Fourier-domain imaging condition for shot-profile migration

Brad Artman
SEP120 pages 325-331
brad@geo.stanford.edu

Goals

- The Fourier-domain imaging condition
- It works ...
- ...but not well
- Why not
- What it teaches us
- Operator aliasing post-migration

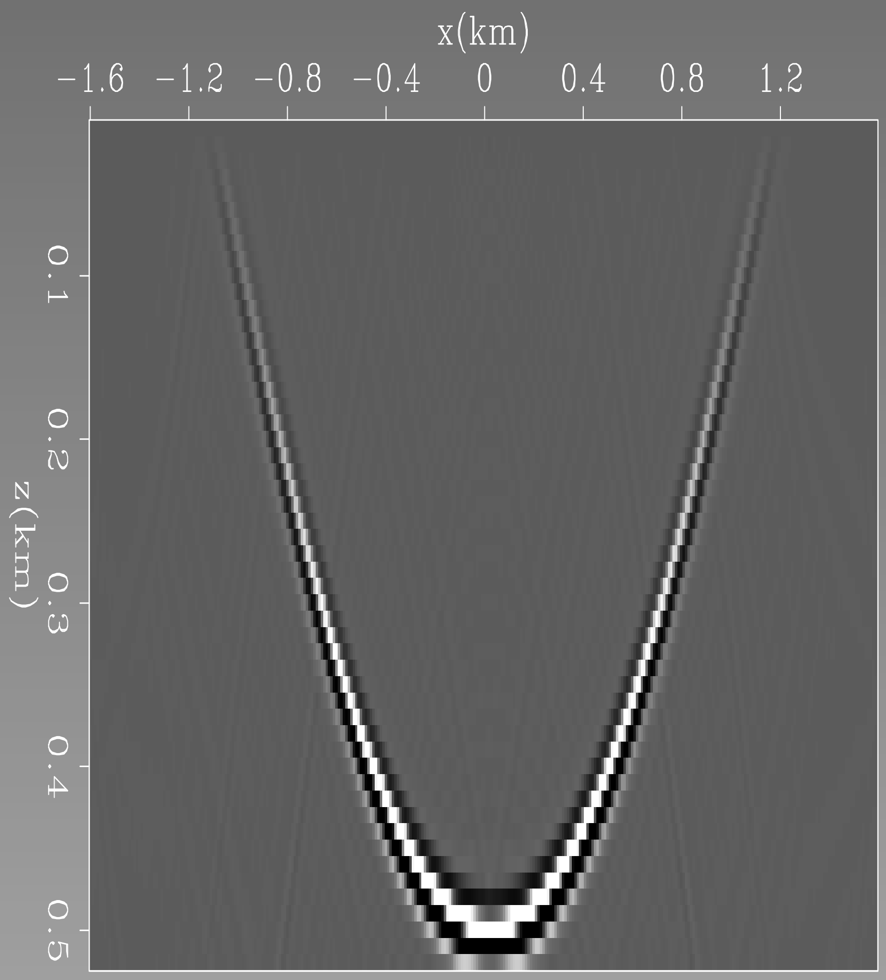
FDIC & aliasing

$$I(x,h)|_{\omega,z} = U(x+h)\,D^*(x-h)$$

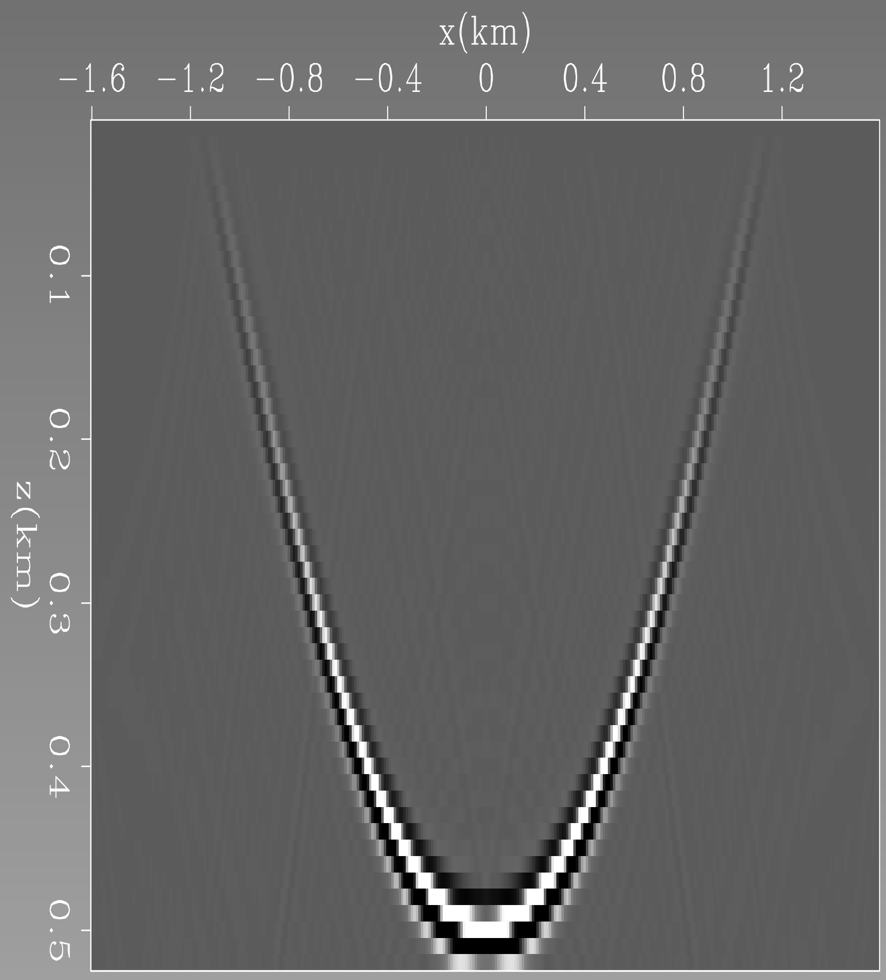
$$\widehat{I}(k_x,k_h)|_{\omega,z} = \frac{1}{2}\hat{U}\left(\frac{k_x+k_h}{2}\right)\,\hat{D}^*\left(\frac{k_x-k_h}{2}\right)$$

