Chapter 3

Solving for the symmetry

In this chapter I derive a source equalization criterion from a general wave equation and show its relation to other processing techniques. The seismic inverse problem can be cast in many different forms: some purely deterministic, some stochastic, still others in an optimization frame work. In the general elastic wave equation

$$\Box\, \mathbf{u}(\mathbf{r},t) \equiv \rho^{-1}\, \nabla\, \underset{\sim}{\mathbf{C}}(\mathbf{r})\, \nabla^T\, \mathbf{u}(\mathbf{r},t) - \partial_t^2\, \mathbf{u}(\mathbf{r},t) \;=\; \mathbf{s}(\mathbf{r}_0,t)\,, \tag{3.1}$$

$\underset{\sim}{\mathbf{C}}$ is the matrix of elastic parameters (stiffness), ρ the density and $\mathbf{u}$ is a vector wave field. The source $\mathbf{s}$ is activated at location $\mathbf{r}_0$. ∇ and its transpose are spatial derivative matrices. I present a more detailed description and implementation of those operators in Chapter 5. The seismic inverse problem in all generality would have to find unknown elastic parameters $\underset{\sim}{\mathbf{C}}$ and the source function $\mathbf{s}$, given only the knowledge of the seismic data $\mathbf{u}$ at the recording locations. In the following, I abbreviate the process of wave propagation generated by a source $\mathbf{s}$ at location $\mathbf{r}_0$ and the extraction of a wave field at location $\mathbf{r}_1$ with the following notation:

$$\mathbf{L}[\underset{\sim}{\mathbf{C}},\rho,\mathbf{s}](\mathbf{r}_0,\mathbf{r}_1,t) = [\Box\, \mathbf{u}(\mathbf{r},t) - \mathbf{s}(\mathbf{r}_0,t)]_{\mathbf{r}_1} \tag{3.2}$$

where $\mathbf{L}$ is the complete operator which parametrically depends on elastic parameters $\underset{\sim}{\mathbf{C}}$ and the source $\mathbf{s}$, as well as explicitly on source location $\mathbf{r}_0$ and receiver location $\mathbf{r}_1$.

3.1 Processing and optimization

In seismic data inversion one aims to find the best subsurface model and source function that explains observed data. In mathematical terms, the optimization procedure finds a minimum norm (|| ||) solution of the functional

$$\min_{\underset{\sim}{\mathbf{C}},\rho,\mathbf{s}} \; || \, \mathbf{L}[\underset{\sim}{\mathbf{C}}, \rho, \mathbf{s}](\mathbf{r}_0, \mathbf{r}_1, t) - \mathbf{d}(\mathbf{r}_0, \mathbf{r}_1, t) \, || \tag{3.3}$$

based on the given data $\mathbf{d}(\mathbf{r}_0, \mathbf{r}_1, t)$. There are examples of this minimization technique, most prominently by Tarantola (1987) and Mora (1987). However, for practical applications, this technique is too costly and often ill conditioned and, thus, not accepted for routine seismic processing.

Another way of attacking the problem is to split up the estimation process into smaller, more manageable pieces and to make a few simplifying assumptions. The density is in many cases a parameter of secondary influence on the wave propagation and is this often assumed to be constant. Of course, this is may not be true, especially in the vicinity of rocks whose pores contain fluids or gases. For the overall propagation, however, it might be a viable assumption.

Following this strategy, the combined estimation of subsurface and source parameters is reduced to determining one and then the other in turn, not simultaneously. Thus, practically, equation (3.3) is recast into

$$\min_{\mathbf{s}} \; || \, \mathbf{L}[\underset{\sim}{\mathbf{C}}, \rho, \mathbf{s}](\mathbf{r}_0, \mathbf{r}_1, t) - \mathbf{d}(\mathbf{r}_0, \mathbf{r}_1, t) \, || \; , \; \underset{\sim}{\mathbf{C}} \text{ known} \tag{3.4}$$

$$\min_{\underset{\sim}{\mathbf{C}}} \; || \, \mathbf{L}[\underset{\sim}{\mathbf{C}}, \rho, \mathbf{s}](\mathbf{r}_0, \mathbf{r}_1, t) - \mathbf{d}(\mathbf{r}_0, \mathbf{r}_1, t) \, || \; , \; \mathbf{s} \text{ known} \tag{3.5}$$

