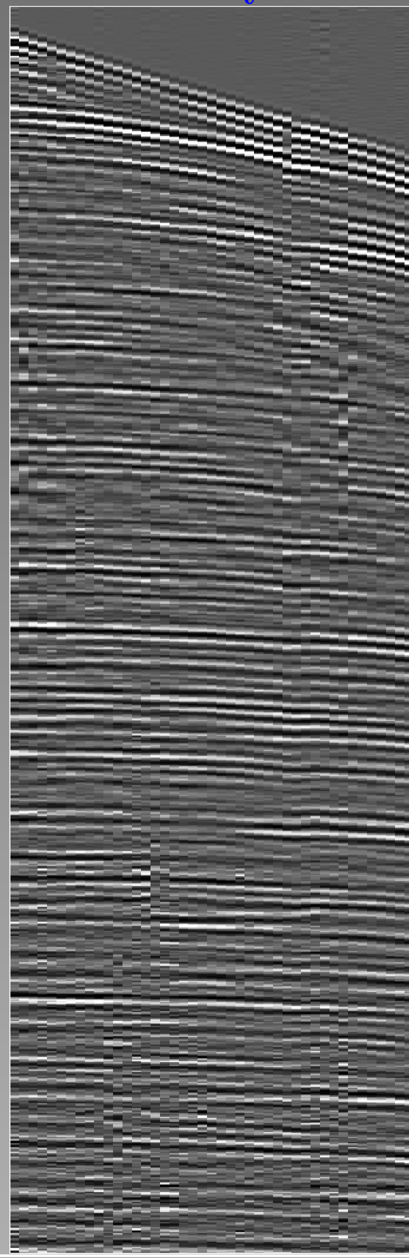
Signal 08

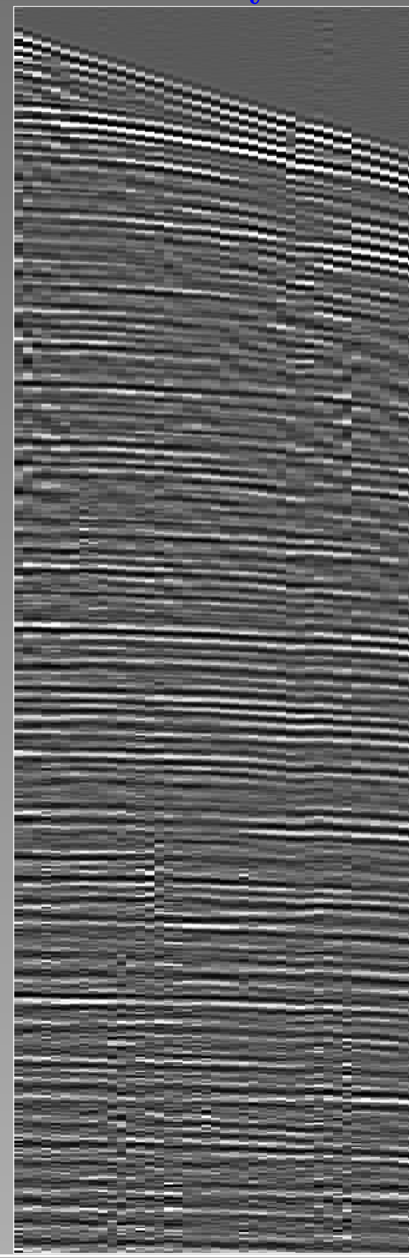
$\mathbf{d} - \mathbf{L}_l \hat{\mathbf{m}}^{BC}$



