Chapter 1

Kirchhoff migration

Migration is the basic image making process in reflection seismology. A simplified conceptual basis is (1) the superposition principle, (2) Pythagoras theorem, and (3) a subtle and amazing analogy called the **exploding reflector concept**. After we see this analogy we'll look at the most basic **migration** method, the Kirchhoff method.

1.1 THE EXPLODING REFLECTOR CONCEPT

Figure 1.1 shows two wave-propagation situations. The first is realistic field sounding. A shot s and a receiving geophone g attached together go to all places on the earth surface and record for us the echo function of t. The second is a thought experiment in which the reflectors in the earth suddenly explode. Waves from the hypothetical explosion propagate up to the earth's surface where they are observed by a hypothetical string of geophones.

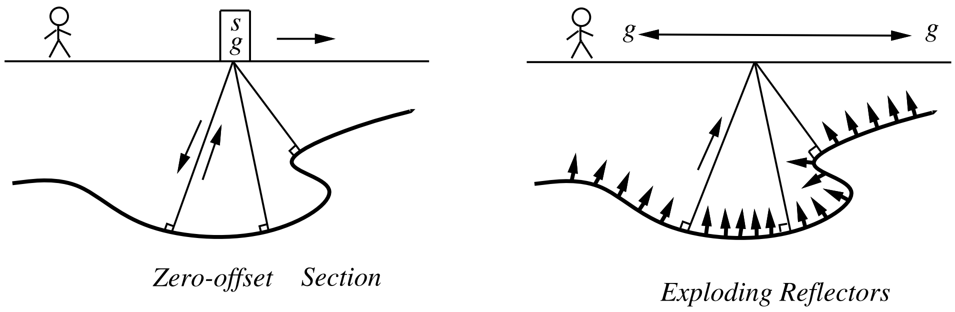

Figure 1.1: Echoes collected with a source-receiver pair moved to all points on the earth's surface (left) and the "exploding-reflectors" conceptual model (right). krch-expref [NR]

Notice in the figure that the ray paths in the field-recording case seem to be the same as those in the **exploding-reflector** case. It is a great conceptual advantage to imagine that the two wavefields, the observed and the hypothetical, are indeed the same. If they are the same, the many thousands of experiments that have really been done can be ignored, and attention can be focused on the one hypothetical experiment. One obvious difference between the two cases is that in the field geometry waves must first go down and then return upward along the same path, whereas in the hypothetical experiment they just go up. Travel time in field experiments could be divided by two. In practice, the data of the field experiments (two-way time) is analyzed assuming the sound velocity to be half its true value.

1.2 THE KIRCHHOFF IMAGING METHOD

There are two basic tasks to be done: (1) make data from a model, and (2) make models from data. The latter is often called **imaging**.

Imagine the earth had just one point reflector at (x_0, z_0). This reflector explodes at $t = 0$. The data at $z = 0$ is a function of location x and travel time t would be an impulsive signal along the hyperbolic trajectory $t^2 = ((x - x_0)^2 + z_0^2)/v^2$. Imagine the earth had more point reflectors in it. The data would then be a superposition of more hyperbolic arrivals. A dipping bed could be represented as points along a line in (x, z). The data from that dipping bed must be a superposition of many hyperbolic arrivals.

Now let us take the opposite point of view: we have data and we want to compute a model. This is called "**migration**". Conceptually, the simplest approach to migration is also based on the idea of an impulse response. Suppose we recorded data that was zero everywhere except at one point (x_0, t_0). Then the earth model should be a spherical mirror centered at (x_0, z_0) because this model produces the required data, namely, no received signal except when the sender-receiver pair are in the center of the semicircle.

This observation plus the superposition principle suggests an algorithm for making earth images: For each location (x, t) on the data mesh $d(x, t)$ add in a semicircular mirror of strength d into the model $m(x, z)$. You need to add in a semicircle for every value of (t, x). Notice again we use the same equation $t^2 = (x^2 + z^2)/v^2$. This equation is the "conic section". A slice at constant t is a circle in (x, z). A slice at constant z is a hyperbola in (x, t).

Examples are shown in Figure 1.2. Points making up a line reflector diffract to a line reflection, and how points making up a line reflection migrate to a line reflector.

Besides the semicircle superposition migration method, there is another migration method that produces a similar result conceptually, but it has more desirable results numerically. This is the "**adjoint modeling**" idea. In it, we sum the data over a hyperbolic trajectory to find a value in model space that is located at the apex of the hyperbola. This is also called the "**pull**" method of migration.