derivation

$x - z$ impulse response

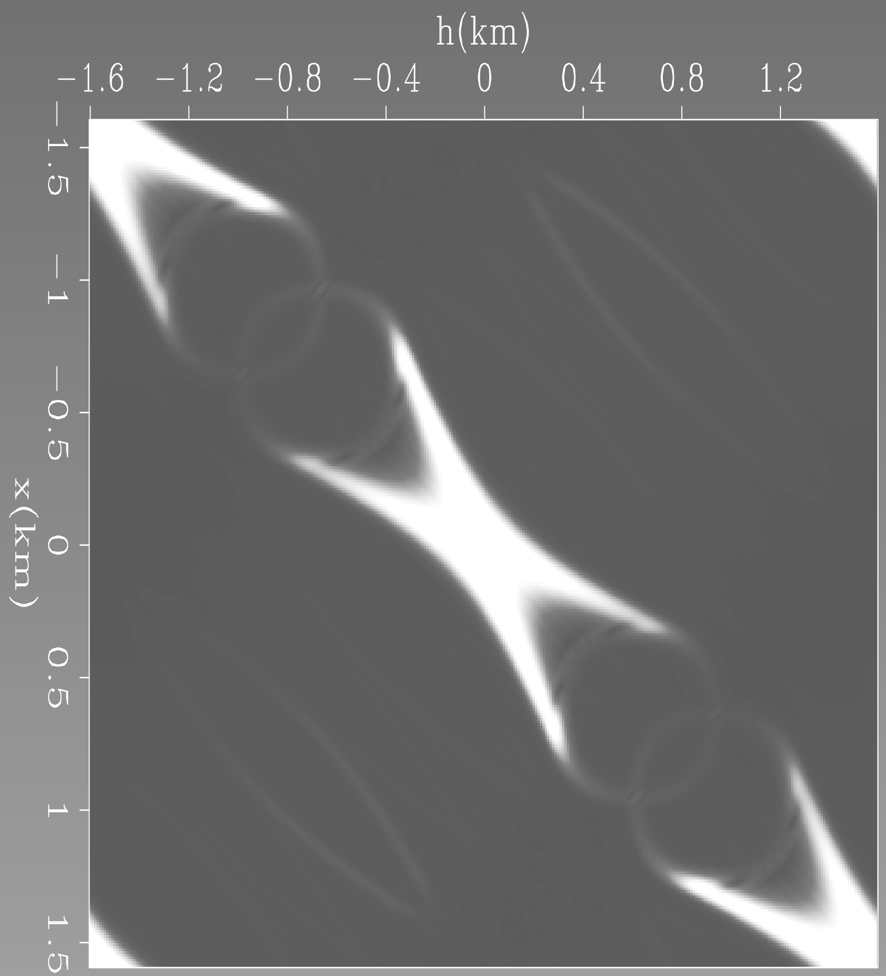


Fourier domain

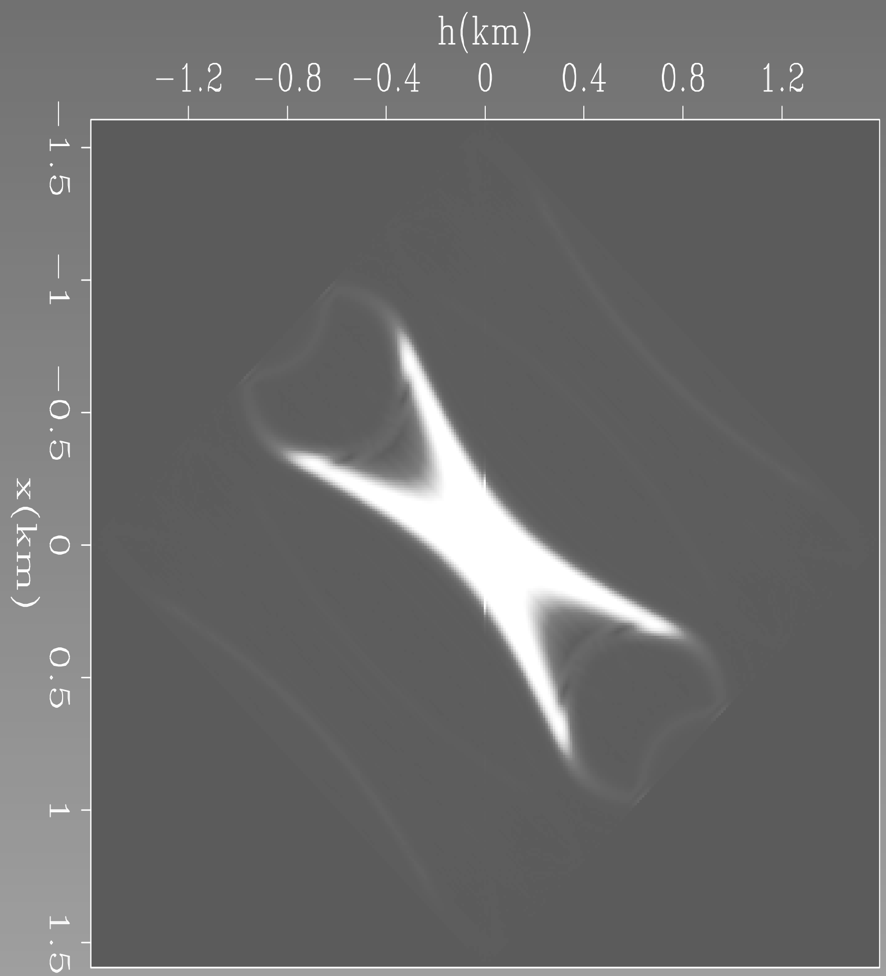


Space domain

$x - h$ impulse response

