Midterm Exam

Math 128A Spring 2002
Sergey Fomel

March 12, 2002

Your Name: ____

- Time: 75 minutes.
- Answer ALL questions.
- Please read carefully every question before answering it.
- If you need extra space, use the other side of the page.

1. (6 points) IEEE defines not only the single-precision and double-precision formats but also the double-extended format: 1 bit for the sign, 15 bits for the exponent, and 63 bits for the mantissa (79 bits in total). A double-extended non-zero number x can be written in this standard as $x = \pm(1.d_1 d_2 \cdots d_{63})_2 \times 2^{n-2^{14}+1}$, with $1 \le n \le 2^{15}$, and $0 \le d_k \le 1$ for $k = 1, 2, \ldots, 63$. Find:

a. The machine epsilon (the smallest positive ϵ such that $1+\epsilon$ has a computer representation and $1+\epsilon > 1$) in the double-extended format.

b. The largest positive double-extended floating-point number. You can express your answer with a formula. Do not use binary in your final answer.

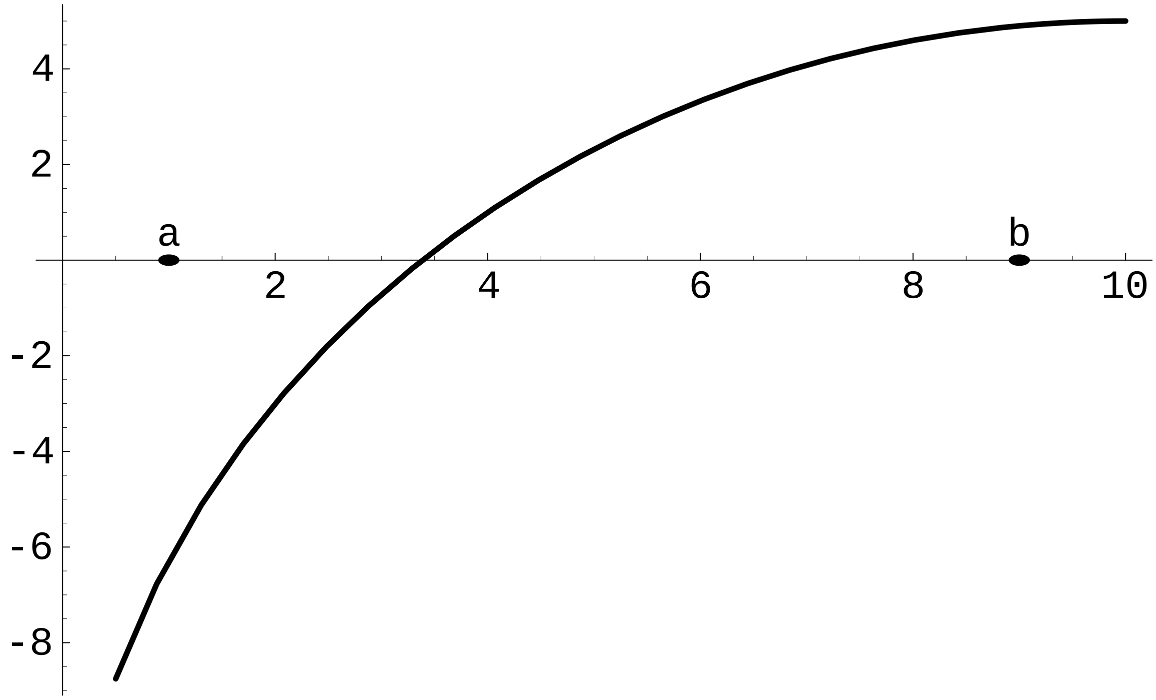
2. (8 points) Some computers do not have a hardware operation for division.

a. Show that one can approximate the inverse square root $c = \frac{1}{\sqrt{a}}$ without doing any divisions by applying Newton's method for solving the equation $f(x) = 0$ with an appropriately selected function $f(x)$.