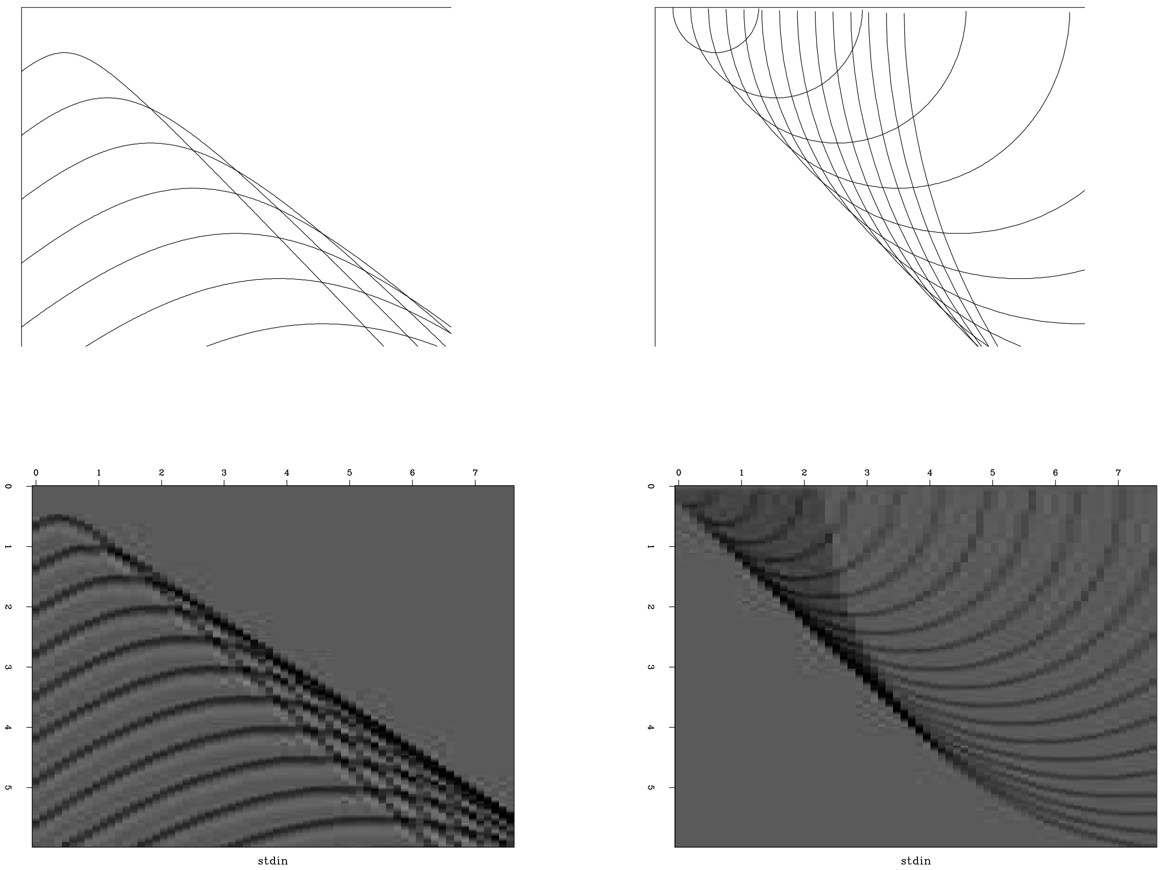

Figure 1.2: Left is a superposition of many hyperbolas. The top of each hyperbola lies along a straight line. That line is like a reflector, but instead of using a continuous line, it is a sequence of points. Constructive interference gives an apparent reflection off to the side. Right shows a superposition of semicircles. The bottom of each semicircle lies along a line that could be the line of an observed plane wave. Instead the plane wave is broken into point arrivals, each being interpreted as coming from a semicircular mirror. Adding the mirrors yields a more steeply dipping reflector. krch-dip [NR]

1.2.1 Tutorial Kirchhoff code

Subroutine `kirchslow()` below is the best tutorial **Kirchhoff migration**-modeling program I could devise. Think of data as a function of traveltime t and the horizontal axis x. Think of model (or image) as a function of traveltime t and the horizontal axis x. The program copies information from data space `data(it,iy)` to model space `modl(iz,ix)` or vice versa.