and both problems are tackled separately. Equation (3.4) describes wavelet and radiation pattern estimation techniques. Numerous techniques exist to determine source wavelets statistically in a seismic data set. Some of the most prominent methods are predictive (Gibson and Larner, 1984) and surface consistent deconvolution as shown by Levin (1987). Directional deconvolution in the F-X domain, as shown by Fokkema et al. (1990) and Roberts and Goulty (1990) emphasize the importance of correcting seismic data for source radiation pattern effects, when one requires increased resolution or one studies amplitude versus offset behavior. Carrion (1990) tries to find the source wavelet's angular spectrum by using a plane wave decomposition technique for precritical reflections and assumes a flat layered earth. Marine source wavelet and radiation pattern estimation is carried out by Keho et al. (1990) and Secrest and Weglein (1990), where a reference medium is assumed and the scattered wave field is removed, resulting in the estimate of an effective source radiation pattern.

Equation (3.5) describes velocity analysis and elastic parameter estimation techniques. The source function is in most cases determined a priori and later used in the estimation procedure. Most standard velocity analyses make no mention of source or receiver radiation patterns and in most cases assume isotropic responses. Standard optimization procedures to determine velocity, such as Toldi (1985) and prestack migration velocity analysis algorithms for arbitrary subsurface structures, as introduced by Al-Yahya (1989) usually neglect source or receiver effects. Migration/Inversion algorithms of seismic data rely on having compensated for source and receiver effects or having applied corrections during the process. Many times those corrections are fixed factors, such as a cosine-like weight for a vertical component and pressure wave type, which is applied throughout the whole data set.

In contrast to equation (3.4) and (3.5), another minimization can be sought: data equalization by reciprocal minimization. It is a somewhat *weaker* minimization at the outset because it determines less from the data, but it is *stronger* in the sense that it does not require knowledge of subsurface parameters $\underset{\sim}{\mathbf{C}}$ or source wavelet $\mathbf{s}$. In the following equation (3.6) $\mathbf{L}^R$ is the reciprocal operator of $\mathbf{L}$. The method consists of two parts, one the original experiment generating data $\mathbf{d}(\mathbf{r}_0, \mathbf{r}_1, t)$ from a source at $\mathbf{r}_0$ and recording at $\mathbf{r}_1$, second an experiment with source and recording position interchanged, generating data $\mathbf{d}^R(\mathbf{r}_0, \mathbf{r}_1, t)$. The following expression

$$|| \; \mathbf{L}[\underset{\sim}{\mathbf{C}}, \rho, \mathbf{s}](\mathbf{r}_0, \mathbf{r}_1, t) - \mathbf{d}(\mathbf{r}_0, \mathbf{r}_1, t) - \Big(\mathbf{L}^R[\underset{\sim}{\mathbf{C}}, \rho, \mathbf{s}](\mathbf{r}_1, \mathbf{r}_0, t) - \mathbf{d}^R(\mathbf{r}_1, \mathbf{r}_0, t)\Big) \; || \tag{3.6}$$

minimizes the difference between those two experiments. If there is no difference, it means that the wave propagation operators and particularly the sources and receivers behaved identically at reciprocal locations and that the data are in perfect agreement. As described in more detail in Chapter 4, operator $\mathbf{L}$ and its reciprocal $\mathbf{L}^R$ are theoretically identical if their elastic parameters $\underset{\sim}{\mathbf{C}}, \rho$ and source $\mathbf{s}$ are identical.

If one indeed finds that differences exist, one has to realize that propagation operators were not identical for the two experiments. That could be due to a seismic noise source or to changes in the medium or to variations in source or receiver properties. In general, noise sources are not reciprocal, because their location, frequency content, and timing change in a non-reciprocal manner, when for instance a truck drives by or there is wind noise. Subsurface medium properties usually do not change while the experiment is carried out. However, near surface properties might exhibit change during an experiment, for example, when the rainfall sets in between a recording and a reciprocal recording. Then near-surface conditions will change, while deeper subsurface conditions remain constant. This change in the near-surface medium parameters is tightly coupled to changes in source and receiver properties. In fact, they might be inseparable, since the immediate near-surface determines the quality of source and receiver coupling, including frequency characteristic and radiation pattern changes. Consequently, the operator **L** and its reciprocal can be parameterized in various ways. As an approximation, I chose to parameterize changes in those operators with convolutional filters and explicitly find filter coefficients by a conjugate gradient minimization algorithm. This minimization procedure essentially removes the reciprocal average from the data in order to minimize residual differences. To know more about behavior of the minimization solution, I cast the data equalization as a general inverse problem and study it in the following section numerically for a small sample experiment.