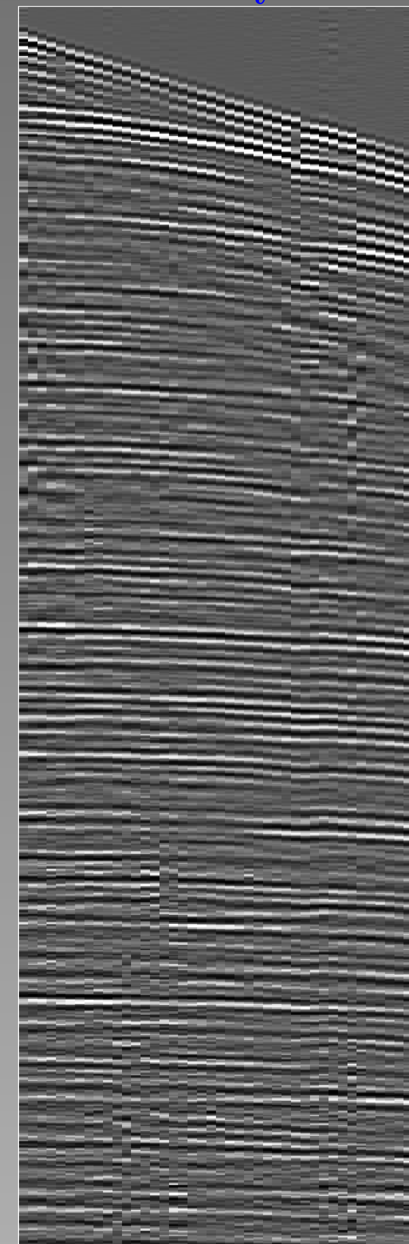
$\mathbf{d} - \mathbf{L}_l \hat{\mathbf{m}}^{C}$



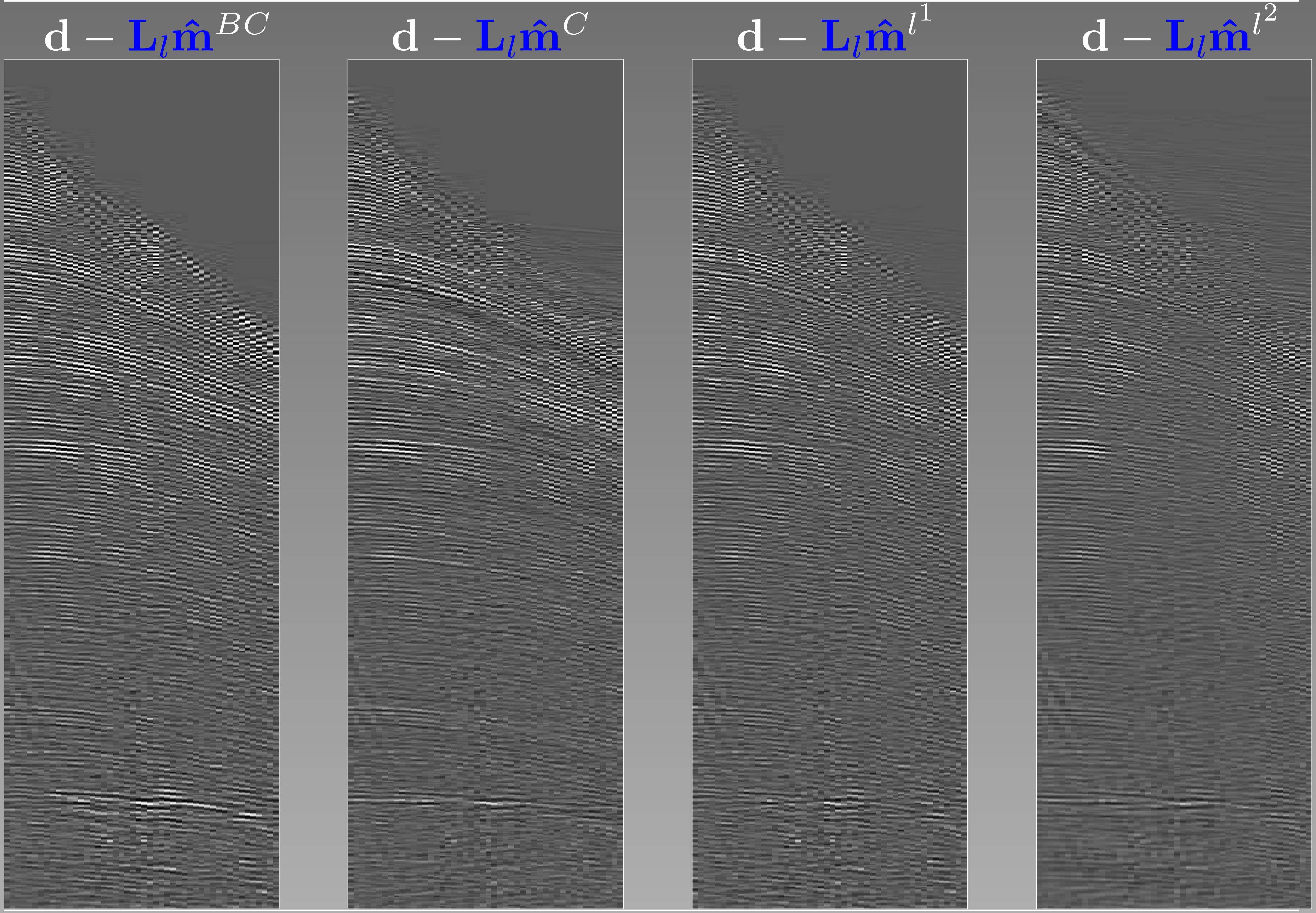
$\mathbf{d} - \mathbf{L}_l \hat{\mathbf{m}}^{l^1}$



