Chapter 1

Multidimensional deconvolution examples

Deconvolution is a well-known process with deep and complex theoretical underpinnings. Here we'll begin with a superficial view of the theory and then jump to some examples in two-dimensional space.

One definition of science that distinguishes it from other views of the human condition is that science is those studies that offer demonstrable predictive power. That direction brings us too far afield. Never-the-less, science can be formalized as the construction of predicted data that can be compared with observed data. This requires a theory. Oft times, we really don't have a theory that we have much confidence in. Fortunately, there is an "all-purpose" theory of statistical signal processing that often allows us some predictability. Basically, it goes like this.

We have some data. From this data we can estimate a spectrum. By assuming the spectrum remains constant in time or space (stationary assumption), we have a certain predictive power. How much power this amounts to in practice depends on lots of things we won't examine in this brief introduction beyond saying that it depends a lot on both the observed data and the validity of the stationary assumption.

It often happens that the item of interest is not the prediction, but the error in the prediction. For example, we might be given the history of the stock market upon which to estimate the spectrum. Then we might be asked to predict (for each moment in time), the market price tomorrow based on today's price and all previous prices. The display that is often of interest is the error in prediction, did we overshoot or undershoot? Theoretically, it turns out that the prediction error is spectally white. If it had any other spectrum, that would imply suboptimum prediction. This leads to the simplest definition of prediction error, the definition we will adopt here. (The examples here come from one of my other books (GEE). These examples are based not on spectra, but on actual prediction error, but we will not examine the distinction, a distinction that is small compared to the essential ideas discussed here.)

1.1 2-D MODELING AND DECON

This chapter shows many examples to give some feeling for multidimensional spectra, deconvolution, and modeling. In each of Figures 1.2-1.7 we see three panels. The left panel is a two-dimensional data set which is an image of something that might be familiar. Let this data $d(x,y)$ have **Fourier Transform** $D(k_x,k_y)$.

Define noise $n(x,y)$ to be a plane of random numbers. In Fourier space, these numbers are $N(k_x,k_y)$. Now compute an amplitude spectrum by smoothing $A = \sqrt{\overline{N(k_x,k_y)}N(k_x,k_y)}$. Using this noise we compute the synthetic data in the center frame. It has the 2-D spectrum $N \times A$ which is the same 2-D *amplitude* spectrum of the given data but a different *phase* spectrum.

For each illustration, notice the similarities and differences between the real data and the synthetic data.

The rightmost frame has Fourier Transform D/A, the original data with its spectrum divided out. What we see is called "deconvolved data". It is also called "**prediction error**". Theoretically, its output is "white". Actually its spectrum is only approximately white because the spectrum is smoothed before it is divided out. (The name prediction error derives from a simple model that can be applied to all data. This simple model is that all data can be modeled as white noise into a filter where the filter has a spectrum that is a smoothed version of that of the data.)

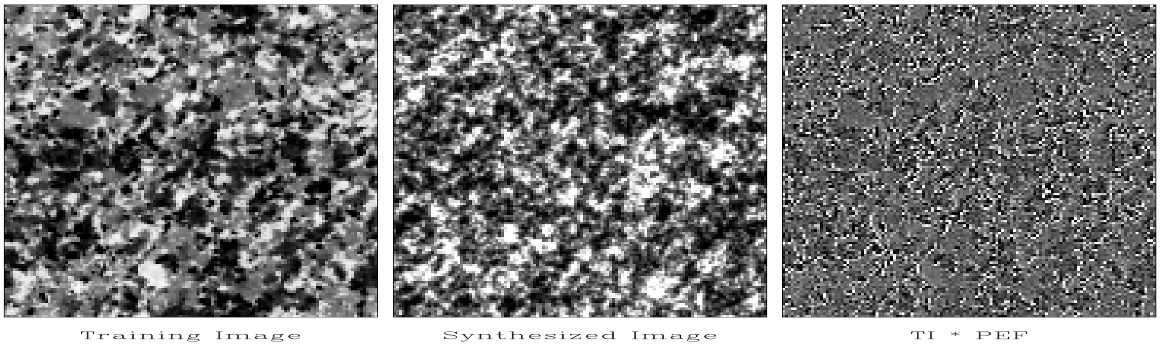

Figure 1.1: Synthetic granite matches the training image quite well. The prediction error is large at crystal grain boundaries and almost seems to outline the grains. $\boxed{\text{mda-granite}}$ [NR]

1.2 DATA RESTORATION

Figures 1.8-1.11 demonstrate the success and lack of success of a process to replace missing data by data of the correct spectrum. Although real data is always better than synthetic data, especially synthetic data produced with a primitive theory involving only spectra, the fact is that synthetic data is often better than zero data.