3.2 Problem structure

In this section I am analyzing the problem of source equalization that is based on minimizing differences of reciprocal trace pairs. I outline the problem structure and the qualitative behavior of solutions by treating it as a generalized inverse problem.

3.2.1 Algorithmic considerations

When there is mismatch between a data trace and its reciprocal mostly resulting from source variations, differences can be described by convolution with an equivalent source radiation pattern

$$\mathbf{w}(\mathbf{s},\mathbf{r},t) * \mathbf{D}_{\mathbf{X}\mathbf{x}}(\mathbf{s},\mathbf{r},t) = \mathbf{r}(\mathbf{r},\mathbf{s},t) * \mathbf{D}_{\mathbf{x}\mathbf{X}}(\mathbf{r},\mathbf{s},t)\ , \tag{3.7}$$

where $\mathbf{w}$ is the radiation pattern at the source location, $\mathbf{r}$ is the radiation pattern at the reciprocal location. The radiation patterns are, in general, dependent on time frequency and take-off angles. I denote the convolution in those domains with "$*$". $\mathbf{w}$ and $\mathbf{r}$ are, generally, vector filters, which act on each of the recorded components. In matrix notation, this equation (3.7) becomes

$$\begin{pmatrix} w_{Xx} & w_{Xy} & w_{Xz} \\ w_{Yx} & w_{Yy} & w_{Yz} \\ w_{Zx} & w_{Zy} & w_{Zz} \end{pmatrix} \circ \begin{pmatrix} d_{Xx} & d_{Xy} & d_{Xz} \\ d_{Yx} & d_{Yy} & d_{Yz} \\ d_{Zx} & d_{Zy} & d_{Zz} \end{pmatrix} = \begin{pmatrix} r_{Xx} & r_{Xy} & r_{Xz} \\ r_{Yx} & r_{Yy} & r_{Yz} \\ r_{Zx} & r_{Zy} & r_{Zz} \end{pmatrix} \circ \begin{pmatrix} d_{xX} & d_{yX} & d_{zX} \\ d_{xY} & d_{yY} & d_{zY} \\ d_{xZ} & d_{yZ} & d_{zZ} \end{pmatrix} \tag{3.8}$$

where $\circ$ denotes element-wise operation. To simplify analysis, radiation patterns can be expanded into a truncated Taylor series which is only dependent on frequency if limited-angle reflections are considered. With this simplifying assumption, one can set up a system of equations to solve for the unknown source radiation patterns $\mathbf{w}$ and $\mathbf{r}$.

3.2.2 Log - spectral domain

I pose the estimation the filters in the log-spectral domain to more clearly delineate the structure of the problem. Transforming equation (3.7) into the frequency domain and taking the natural logarithm results in

$$\mathbf{W}(\mathbf{s},\mathbf{r},\omega) - \mathbf{R}(\mathbf{r},\mathbf{s},\omega) = \mathbf{D}_{Xx}(\mathbf{s},\mathbf{r},\omega) - \mathbf{D}_{xX}(\mathbf{r},\mathbf{s},\omega)\,. \tag{3.9}$$

Transformation into the logarithmic spectral domain is useful for studying the problem and its solutions. This domain is, in general, not recommended for the actual filter design because of the distortion of additive noise. The log operation weighs small numbers in the data set, such as ambient noise, much heavier than the actual signal. For each location where there exists a fully reciprocal trace pair, there is an equation with the filters $\mathbf{W}$ and $\mathbf{R}$ as unknown and the trace pairs as known quantities. The resulting linear system of equations is given in the log-spectral domain by

$$\mathbf{A}\ \mathbf{m} = \mathbf{b} \tag{3.10}$$

where $\mathbf{A}$ is a sparse and banded matrix, such as given in equation (3.12), $\mathbf{m} = [w, r]$ is the vector of the unknown filter coefficients, and the elements of $\mathbf{b}$ are the differences between the trace and its reciprocal in the log-spectral domain.

