The Dichotomy

Gassmann's equations are low frequency (quasi-static) and predict that

$$K^{sat} = \Lambda_{sat}(G_{dr}) \qquad \text{and} \qquad G^{sat} = G_{dr}.$$

But, the effective medium theory (which is also for low frequencies), for apparently the same problem, predicts instead that

$$K^{eff}_{sat} = \Lambda_{sat}(G^{eff}_{sat}) \qquad \text{and} \qquad G^{eff}_{sat} = \Gamma_{sat}(F^{eff}_{sat}).$$

The Dichotomy (2)

Furthermore, because the canonical functions Λ and Γ are monotonic, it is easy to show that, whenever $K_f > K_{air} \simeq 0$, we must have

$$G^{sat} < G^{eff}_{sat}$$

and, therefore,

$$K^{sat} < K^{eff}_{sat}.$$

The Dichotomy (3)

How do we explain that the predictions of these two low frequency theories clearly differ? Even if the numerical difference is not great, the mere existence of the difference (assuming both theories are correct, so it is a real difference) shows that there must be dispersion in such systems. Dispersion also implies attenuation, because of Kramers-Kronig relations. So this difference, if true, guarantees that there is more attenuation of sound waves in a poroelastic system than we might expect from other considerations.

The Dichotomy (4)

The physical reason for this dispersion is that the Gassmann approach is really quasi-static, and therefore applies at extremely low frequencies, whereas the effective medium theory is clearly not formulated to apply at such low frequencies. The difference lies in how fluid pressure is treated in the two approaches. Gassmann allows the fluid pressure sufficient time to equilibrate throughout the medium, however long it takes – perhaps very long times indeed. The effective medium theory does not preclude the fluid from equilibrating, but does not necessarily allow enough time for equilibration to happen.