Math 128A Spring 2002 Handout # 2
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Homework 1: Computer Arithmetics (due on January 31)

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1. The number $\pi = 3.14159265358979\ldots$ If we use the approximation $\pi \approx 3.14$, what is the absolute error? Express your answer using chopping to a decimal normalized floating-point representation with 5 significant digits.

2. The hexadecimal (base 16) counting system uses letters A, B, C, D, E, and F in addition to decimal digits (from 0 to 9) to represent digits for 10, 11, 12, 13, 14, and 15, correspondingly. Find the hexadecimal representation of the decimal number 2989.

3. In the IEEE double-precision floating-point standard, 64 bits (binary digits) are used to represent a real number: 1 bit for the sign, 11 bits for the exponent, and 52 bits for the mantissa. A double-precision normalized non-zero number x can be written in this standard as $x = \pm(1.d_1 d_2 \cdots d_{52})_2 \times 2^{n-1023}$, with $1 \le n \le 2046$, and $0 \le d_k \le 1$ for $k = 1, 2, \ldots, 52$.

   (a) What is the smallest positive number in this system?

   (b) What is the smallest negative number in this system?

   (c) How many real numbers are in this system?

   Note: You may express your answers with formulas. Note that the correct answers are different from the ones in the textbook.

4. (Programming) In this assignment, you will evaluate the accuracy of Stirling's famous approximation

$$n! \approx n^n e^{-n} \sqrt{2\pi n} \,. \tag{1}$$

Write a program to output a table of the form

n	$n!$	Stirling's approximation	Absolute error	Relative error

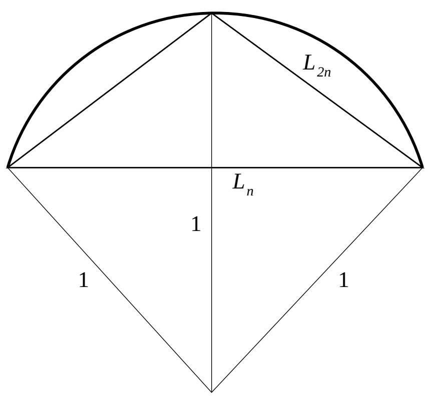
for $n = 1, 2, \ldots, 10$.

Judging from the table, does the accuracy increase or decrease with increasing n?

Hint: If your computer system does not have a predefined value for π, use $\pi =$ `acos(-1.0)` or $\pi =$ `4.0*atan(1.0)`.

5. (Programming) In this assignment, you will compute the number π using an iterative method. An equilateral regular polygon, inscribed in a circle of radius 1, has the perimeter $n\,L_n$, where n is the number of sides of the polygon, and L_n is the length of one side. This can serve as an approximation for the circle perimeter 2π. Therefore, $\pi \approx \frac{n\,L_n}{2}$. From trigonometry (see the figure), a polynomial with twice as many sides, inscribed in the same circle, has the side length

$$L_{2n} = \sqrt{2 - \sqrt{4 - L_n^2}} \,. \tag{2}$$



(a) Write a program to iteratively compute approximations for π using equation (2) and starting from $n = 6$ and $L_6 = 1$ (regular hexagon). You need to do the computations using double precision floating-point numbers. Output a table of the form

n	L_n	Absolute error in approximating π

for $n = 6, 6 \times 2, 6 \times 4, \ldots, 6 \times 2^{20}$.

(b) Use the formula $b - \sqrt{b^2 - a} = \frac{a}{b + \sqrt{b^2 - a}}$ to derive a different form of equation (2).

(c) Modify your program using the new equation and repeat the computation to produce a new table.

(d) Compare the tables and explain the source of the difference.