The solution to system (3.10) is formally given by

$$\mathbf{m} = \mathbf{A}^{\dagger}\mathbf{b}\,, \tag{3.11}$$

where $\mathbf{A}^{\dagger}$ is the pseudo-inverse of the system of equations. Singular value decomposition of $\mathbf{A}$ shows that there exists a well-constrained but non-unique solution. The non-uniqueness

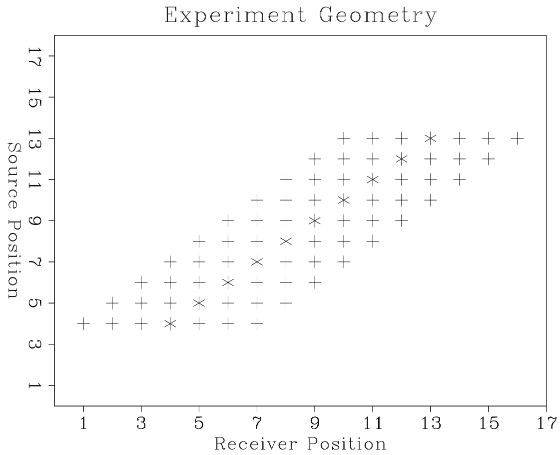

FIG. 3.1. This plot outlines the shooting geometry for 10 shots with 7 traces recorded each shot, and is used for the example problem. Equation (3.12) reflects in its structure this source and receiver layout, in that it is sparse and banded. solving-sgplot [R]

lets one estimate only relative reciprocal trace pair differences, not absolute ones. A is a sparse structured matrix as in the case of 10 shots with 7 traces per shot, outlined in Figure 3.1. The figure shows surface location coordinates for source and receivers in a numerical experiment. The source location denoted with a "$*$" is sandwiched between three receiver locations on either side. Receivers are denoted by a "$+$". The first shot starts at source location four and the receiver spread extends from surface location one to seven. Subsequent shots are shifted by exactly one spatial unit to the next location. Such an experiment geometry results in the structure of the corresponding matrix $\mathbf{A}$, as given in equation (3.12). The symbol "$\cdot$" in the matrix A means a zero entry for that element. Therefore it is extremely sparse. Each row in the matrix $\mathbf{A}$ determines one reciprocal trace pair, and the position of the factors 1 and -1 corresponds to the locations of the trace and its reciprocal. One sees that typically there are more equations than unknown filters. That means, that the system of equations is usually over-determined. This redundancy is beneficial in that it gives a more reliable estimate under noisy conditions. The redundancy is easily determined by merely counting all the 1 or -1 appearing in a certain column. At the edges of the experiment redundancy tapers off, while in the middle, it remains constant.