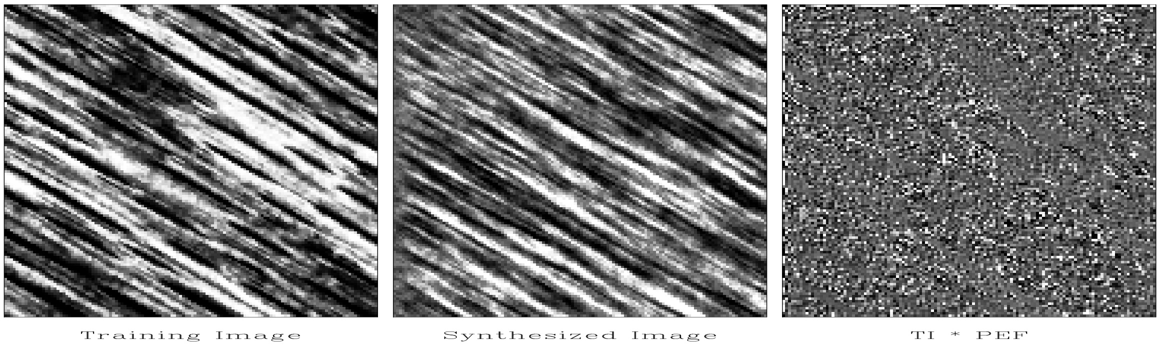

Figure 1.2: Synthetic wood grain has too little white. This is because of the nonsymmetric brightness histogram of natural wood. Again, the deconvolution looks random as expected. mda-wood [NR]

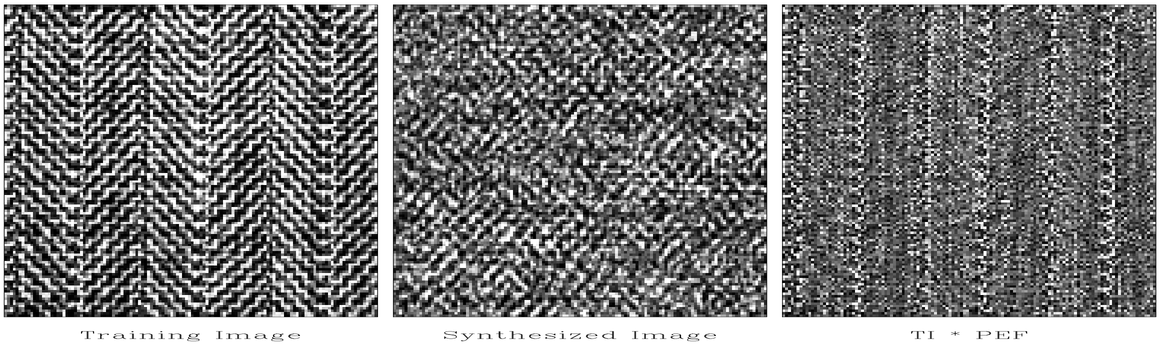

Figure 1.3: A banker's suit (left). A student's suit (center). My suit (right). The deconvolution is large where the weave changes direction (herring bone spine). mda-herr [NR]

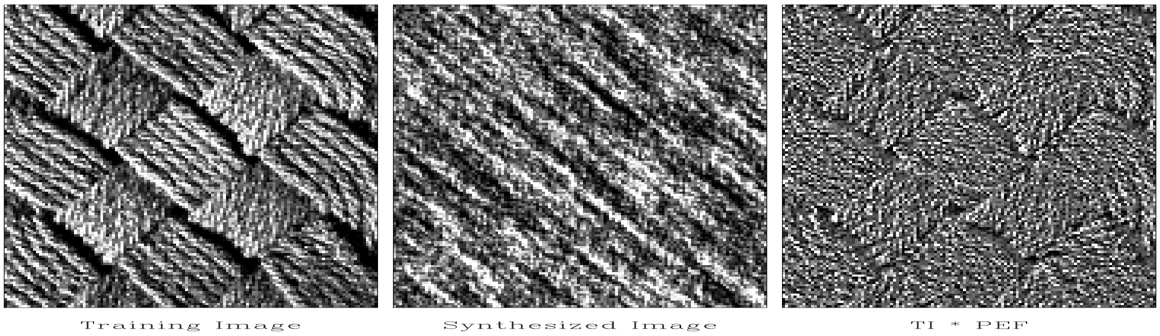

Figure 1.4: Basket weave. The synthetic data fails to segregate the two dips into a checkerboard pattern. The deconvolution looks structured. mda-basket [NR]