$\mathbf{d} - \mathbf{L}_l \hat{\mathbf{m}}^{l^2}$

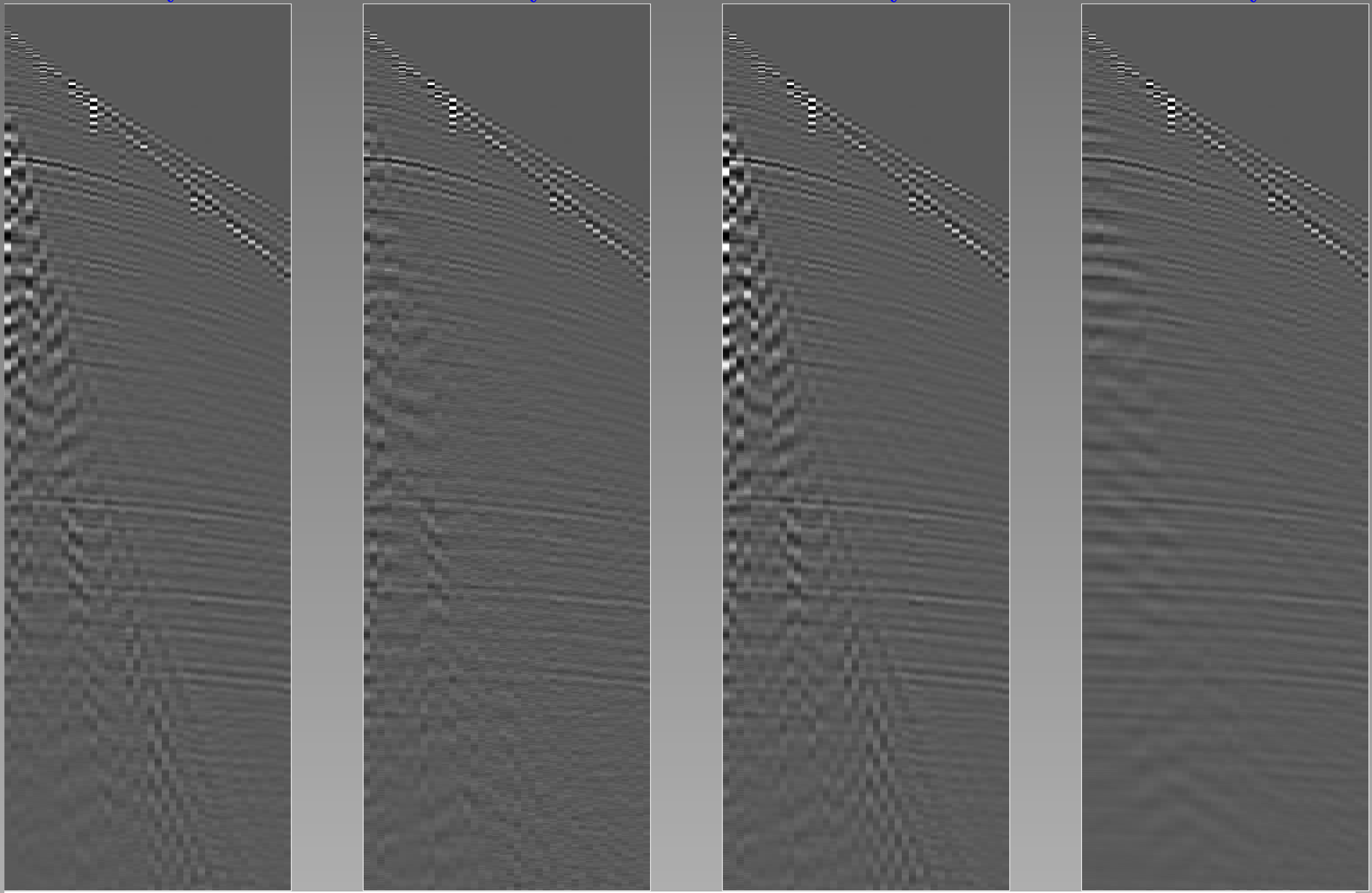


Signal 14


$\mathbf{d} - \mathbf{L}_l\hat{\mathbf{m}}^{BC}$
$\mathbf{d} - \mathbf{L}_l\hat{\mathbf{m}}^{C}$
$\mathbf{d} - \mathbf{L}_l\hat{\mathbf{m}}^{l^1}$
$\mathbf{d} - \mathbf{L}_l\hat{\mathbf{m}}^{l^2}$

Signal 25

$\mathbf{d} - \mathbf{L}_l\hat{\mathbf{m}}^{BC}$	$\mathbf{d} - \mathbf{L}_l\hat{\mathbf{m}}^{C}$	$\mathbf{d} - \mathbf{L}_l\hat{\mathbf{m}}^{l^1}$	$\mathbf{d} - \mathbf{L}_l\hat{\mathbf{m}}^{l^2}$



Conclusions

- Sparse inversions have too little freedom
- Sparse inversions good for analysis
- Sparse inversions are poor for synthesis
- Signal/Noise separation w/ dual operators:
  Use least-squares
  - ⋆ (besides, it's much much cheaper)

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\def\modhead{\vskip -.34in {\small
\hskip 0.90in $\hat{\bf{m}}^{BC}$
\hskip 1.80in $\hat{\bf{m}}^{C}$
\hskip 1.95in $\hat{\bf{m}}^{l^1}$
\hskip 1.95in $\hat{\bf{m}}^{l^2}$ }}

\foilhead{Model space 08}
\modhead
\includegraphics[height=6.5in,width=10in]{./Fig/m08}
\foilhead{Model space 14}
\modhead
\includegraphics[height=6.5in,width=10in]{./Fig/m14}
\foilhead{Model space 25}
\modhead
\includegraphics[height=6.5in,width=10in]{./Fig/m25r}

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\def\reshead{\vskip -.3in {\small
\hskip .3in ${\bf d-L\hat{m}}^{BC}$
\hskip 1.in ${\bf d-L\hat{m}}^{C}$
\hskip 1.in ${\bf d-L\hat{m}}^{l^1}$
\hskip 1.15in ${\bf d-L\hat{m}}^{l^2}$ }}

\foilhead{Data space residual 08}
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