$$\mathbf{A} = \begin{pmatrix}
1 & . & . & . & . & . & . & . & . & . & . & -1 & . & . & . & . & . & . & . & . \\
1 & . & . & . & . & . & . & . & . & . & . & . & -1 & . & . & . & . & . & . & . \\
1 & . & . & . & . & . & . & . & . & . & . & . & . & -1 & . & . & . & . & . & . \\
. & 1 & . & . & . & . & . & . & . & . & -1 & . & . & . & . & . & . & . & . & . \\
. & 1 & . & . & . & . & . & . & . & . & . & . & -1 & . & . & . & . & . & . & . \\
. & 1 & . & . & . & . & . & . & . & . & . & . & . & -1 & . & . & . & . & . & . \\
. & 1 & . & . & . & . & . & . & . & . & . & . & . & . & -1 & . & . & . & . & . \\
. & . & 1 & . & . & . & . & . & . & . & -1 & . & . & . & . & . & . & . & . & . \\
. & . & 1 & . & . & . & . & . & . & . & . & -1 & . & . & . & . & . & . & . & . \\
. & . & 1 & . & . & . & . & . & . & . & . & . & . & -1 & . & . & . & . & . & . \\
. & . & 1 & . & . & . & . & . & . & . & . & . & . & . & -1 & . & . & . & . & . \\
. & . & 1 & . & . & . & . & . & . & . & . & . & . & . & . & -1 & . & . & . & . \\
. & . & . & 1 & . & . & . & . & . & . & -1 & . & . & . & . & . & . & . & . & . \\
. & . & . & 1 & . & . & . & . & . & . & . & -1 & . & . & . & . & . & . & . & . \\
. & . & . & 1 & . & . & . & . & . & . & . & . & -1 & . & . & . & . & . & . & . \\
. & . & . & 1 & . & . & . & . & . & . & . & . & . & . & -1 & . & . & . & . & . \\
. & . & . & 1 & . & . & . & . & . & . & . & . & . & . & . & -1 & . & . & . & . \\
. & . & . & 1 & . & . & . & . & . & . & . & . & . & . & . & . & -1 & . & . & . \\
. & . & . & . & 1 & . & . & . & . & . & . & -1 & . & . & . & . & . & . & . & . \\
. & . & . & . & 1 & . & . & . & . & . & . & . & -1 & . & . & . & . & . & . & . \\
. & . & . & . & 1 & . & . & . & . & . & . & . & . & -1 & . & . & . & . & . & . \\
. & . & . & . & 1 & . & . & . & . & . & . & . & . & . & . & -1 & . & . & . & . \\
. & . & . & . & 1 & . & . & . & . & . & . & . & . & . & . & . & -1 & . & . & . \\
. & . & . & . & 1 & . & . & . & . & . & . & . & . & . & . & . & . & -1 & . & . \\
. & . & . & . & . & 1 & . & . & . & . & . & . & -1 & . & . & . & . & . & . & . \\
. & . & . & . & . & 1 & . & . & . & . & . & . & . & -1 & . & . & . & . & . & . \\
. & . & . & . & . & 1 & . & . & . & . & . & . & . & . & -1 & . & . & . & . & . \\
. & . & . & . & . & 1 & . & . & . & . & . & . & . & . & . & . & -1 & . & . & . \\
. & . & . & . & . & 1 & . & . & . & . & . & . & . & . & . & . & . & -1 & . & . \\
. & . & . & . & . & 1 & . & . & . & . & . & . & . & . & . & . & . & . & -1 & . \\
. & . & . & . & . & . & 1 & . & . & . & . & . & . & -1 & . & . & . & . & . & . \\
. & . & . & . & . & . & 1 & . & . & . & . & . & . & . & -1 & . & . & . & . & . \\
. & . & . & . & . & . & 1 & . & . & . & . & . & . & . & . & -1 & . & . & . & . \\
. & . & . & . & . & . & 1 & . & . & . & . & . & . & . & . & . & . & -1 & . & . \\
. & . & . & . & . & . & 1 & . & . & . & . & . & . & . & . & . & . & . & -1 & . \\
. & . & . & . & . & . & 1 & . & . & . & . & . & . & . & . & . & . & . & . & -1 \\
. & . & . & . & . & . & . & 1 & . & . & . & . & . & . & -1 & . & . & . & . & . \\
. & . & . & . & . & . & . & 1 & . & . & . & . & . & . & . & -1 & . & . & . & . \\
. & . & . & . & . & . & . & 1 & . & . & . & . & . & . & . & . & -1 & . & . & . \\
. & . & . & . & . & . & . & 1 & . & . & . & . & . & . & . & . & . & . & -1 & . \\
. & . & . & . & . & . & . & 1 & . & . & . & . & . & . & . & . & . & . & . & -1 \\
. & . & . & . & . & . & . & . & 1 & . & . & . & . & . & . & -1 & . & . & . & . \\
. & . & . & . & . & . & . & . & 1 & . & . & . & . & . & . & . & -1 & . & . & . \\
. & . & . & . & . & . & . & . & 1 & . & . & . & . & . & . & . & . & -1 & . & . \\
. & . & . & . & . & . & . & . & 1 & . & . & . & . & . & . & . & . & . & . & -1 \\
. & . & . & . & . & . & . & . & . & 1 & . & . & . & . & . & . & -1 & . & . & . \\
. & . & . & . & . & . & . & . & . & 1 & . & . & . & . & . & . & . & -1 & . & . \\
. & . & . & . & . & . & . & . & . & 1 & . & . & . & . & . & . & . & . & -1 & .
\end{pmatrix} \tag{3.12}$$

To see how this structure affects the solution vector **m**, I perform a singular value decomposition of this matrix. One finds, as Figure 3.2 shows, that even in the case of having all reciprocal trace pairs available, the solution will be non-unique. At least one of the singular values is zero while the remaining singular values are comparable in magnitude. The singular values are not normalized to unity, since matrix A was not normalized to start with. The values are ordered from highest to lowest and directly correspond to the ordering of the eigenvectors, shown in figures 3.3 and 3.4. Figure 3.3 shows the plots of the normalized right hand eigenvectors. This result is very similar to the one obtained by Wiggins et al. (1976) in which the solution of the statics problems was treated as a generalized inverse problem. Right eigenvectors are associated with the model space **m**, while left eigenvectors are associated with the data space **b**. Since I want to analyze the solution behavior, I concentrate on the right singular vectors.

