Introducing the Adjoint Method

Now, we introduce another method, not as well known as some of the others we have discussed, but becoming more common: the adjoint approximation to the inverse. Start by recalling the formal solutions to the over- and underdetermined least squares problems.

$$\text{Overdetermined: } s \simeq (M^T M)^{-1} M^T t$$

$$\text{Underdetermined: } s \simeq M^T (M M^T)^{-1} t$$

Both formulas contain a factor of M^T (which is the adjoint of M when M is a real matrix), and a matrix inverse that may be very hard or impossible to compute (if the matrix is singular) multiplying the data vector.

The Adjoint Approximation

The form of these equations suggests that it might prove effective to try replacing the unknown inverses with either a diagonal matrix, or even a constant scalar. Then,

$$\text{Adjoint approximation: } s \simeq \alpha M^T t$$

The value of the constant α should be chosen to give the formula the correct dimensions (of slowness in this case) and to optimize some other choice of functional. Note that this procedure is better justified for the underdetermined case than for the overdetermined one.