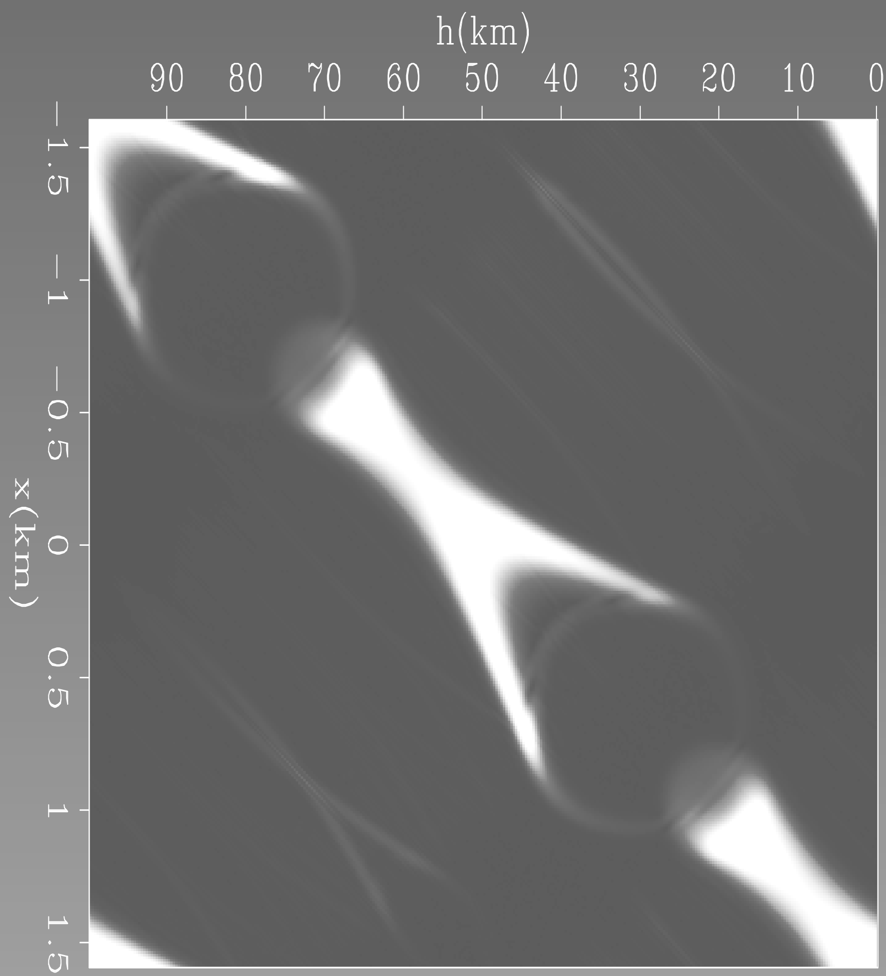


Fourier domain

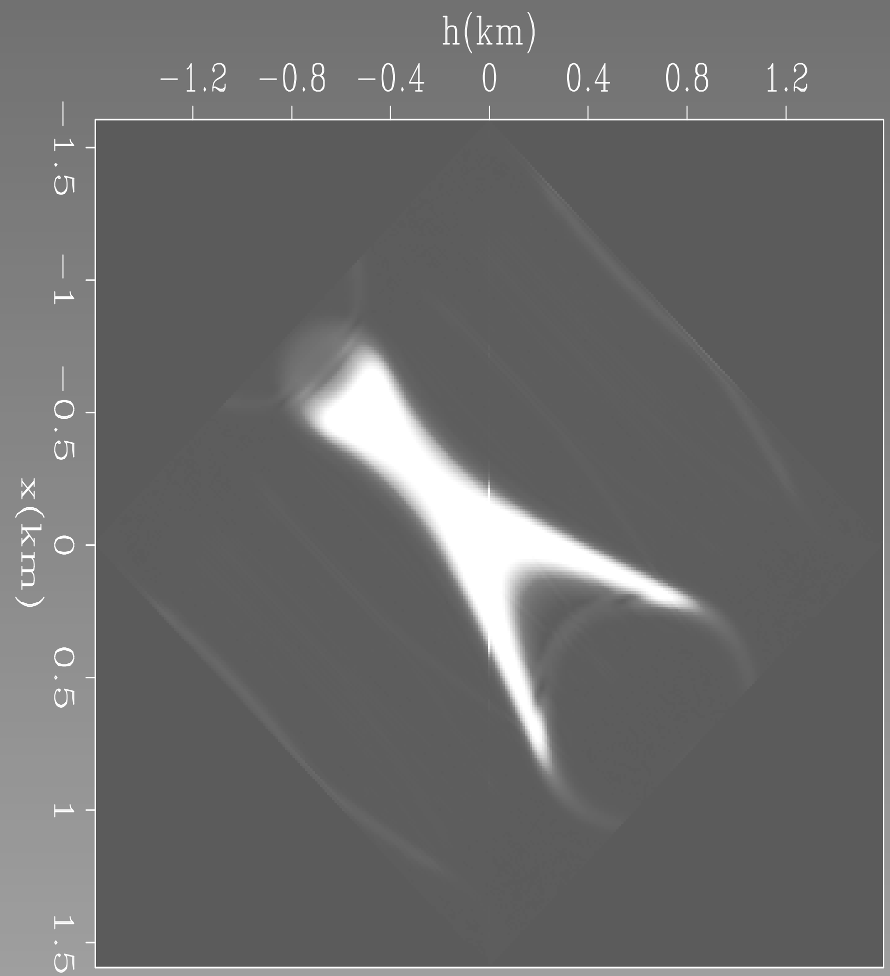


Space domain

$x - h$ impulse response



Fourier domain



Space domain