Data space and model space each have two axes. Of the four axes, three are independent (stated by loops) and the fourth is derived by the circle-hyperbola relation $t^2 = \tau^2 + x^2/v^2$. Subroutine `kirchslow()` for `adj=0` copies information from model space to data space, i.e. from the hyperbola top to its flanks. For `adj=1`, data summed over the hyperbola flanks is put at the hyperbola top.

```
# Kirchhoff migration and diffraction.  (tutorial, slow)
#
subroutine kirchslow(   adj, add,  velhalf, t0,dt,dx, modl,nt,nx,  data)
integer ix,iy,it,iz,nz, adj, add,                          nt,nx
real x0,y0,dy,z0,dz,t,x,y,z,hs,    velhalf, t0,dt,dx, modl(nt,nx), data(nt,nx)
call adjnull(           adj, add,                     modl,nt*nx,  data,nt*nx)
x0=0.;  y0=0;  dy=dx;  z0=t0;  dz=dt; nz=nt
do ix= 1, nx {  x = x0 + dx * (ix-1)
do iy= 1, nx {  y = y0 + dy * (iy-1)
do iz= 1, nz {  z = z0 + dz * (iz-1)            # z = travel-time depth
        hs=      (x-y) / velhalf
        t = sqrt( z * z  +  hs * hs )
        it = 1.5 + (t-t0) / dt
        if( it <= nt )
                if( adj == 0 )
                        data(it,iy) = data(it,iy) + modl(iz,ix)
                else
                        modl(iz,ix) = modl(iz,ix) + data(it,iy)

        }}}
return; end
```

Figure 1.3 shows an example. The model includes dipping beds, syncline, anticline, fault, unconformity, and buried focus. The result is as expected with a "**bow tie**" at the buried focus. On a video screen, I can see hyperbolic events originating from the unconformity and the fault. At the right edge are a few faint **edge artifacts**. We could have reduced or eliminated these edge **artifacts** if we had extended the model to the sides with some empty space.

1.2.2 Kirchhoff artifacts

Given one of our pair of Kirchoff operations, we can manufacture data from models. With the other, we try to reconstruct the original model. This is called imaging. It does not work perfectly. Observed departures from perfection are called "artifacts".

Reconstructing the earth model with the adjoint option yields the result in Figure 1.4. The reconstruction generally succeeds but is imperfect in a number of interesting ways. Near the

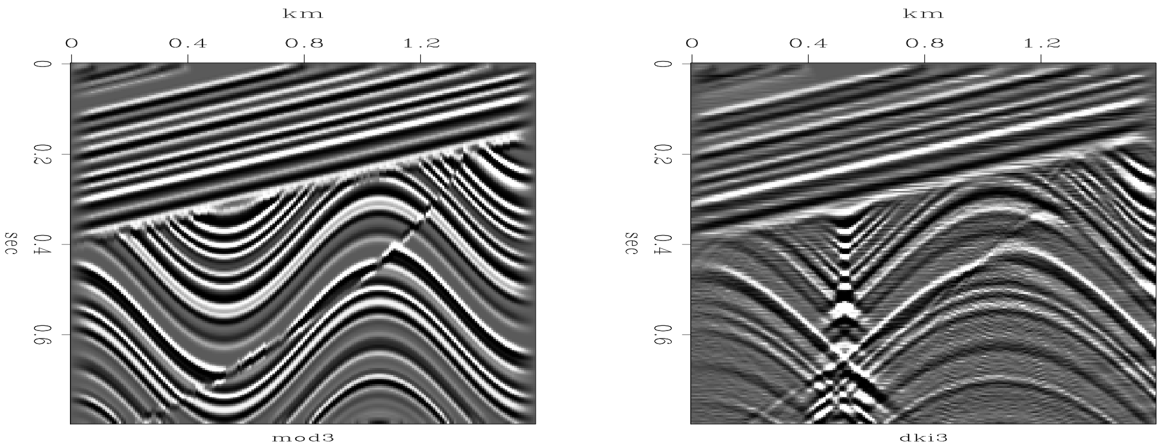

Figure 1.3: Left is the model. Right is diffraction to synthetic data. We notice the "syncline" (depression) turns into a "bow tie" whereas the anticline (bulge up) broadens. krch-kfgood [NR]