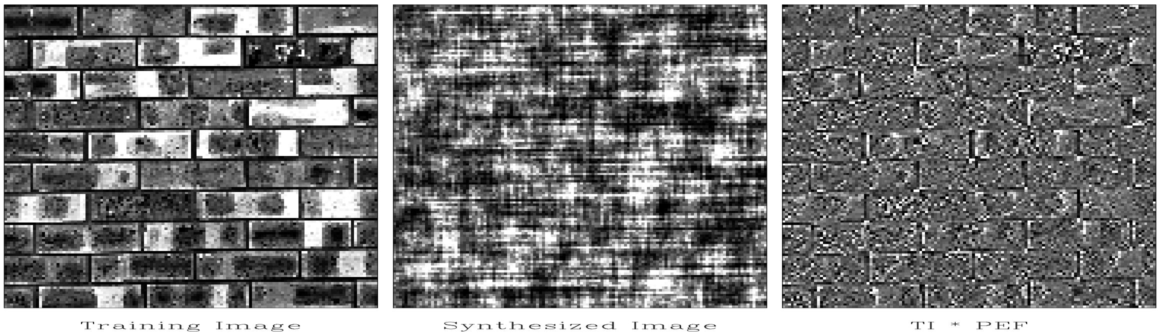

Figure 1.5: Brick. Synthetic brick edges are everywhere and do not enclose blocks containing a fixed color. PEF output highlights the mortar. mda-brick [NR]

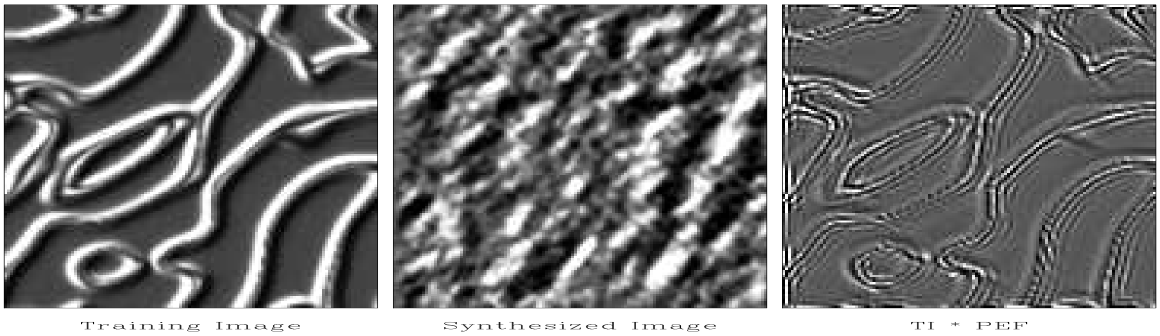

Figure 1.6: Ridges. A spectacular failure of the stationarity assumption. All dips are present but in different locations. The ridges have been sharpened by the deconvolution. mda-ridges [NR]