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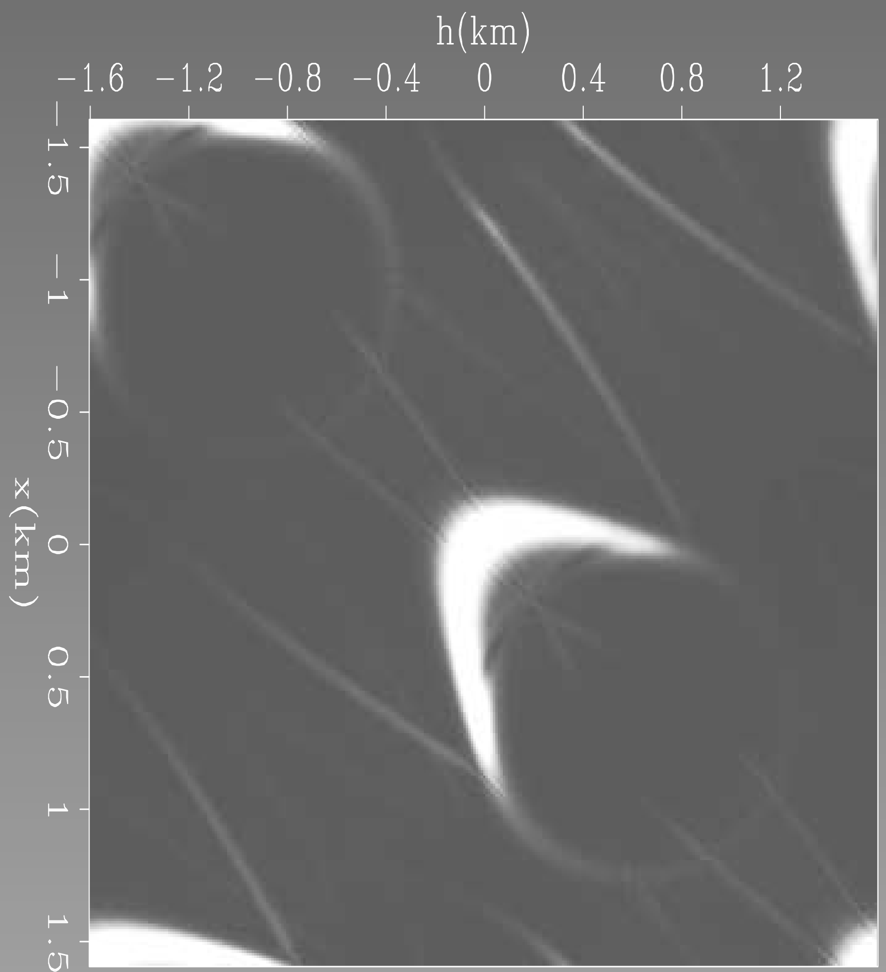
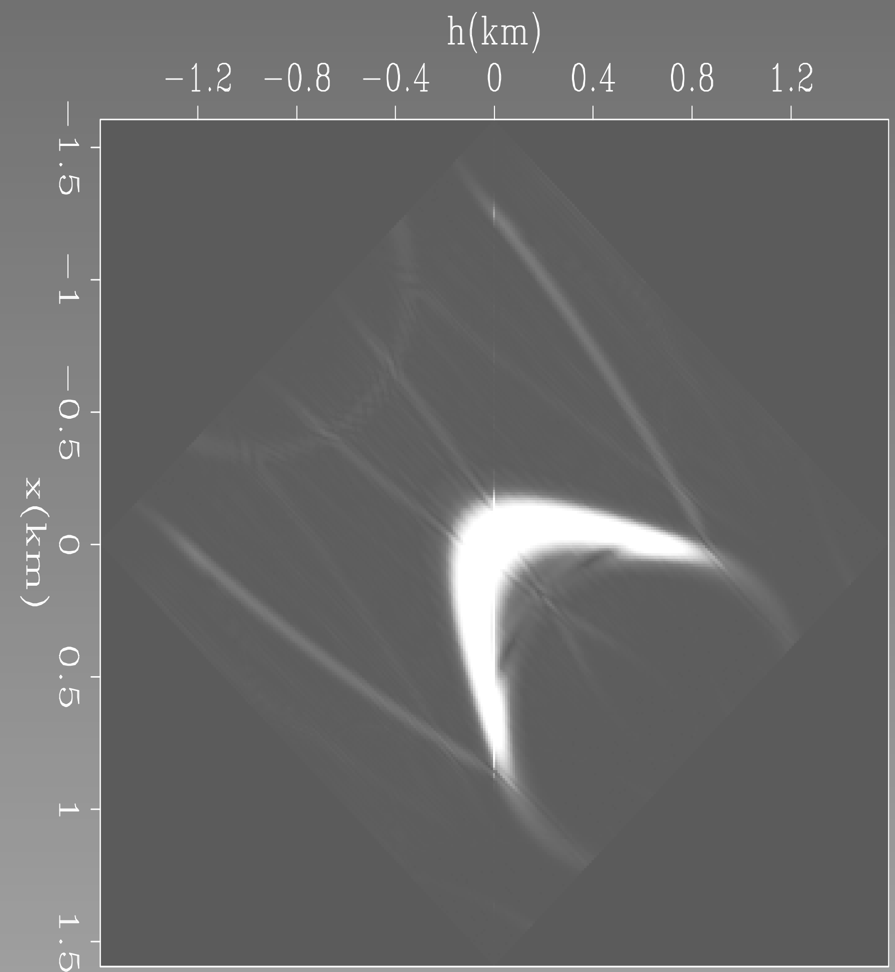
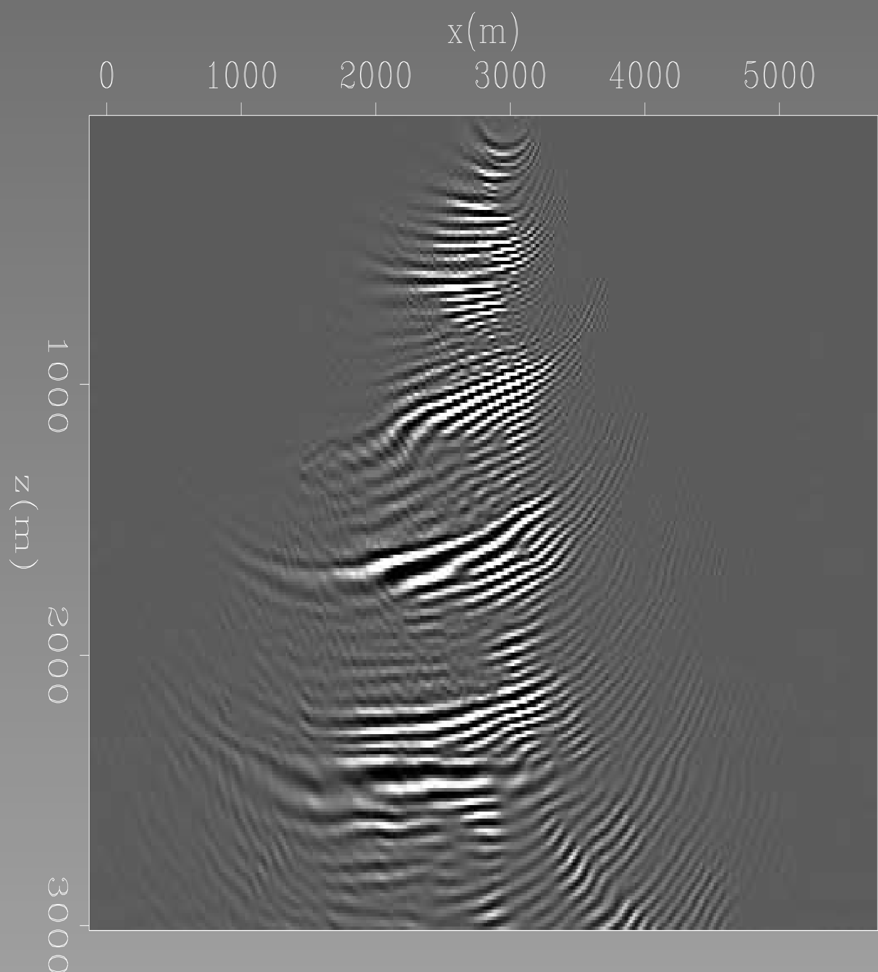