The right singular vectors all have no pure DC component, but are symmetric or anti-symmetric around the center. The solution, then, is determined within that reciprocal cable length, but not beyond. Fluctuations which are on a spatial wave length longer than that length are not detectable, but are in the null space of the problem. In particular, a true DC component cannot be determined. Consequently, absolute differences are not resolvable, but only display relative differences in the reciprocal trace pairs.

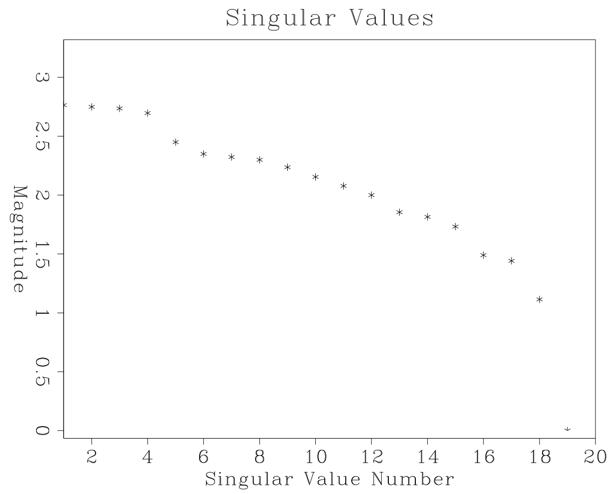

FIG. 3.2. The source equalization problem mathematically presented in equation (3.12) can be numerically analyzed using singular value decomposition (SVD). At least one singular value of zero exists in the decomposition. It happens that this zero singular value corresponds to the DC singular vector. solving-singular [R]

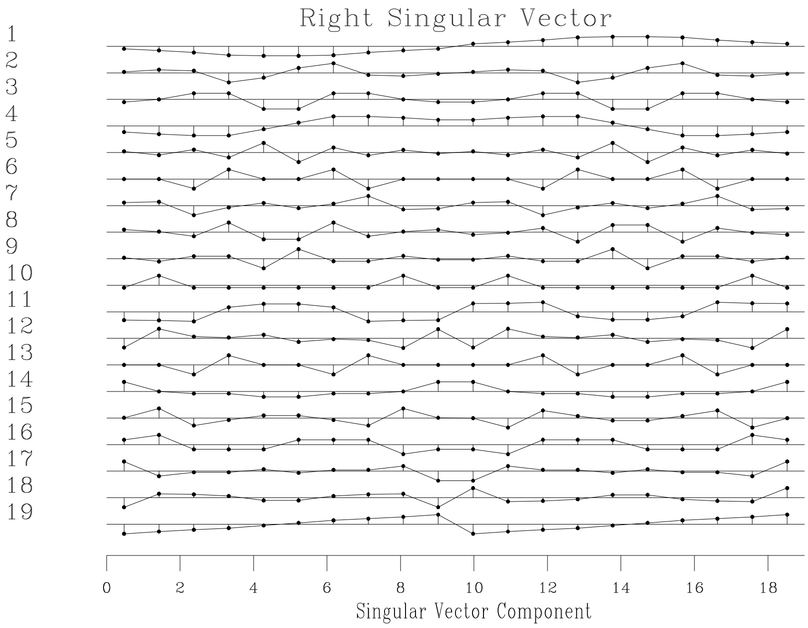

FIG. 3.3. The source equalization problem presented in equation (3.12) can be numerically analyzed using singular value decomposition. The right singular vectors all have no pure DC component, but are symmetric or anti-symmetric around the center. solving-right [R]

