Phase Matching at Interfaces

Frequency content is preserved so $f =$ constant.

Wavelength changes so

$\lambda_0 = c/f$ and $\lambda = c/nf$

where n is the index of refraction.

If D is the spacing between crests at the interface,

$\lambda_0 = D \cdot sin\,\theta_0$ and $\lambda = D\,sin\,\theta$.

Eliminating D gives:

$sin\,\theta_0/\lambda_0 = sin\,\theta/\lambda$ or $sin\,\theta_0 = n \cdot sin\,\theta$