Fourier domain

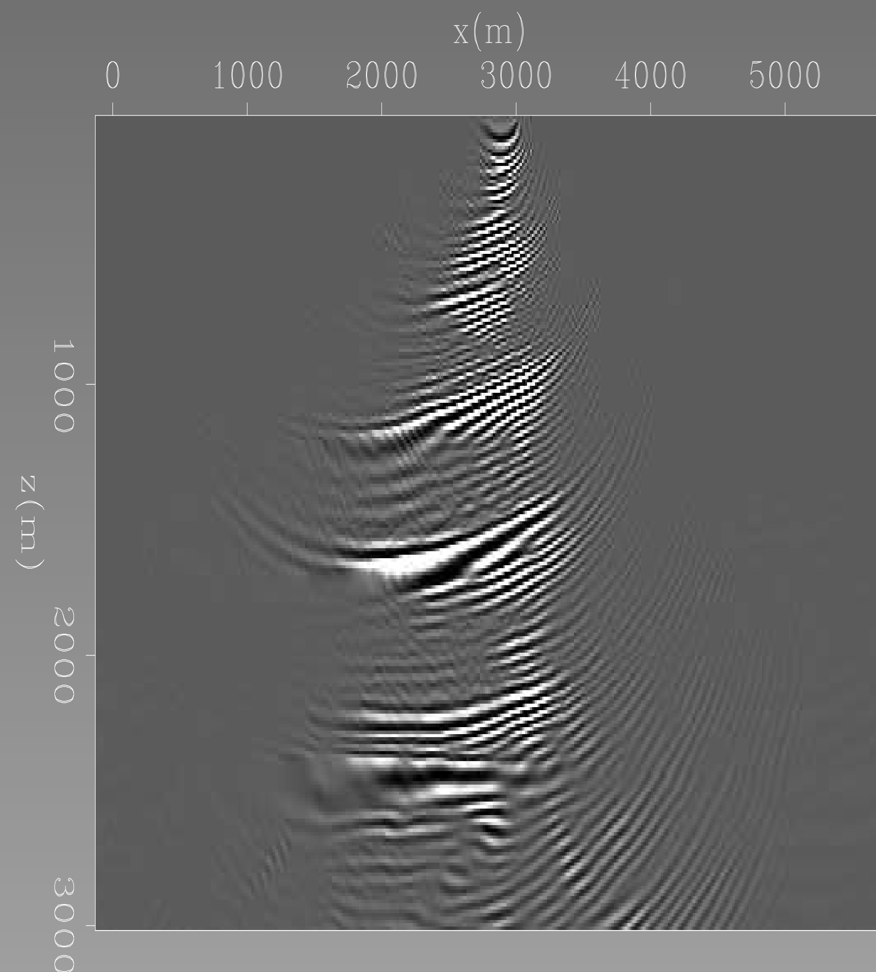


Space domain

Marmousi shot

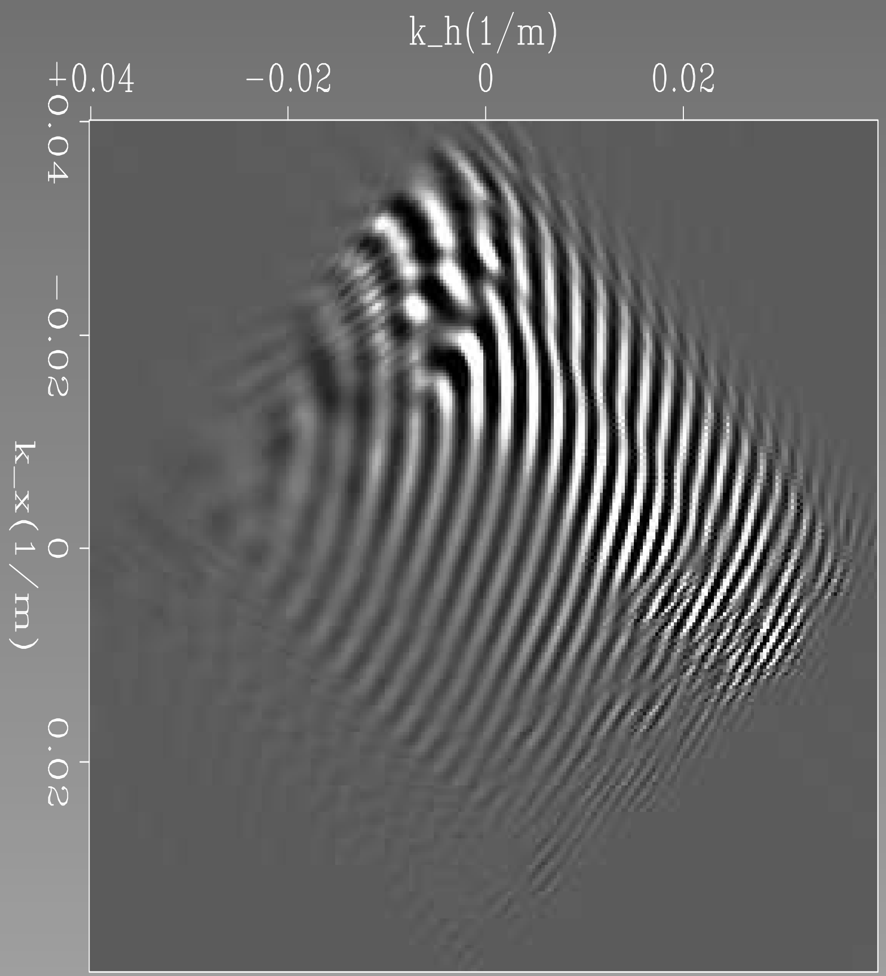


Fourier domain