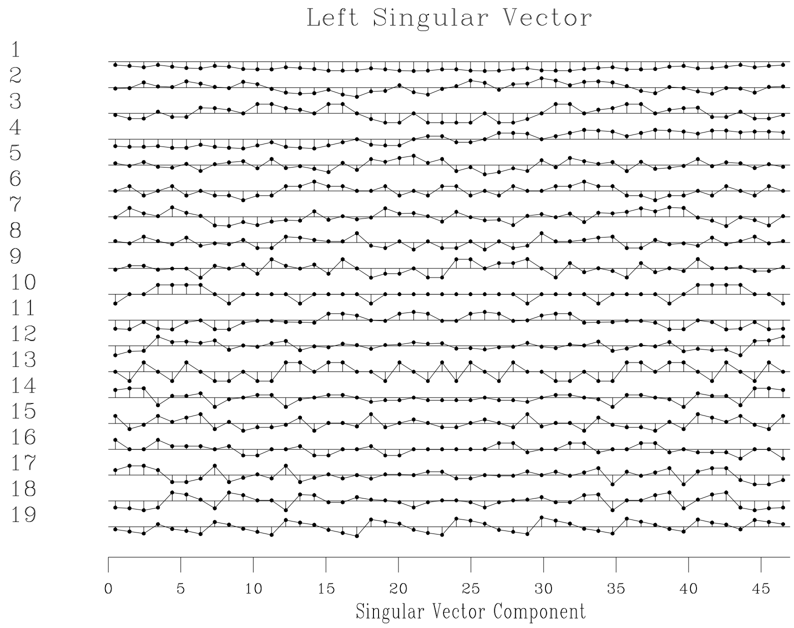

FIG. 3.4. The source equalization problem presented in equation (3.12) can be numerically analyzed using singular value decomposition. For completeness, the left singular vectors are shown here. solving-left [R]

3.3 Relation to other methods

In order to relate this source and receiver equalization by reciprocal difference minimization to other data processing methods, such as statics estimation or surface consistent deconvolution, I introduce here a data model that is usually used in conjunction with those approaches. I would like to stress that it is not inherent to the source equalization scheme, but rather serves the purpose of highlighting differences and similarities between those methods.

3.3.1 A useful data model

A seismic record can be described as a sequence of operators applied to a reflective subsurface medium. I am following the notation of Claerbout (1985) where "wav" is the operation for convolving (in multiple space and the time dimension) the data with a wavelet.

On its way to the earth surface, an intended wavelet is filtered by various mechanisms. The source device itself will filter the wavelet and one can model this behavior by applying the device transfer function $(\mathrm{I} - \mathrm{dev})^{-1}$. The nonlinear interaction of the device with the free surface of the earth comes next. Within a small region of the source location wave propagation will not be linear. After a critical distance the propagation is linear again, but carries with it its nonlinear history. This can be regarded as the creation of a source radiation pattern. Describing the resulting effective radiation with a convolutional filter, we convolve the model trace further with the wavelet $(\mathrm{I} - \mathrm{nonl})^{-1}$ as a macro description for the equivalent radiation pattern in the far-field. These effects are independent of what is called near-surface effects, which is mainly wave propagation in the weathering layer, with effects such as static shifts or reverberations. These effects are described by the operation with the wavelet $(\mathrm{I} - \mathrm{near})^{-1}$. Wave propagation proceeds and is modeled by $(\mathrm{I} - \mathrm{prop})^{-1}$, the transfer function of the earth in the far-field. At location 2 one roughly reverses the above described sequence. The end result is then the recorded trace, when one splits the propagation into a portion within the near surface layer, ground coupling and geophone transfer. The model equation for this trace is

$$\begin{aligned}&(\mathrm{I} - \mathrm{rec_r})^{-1}\ (\mathrm{I} - \mathrm{coup_r})^{-1}\ (\mathrm{I} - \mathrm{near_r})^{-1}\ (\mathrm{I} - \mathrm{prop})^{-1}\\ &(\mathrm{I} - \mathrm{near_s})^{-1}\ (\mathrm{I} - \mathrm{nonl_s})^{-1}\ (\mathrm{I} - \mathrm{dev_s})^{-1}\ (\mathrm{I} - \mathrm{wav_s})^{-1}\quad \mathrm{refl}\ =\ \mathrm{data},\end{aligned} \tag{3.13}$$

where the indices s and r are source and receiver location, respectively. A reciprocal trace is recorded if source and receiver location are interchanged. This convolutional model can be used for scalar as well as vector data and it describes a completely noise-free propagation.

This convolutional model tries to describe wave propagation as the successive application of several operators to the original subsurface reflectivity model with a given source wavelet.

3.3.2 Source estimation and statics

The simplest subsurface correction is the time shift; the simplest source correction is a gain change. Most statics programs generate shot and geophone corrections that are not constrained to be the same. They are shot and geophone consistent, but not, properly speaking, surface consistent. This is, of course, because they are intended to correct not only for subsurface effects but also for differences in the shot wavelets. In fact, it would make sense if statics programs distinguished explicitly between their functions and reported out independently the near-surface corrections and shot corrections. This will be particularly important in the case of vector field processing, when source effects must be kept quite separate from subsurface effects because they represent quite different models. The source waveform can truly be represented as a convolution over time and the surface coordinates, while for the subsurface, convolution is an approximation to a medium replacement scheme.