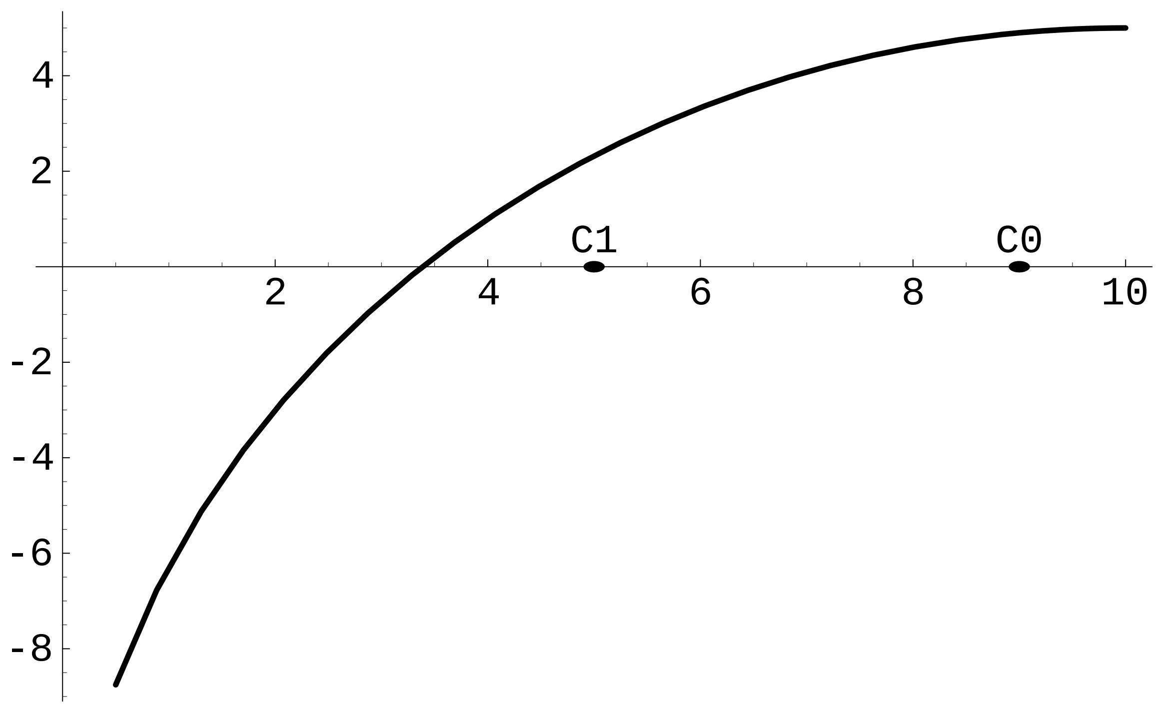
b. Starting with $c_0 = 1$, find the next two iterations for approximating $c = \frac{1}{\sqrt{2}} \approx 0.707$.

3. (4 points)

a. The figure shows a function $f(x)$ and the initial interval $[a, b]$. Sketch the first two iterations of the *regula falsi* method.



b. The figure shows a function $f(x)$ and the initial root estimates c_0 and c_1. Sketch the next two iterations of the secant method.



4. (10 points) The goal of the *Hermite* polynomial interpolation is to construct a polynomial $H(x)$ of order $2n-1$ that satisfies the conditions $H(x_k) = f(x_k)$ and $H'(x_k) = f'(x_k)$ for $k = 1, 2, \ldots, n$. Prove that you can solve the Hermite interpolation problem for two nodes x_1 and x_2 by Newton's interpolation with the divided difference table, where each interpolation node is entered twice:

x	$f[\,]$	$f[\,,\,]$	$f[\,,\,,\,]$	$f[\,,\,,\,]$
x_1	f_1			
x_1	f_1	$f[x_1,x_1] = f'(x_1)$		
x_2	f_2	$f[x_1,x_2]$	$f[x_1,x_1,x_2]$	
x_2	f_2	$f[x_2,x_2] = f'(x_2)$	$f[x_1,x_2,x_2]$	$f[x_1,x_1,x_2,x_2]$

$$H(x) \quad = \quad f_1 + f'(x_1)(x-x_1) + f[x_1,x_1,x_2](x-x_1)^2 + f[x_1,x_1,x_2,x_2](x-x_1)^2(x-x_2)$$

5. (8 points) Interpolate the function $f(x) = \cos(\pi x)$ at the nodes $x_1 = 0$ and $x_2 = 1$ with the cubic *Hermite* polynomial $H(x)$ (see the previous problem for the definition of the Hermite interpolation). Find the absolute error of $H(1/2)$ and $H'(1/2)$.

6. (4 points) A function $S(x)$ defined on the interval $[a,b]$ is a spline of order m if $S(x)$ and all its derivatives up to the order $m-1$ are continuous ($S(x) \in C^{m-1}[a,b]$) and the portion of $S(x)$ on each of the subintervals $[x_k, x_{k+1}]$ is a polynomial of order m ($k = 1,2,\ldots,n-1$ and $a = x_1 < x_2 < \cdots < x_n = b$). How many boundary conditions are necessary for specifying the spline of order m that interpolates $f(x)$ at the nodes $x_1, x_2, \ldots, x_n$?