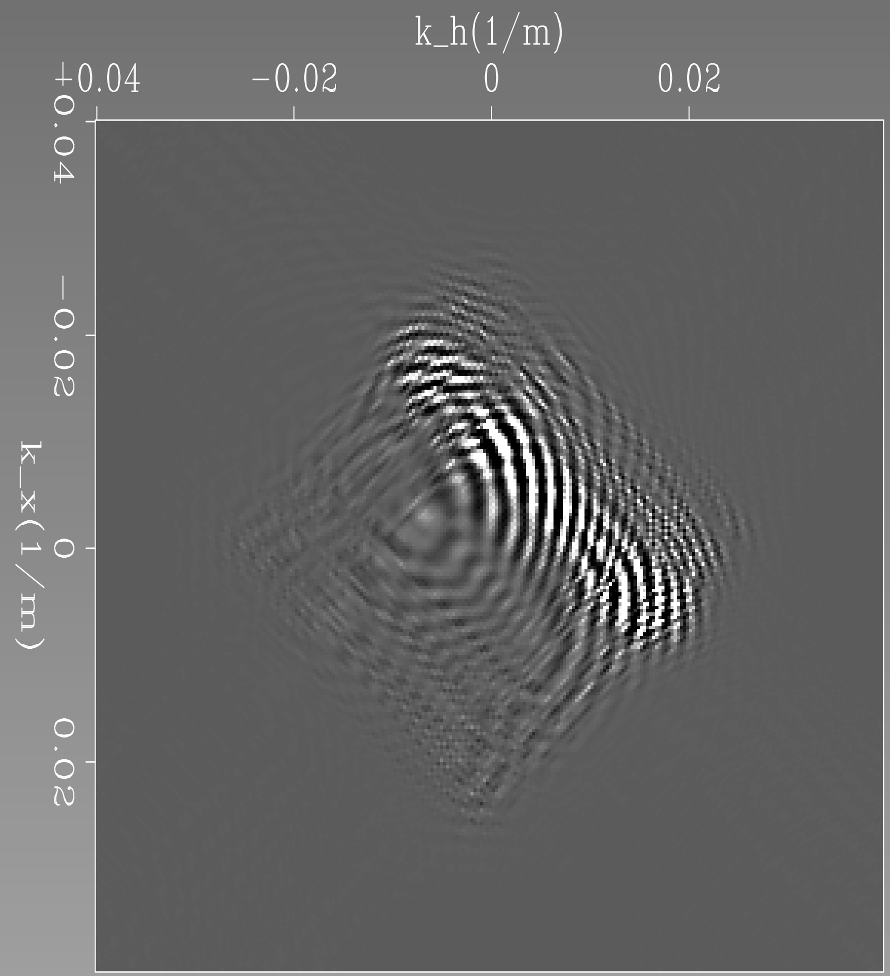


Space domain

Evanescence filter



shallow

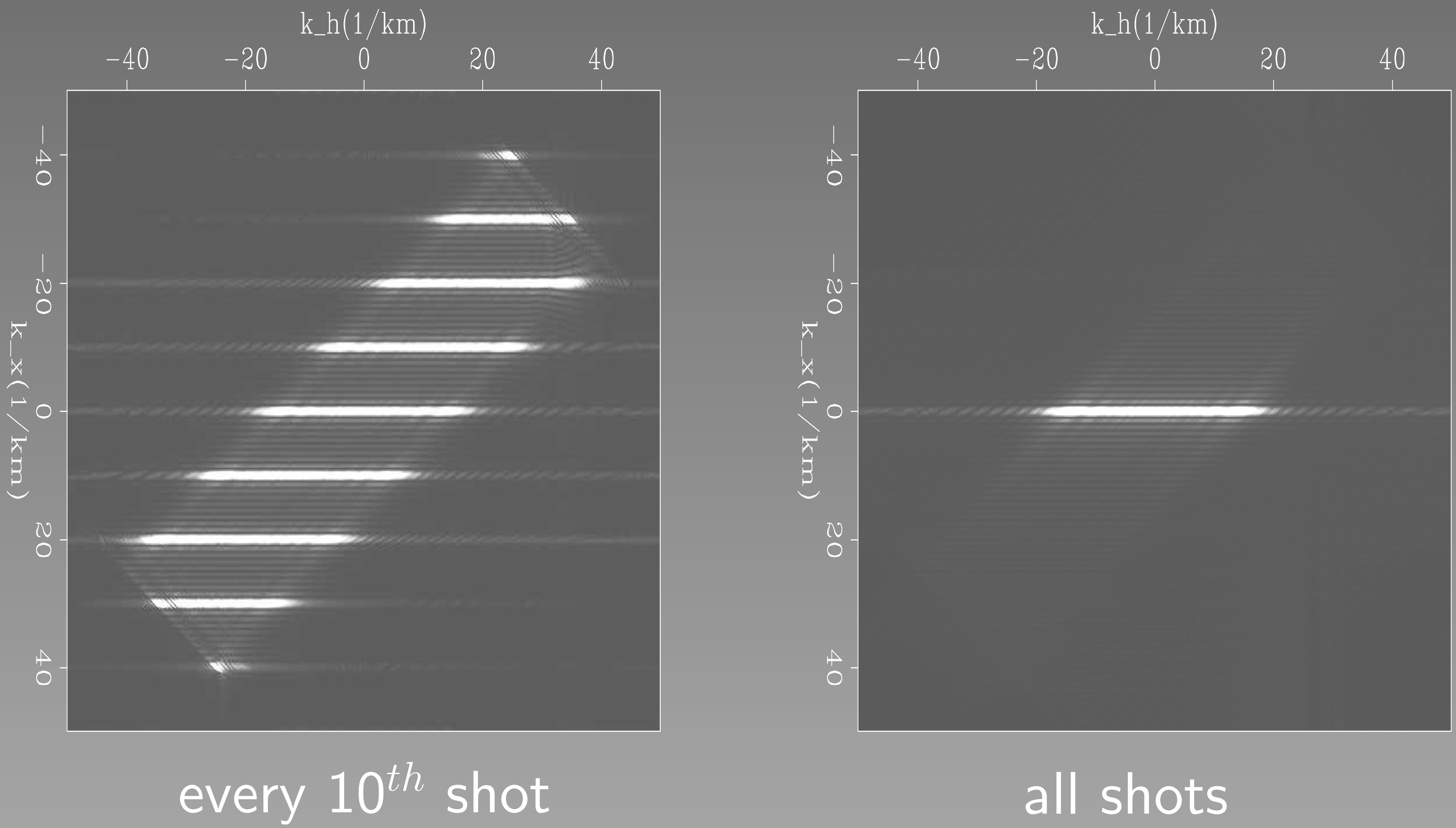