A common procedure for increasing the stack power of CDP data is to approximate the convolutional filters in Equation (3.13) by simple time shifts. In residual statics analysis one tries to compensate primarily for time shifts caused by the near-surface weathering layer. These delays occur near the source and receiver location. In order to estimate these time anomalies, one usually assumes that the seismic reflection times are composed of a sum of surface consistent source and receiver static terms, and structural terms, such as normal incidence two-way travel time and residual normal moveout. Irregardless the procedure used to estimate relative consistent time shifts, traveltime picks (Wiggins et al., 1976) or stack power (Ronen and Claerbout, 1985), one has to assume the structure as known, i.e, the velocity model to be given. In this case, we take the operators **prop** as given and approximate $\mathbf{near_s}$ and $\mathbf{near_r}$ by simple time shifts. The radiation pattern is assumed not to influence the estimation procedure. No assumptions are made about the remaining convolutional operators. A key assumption, then, is that the cross correlation of two reflection events is maximized when the events are aligned. This holds true only, if the two events have the same wavelet attached to them. If the wavelets differ, we have to appeal to surface consistent spectral balancing in some form.

3.3.3 Source estimation and surface consistent deconvolution

Surface consistent deconvolution is usually employed to balance the spectra of seismic traces, thus suppressing effects which are not due to changes in the reflectivity series of the earth. In light of the convolutional representation in Equation (3.13), surface consistent deconvolution lumps together the effects of the individual convolutions. It tries to factor out wavelets which are consistent with source and receiver location, thus simplifying Equation (3.13) to

$$(\mathrm{I}-\mathrm{g})^{-1}\ (\mathrm{I}-\mathrm{s})^{-1}\ \mathrm{refl}\ =\ \mathrm{data}. \tag{3.14}$$

s is a filter which removes source inconsistent effects from the data trace, while g removes inconsistent effects at the receiver location. These inconsistencies are deduced from the data itself. Usually, in surface consistent deconvolution, information about the subsurface structure is not used, although there are applications such as simultaneous pre-normal and post-normal moveout deconvolution (Ronen and Claerbout, 1985) which try to incorporate information about the nature of inconsistencies. Deconvolution is based on optimizing some objective function; nothing is assumed about the physics leading to inconsistencies. A surface consistent prediction error filter would minimize the norm of f:

$$\mathrm{f}_{\mathrm{ij}}\ =\ (\mathrm{I}-\mathrm{s}_{\mathrm{i}})\ (\mathrm{I}-\mathrm{g}_{\mathrm{j}})\ \mathrm{d}_{\mathrm{ij}} \tag{3.15}$$

where i is a unique receiver index and j a unique shot index. If there are no other inconsistencies other than discrepancies produced by the source wavelet and geophone transfer function, surface consistent deconvolution gives an accurate estimation of both.

3.3.4 Formal similarity

The formally mathematical similarity of source equalization with surface-consistent deconvolution or statics estimation, appears in the structure of the problem. In the above convolutional data model the relationship between the data and its reciprocal is, under the assumption of perfect receivers, given by:

$$(\mathrm{I}-\mathrm{nonl}_{\mathrm{s}})(\mathrm{I}-\mathrm{nonl}_{\mathrm{r}})^{-1}\mathrm{recip}=\mathrm{data}. \tag{3.16}$$

In actual filter design this amounts to minimizing an objective function:

$$\mathrm{f_{ij}} \;=\; (\mathrm{T} - \mathrm{nonl_i})\;(\mathrm{T} - \mathrm{nonl_j})^{-1}\;\mathrm{d_{ij}}. \tag{3.17}$$

Here T denotes the reciprocity operator, nonl is an operator with indices i and j for source and receiver location, respectively.

3.4 Summary

Data equalization based on reciprocal minimization can be treated as a general inverse problem. I analyze its solution behavior using a standard numerical technique (SVD), which shows that the over-determined system of equations can give well-conditioned solutions, but they will be non-unique. Only relative differences can be resolved within the reciprocal length scale. This equalization technique is formally similar to predictive deconvolution and statics problems; however, it does not try to shape the wavelets in the data set. In contrast to subsurface parameter estimation or explicit source function determination, this data equalization needs no a priori assumptions, except a (nearly) reciprocal acquisition geometry and it thus can be used as a data preprocessing method.