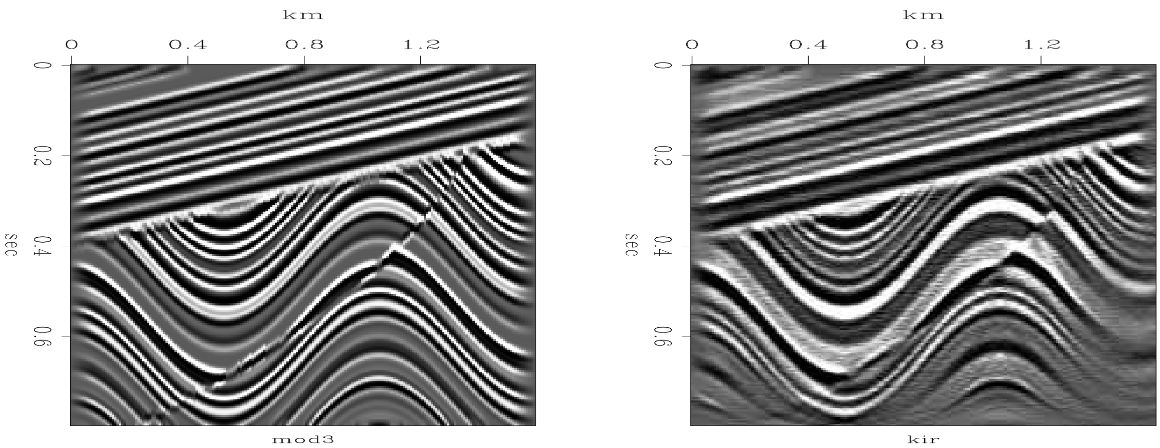

Figure 1.4: Left is the original model. Right is the reconstruction. krch-skmig [NR]

bottom and right side, the reconstruction fades away, especially where the dips are steeper. Bottom fading results because in modeling the data we abandoned arrivals after a certain maximum time. Thus energy needed to reconstruct dipping beds near the bottom was abandoned. Likewise along the side we abandoned rays shooting off the frame.

Unlike the example shown here, real data is notorious for producing **semicircular artifacts**. The simplest explanation is that real data has impulsive noises that are unrelated to primary reflections. Here we have controlled everything fairly well so no such semicircles are obvious, but on a video screen I can see some semicircles.

1.3 SAMPLING AND ALIASING

Spatial aliasing is a vexing issue of numerical analysis. The Kirchhoff codes shown here do not work as expected unless the space mesh size is suitably more refined than the time mesh.

*Spatial **alias**ing* means insufficient sampling of the data along the space axis. This difficulty is so universal, that all migration methods must consider it.

Data should be sampled at more than two points per wavelength. Otherwise the wave arrival direction becomes ambiguous. Figure 1.5 shows synthetic data that is sampled with insufficient density along the x-axis. You can see that the problem becomes more acute at

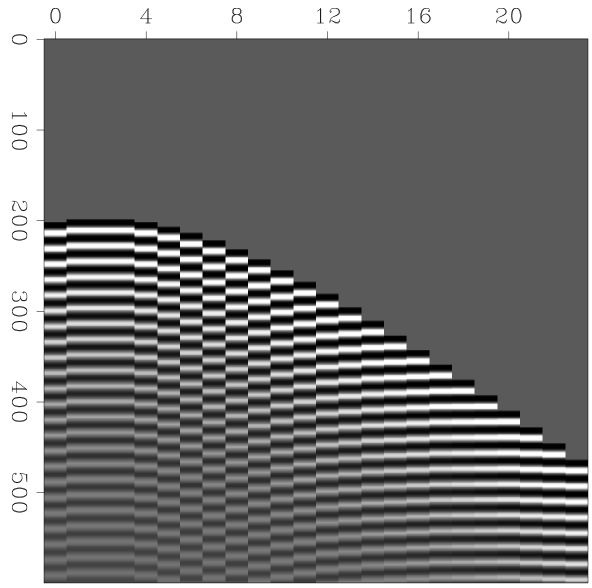

Figure 1.5: Insufficient spatial sampling of synthetic data. To better perceive the ambiguity of arrival angle, view the figures at a grazing angle from the side. krch-alias [NR]

high frequencies and steep dips.

There is no generally-accepted, automatic method for migrating spatially aliased data. In such cases, human beings may do better than machines, because of their skill in recognizing true slopes. When the data is adequately sampled however, computer migrations give better results than manual methods.

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