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ELASTIC RESPONSE OF GRANULAR SOILS WITH MULTISCALE SUBSTRUCTURE

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REFERENCES (1) – RECENT PAPERS

◦ J. G. Berryman, "Bounds and self-consistent estimates for elastic constants of granular polycrystals composed of orthorhombics or crystals with higher symmetries," *Phys. Rev. E*, to appear, 2011.

◦ J. G. Berryman, "Mechanics of layered anisotropic poroelastic media with applications to effective stress for fluid permeability," *Int. J. Engng. Sci.* **48**, 446–459 (2011).

REFERENCES (2) – SOME RELATED PAPERS

◦ S. R. Pride & J. G. Berryman, "Goddard rattler-jamming mechanism for quantifying pressure dependence of elastic moduli of grain packs," *Acta Mechanica* **205**, 185–196 (2009).

◦ J. G. Berryman, "Bounds and self-consistent estimates for elastic constants of random polycrystals with hexagonal, trigonal, and tetragonal symmetries," *J. Mech. Phys. Solids* **53**, 2141–2173 (2005).

◦ J. Dvorkin, J. Berryman, A. Nur, "Elastic moduli of cemented sphere packs," *Mech. Mat.* **31**, 461–469 (1999).

OUTLINE

- Bounds for random polycrystals of anisotropic grains
  - Uses work of Hashin and Shtrikman, also Peselnick, Meister, and Watt for less symmetric cases.
  - Combine this work with earlier work on self-consistent estimates to produce results for a variety of mixtures.
- Brief discussion of some hybrid methods
- Conclusions

BOUNDS ON K FOR POLYCRYSTALS (1)

Hashin-Shtrikman-type bounds for elastic constants of isotropic random polycrystals are known, and given first in detail by Peselnick and Meister (1965), later improved by Watt and Peselnick (1980).

In this presentation, formulas will be limited to those for hexagonal symmetry, since the corresponding ones for orthorhombic symmetry are much more complicated, and therefore well beyond our time constraints.

BOUNDS ON K FOR POLYCRYSTALS (2)

Bounds for the bulk modulus can be expressed in terms of these uniaxial shear energies per unit volume as

$$K_{PM}^{\pm} = K_V \frac{G_{eff}^r + \zeta_\pm}{G_{eff}^v + \zeta_\pm},$$

where

$$\zeta_\pm = \frac{G_\pm}{6}\left(\frac{9K_\pm + 8G_\pm}{K_\pm + 2G_\pm}\right).$$

Parameters $G_\pm$, $K_\pm$ were defined by Watt and Peselnick. They depend on details of the elastic matrix of the crystal of which the polycrystal is composed.

BOUNDS ON G FOR POLYCRYSTALS (1)

Bounds on shear modulus for hexagonal symmetry can be expressed similarly as

$$\frac{5}{G^{\pm}_{PM}+\zeta_\pm} = \frac{1-X_\pm}{G^{v}_{eff}+\zeta_\pm+Y_\pm} + \frac{2}{c_{44}+\zeta_\pm} + \frac{2}{c_{66}+\zeta_\pm},$$

where $X_\pm$ and $Y_\pm$ are additional parameters depending on certain well-defined parameters $G_\pm$ and $K_\pm$.

Note that in both cases when $\zeta_- \to 0$ the bounds are limited by the Reuss average (lower bound), and when $\zeta_+ \to \infty$ the bounds go to the Voigt average (upper bound).

BOUNDS ON G FOR POLYCRYSTALS (2)

For example,

$$K_{PM}^{-} \to K_V G_{eff}^{r}/G_{eff}^{v} \equiv K_R$$

from certain product formulas (true for hexagonal, tetragonal, and trigonal symmetry) relating special shear moduli to the uniaxial shear components: G_{eff}^{r} and G_{eff}^{v}, depending on whether the uniaxial shear loading was applied via a pure stress (applied force using pure compression or tension), or a pure strain (applied deformation using pure compaction or stretch).

SELF-CONSISTENT MODULI K^*_{SC} AND G^*_{SC} (1)

Self-consistent estimates are obtained (approximately) by taking $K_\pm \to K^*_{SC}$ and $G_\pm \to G^*_{SC}$.

The resulting hexagonal symmetry formulas are:

$$K^*_{SC} = K_V \frac{G^r_{eff} + \zeta^*_{SC}}{G^v_{eff} + \zeta^*_{SC}},$$

where

$$\zeta^*_{SC} = \frac{G^*_{SC}}{6}\left(\frac{9K^*_{SC} + 8G^*_{SC}}{K^*_{SC} + 2G^*_{SC}}\right),$$

and

$$\frac{5}{G^*_{SC} + \zeta^*_{SC}} = \frac{1 - X^*_{SC}}{G^v_{eff} + \zeta^*_{SC}} + \frac{2}{c_{44} + \zeta^*_{SC}} + \frac{2}{c_{66} + \zeta^*_{SC}}.$$

SELF-CONSISTENT MODULI K^*_{SC} AND G^*_{SC} (2)

What we have accomplished so far is to find the effective constants for the gritty or gluey stuff between the bigger particles in the complicated soil we are trying to study.

But it is now straightforward to incorporate this information into a formalism that includes all the particles, and gives estimates of the overall (or total) elastic behavior of our messy composite granular material.

SELF-CONSISTENT MODULI K^T_{SC} AND G^T_{SC} (3)

The previous formulas can then be used together with the bounds to compute both bounds and estimates on the elastic constants of mixtures. For example, in isotropic composites of isotropic components, we have (for any number of components N):

$$\frac{1}{K^T_{SC}+\frac{4}{3}G^T_{SC}} = \sum_{n=1,N} \frac{v_n}{K^{(n)}_{SC}+\frac{4}{3}G^T_{SC}},$$

and, similarly,

$$\frac{1}{G^T_{SC}+\frac{4}{3}\zeta^T_{SC}} = \sum_{n=1,N} \frac{v_n}{G^{(n)}_{SC}+\zeta^T_{SC}},$$

SELF-CONSISTENT MODULI K_{SC}^T AND G_{SC}^T (4)

where, for each value of n, we have $K_{SC}^{(n)} = K_{SC}^*$ and $G_{SC}^{(n)} = G_{SC}^*$ for one particular choice of anisotropic constituent K_{SC}^* and G_{SC}^*, as expressed previously. And where, for the overall ($T =$ total) system, we again have the ζ parameter definition:

$$\zeta_{SC}^T = \frac{G_{SC}^T}{6}\left(\frac{9K_{SC}^T + 8G_{SC}^T}{K_{SC}^T + 2G_{SC}^T}\right).$$

The volume fractions v_n sum to unity, including the volume fractions for any voids having null moduli.

SUMMARY AND CONCLUSIONS (1)

- Progress has been made on the analysis of effective constants for granular polycrystals of anisotropic material, up to and including orthorhombic symmetries. (The only cases not included at the moment are monoclinic and triclinic symmetries.)
- One key idea has been to incorporate the values of the (very robust) self-consistent estimates into the search routines for finding the optimum Hashin-Shtrikman bounds for orthorhombic materials.

SUMMARY AND CONCLUSIONS (2)

- Mixing rules for effective constants of heterogeneous mixtures of anisotropic granular media can be handled in a straightforward way once these effective constants for the individual components of the mixture are known.
- Issues that remain concern known (but not necessarily resolved) matters concerning the importance of grain-to-grain contacts and how these coating materials may strengthen the grain-packs via a rather complicated type of cementing mechanism. Relative particle-particle arrangement information is then also needed.

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