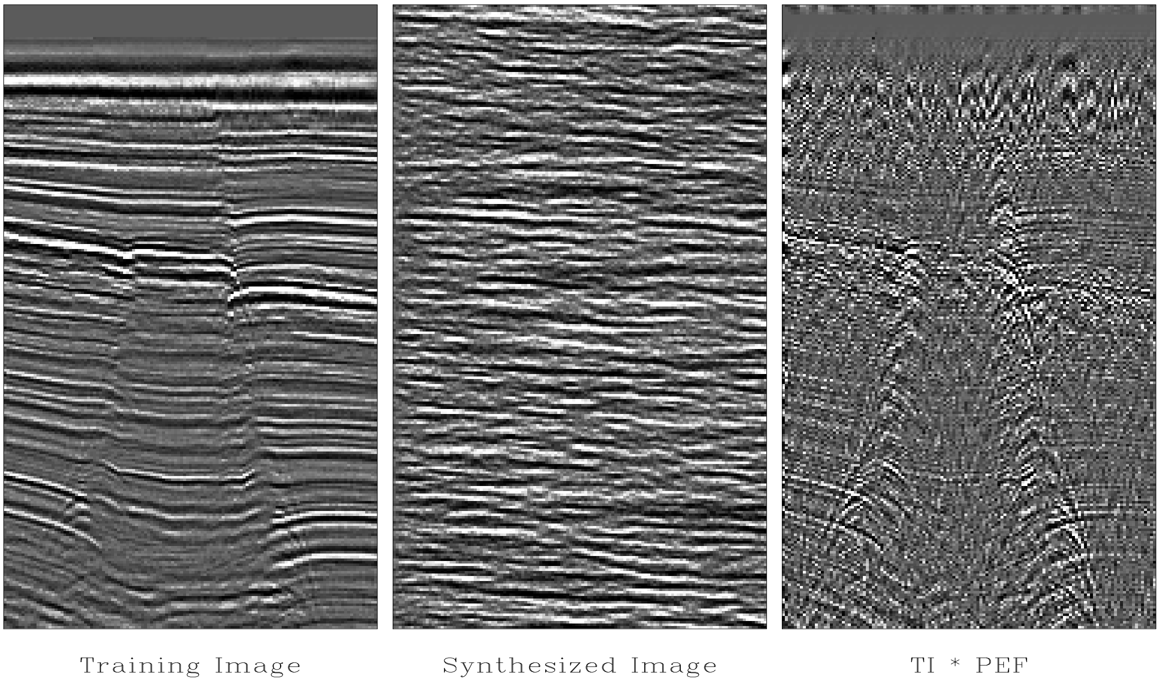

Figure 1.7: Gulf of Mexico seismic section, modeled, and deconvolved. Do you see any drilling prospects in the synthetic data? The deconvolution suppresses the strong horizontal layering giving a better view of the hyperbolas. mda-WGstack [NR]

In geophysical parameter fitting (inversion) you would not want to fit parameters to synthetic data, but in imaging processes like migration, synthetic data is better than zero data. This would be true for Figure 1.11.

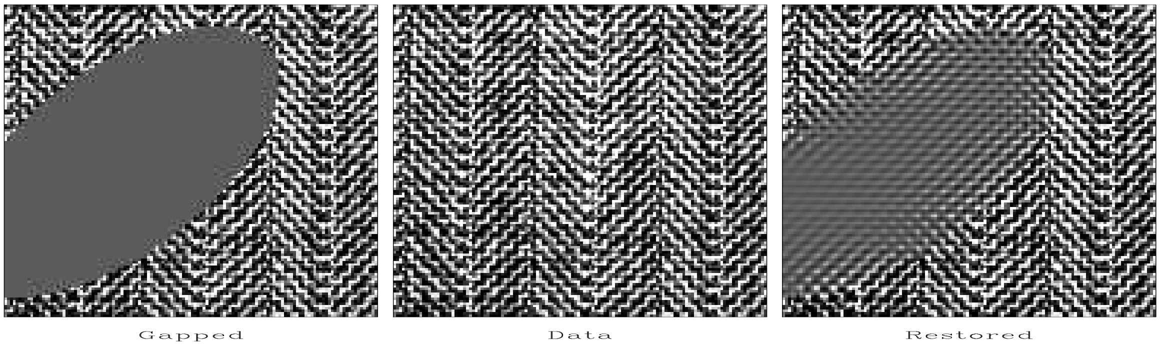

Figure 1.8: The herringbone texture is a patchwork of two textures. We notice that data missing from the hole tends to fill with the texture at the edge of the hole. The spine of the herring fish, however, is not modeled at all. $\boxed{\text{mda-herr-hole-fill}}$ [NR,M]

Figure 1.9: The brick texture has a mortar part (both vertical and horizontal joins) and a brick surface part. These three parts enter the empty area but do not end where they should. $\boxed{\text{mda-brick-hole-fill}}$ [NR,M]