deep

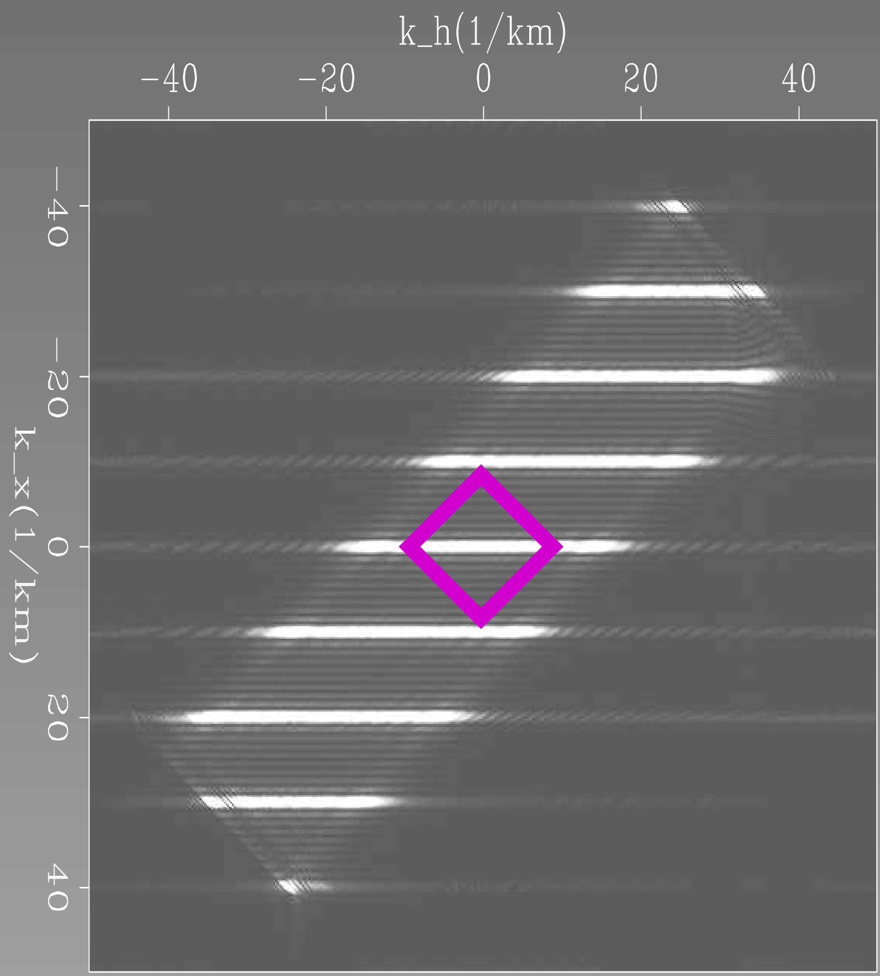
Operator aliasing

"No hyperbolas have been harmed
during the filming of this motion picture."

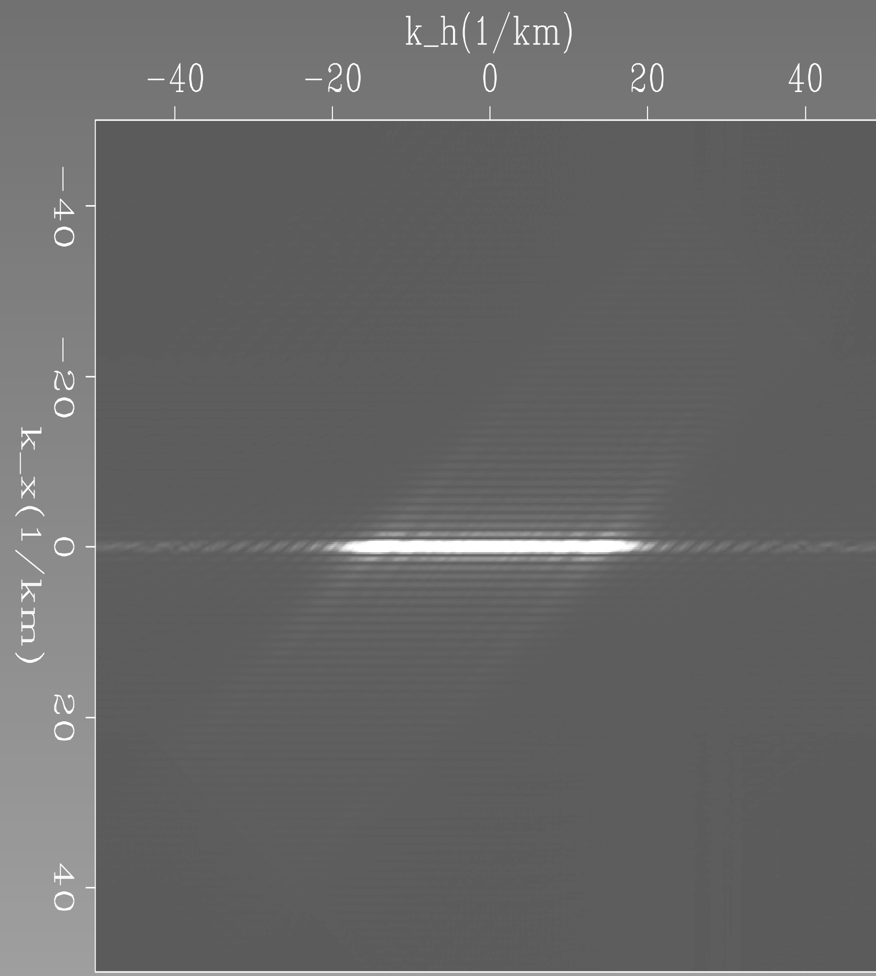
Operator aliasing



Operator aliasing



every 10^{th} shot



all shots

FDIC & aliasing

$$\widehat{I}(k_x, k_h) = \frac{1}{2}\hat{U}\left(\frac{k_x + k_h}{2}\right)\hat{D}^*\left(\frac{k_x - k_h}{2}\right)$$

k_x \ k_h	-1	0	1
-2	$\hat{U}(\frac{-3}{2})\hat{D}^*(\frac{-1}{2})$	$\hat{U}(\frac{-2}{2})\hat{D}^*(\frac{-2}{2})$	$\hat{U}(\frac{-1}{2})\hat{D}^*(\frac{-3}{2})$
-1	$\hat{U}(\frac{-2}{2})\hat{D}^*(0)$	$\hat{U}(\frac{-1}{2})\hat{D}^*(\frac{-1}{2})$	$\hat{U}(0)\hat{D}^*(\frac{-2}{2})$
0	$\hat{U}(\frac{-1}{2})\hat{D}^*(\frac{1}{2})$	$\hat{U}(0)\hat{D}^*(0)$	$\hat{U}(\frac{1}{2})\hat{D}^*(\frac{-1}{2})$
1	$\hat{U}(0)\hat{D}^*(\frac{2}{2})$	$\hat{U}(\frac{1}{2})\hat{D}^*(\frac{1}{2})$	$\hat{U}(\frac{2}{2})\hat{D}^*(0)$
2	$\hat{U}(\frac{1}{2})\hat{D}^*(\frac{3}{2})$	$\hat{U}(\frac{2}{2})\hat{D}^*(\frac{2}{2})$	$\hat{U}(\frac{3}{2})\hat{D}^*(\frac{1}{2})$

Conclusion

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  - ⋆ Post-migration!

Thanks

Sergey Fomel & Biondo

Derivation 1

$$I(x,h)|_{\omega,z} = U(x_r - h)D(x_s + h) \quad (1)$$

Fourier transform D to $\hat{D}$

$$\hat{I}(x,h) = U(x-h)\int \hat{D}(k_s)e^{ik_s(x+h)}dk_s \quad (2)$$

FT all the x's

$$\hat{I}(k_x,h) = \int U(x-h)\int \hat{D}(k_s)e^{ik_s(x+h)}dk_s e^{-ixk_x}dx \quad (3)$$

Derivation 2

Introduce (Sergey inspired) variable flip-flop/reorder

$$\hat{I}(k_x, h) = \int \hat{D}(k_s) e^{ik_s h} \int U(x-h) e^{-ix(k_x - k_s)} dx dk_s$$

$$\hat{I}(k_x, h) = \int \hat{D}(k_s) e^{ih(2k_s - k_x)} \int U(x-h) e^{-i(x-h)(k_x-k_s)} d(x-h) dk_s \quad (4)$$

Note salient details: Inner integral is FT of U,

$$\hat{I}(k_x, h) = \int \hat{U}(k_x - k_s) \hat{D}(k_s) e^{ih(2k_s - k_x)} dk_s \quad (5)$$

Derivation 3

knowing $k_h = 2k_s - k_x$,

$$\hat{I}(k_x, h) = \frac{1}{2}\int \hat{U}(\frac{k_x - k_h}{2})\hat{D}(\frac{k_x + k_h}{2})e^{-ihk_h}dk_h \qquad (6)$$

This we see is FT over offset, so the complete FT of the IC is

$$\widehat{I}(k_x, k_h) = \frac{1}{2}\hat{U}(\frac{k_x - k_h}{2})\hat{D}(\frac{k_x + k_h}{2}). \qquad (7)$$

return