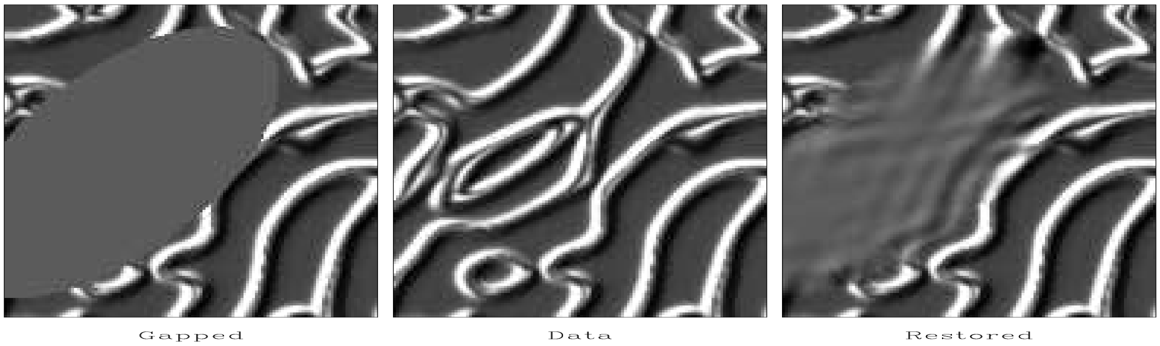

Figure 1.10: The theoretical model is a poor fit to the ridge data since the prediction must try to match ridges of all possible orientations. This data requires a broader theory which incorporates the possibility of nonstationarity (space variable slope). $\boxed{\text{mda-ridges-hole-fill}}$ [NR,M]

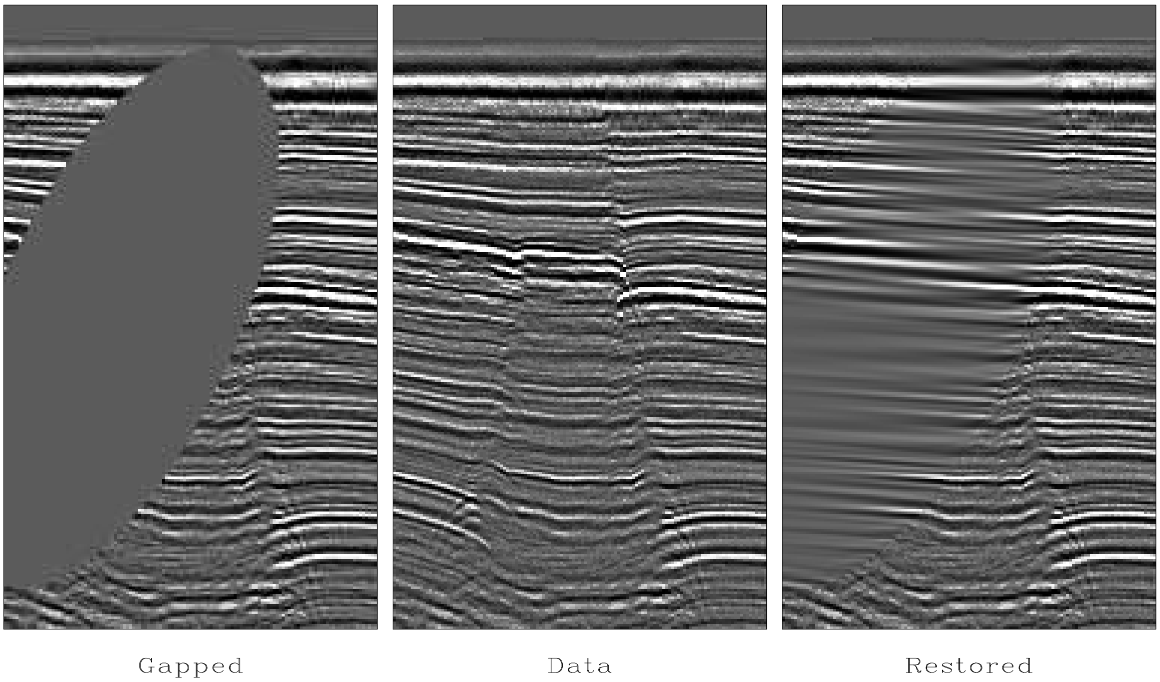

Figure 1.11: Filling the missing seismic data. The imaging process known as "migration" would suffer diffraction artifacts in the gapped data that it would not suffer on the restored data. mda-WGstack-hole-fill [NR,M]

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