Introduction

0.1 THEMES

These four long lectures on Fourier Transforms and waves follow two general themes,

First, instead of drilling down into analytical details of one-dimensional Fourier analysis, these lectures scan the basic definitions and concepts focusing on the concrete, namely, computation. Then we move on to the basic principles of multidimensional spectra, a field fundamental to wavefield analysis and practical imaging.

The second theme of these lectures is examples. There are 60 illustrations here, the majority of them being computed illustrations of the concepts. The theory here is mostly that required to explain the examples.

All of the illustrations here come from my earlier books

1. Fundamentals of Geophysical Data Processing (**FGDP**)
2. Earth Soundings Analysis, Processing versus Inversion (**PVI**)
3. Basic Earth Imaging (**BEI**)
4. Geophysical Estimation by Example (**GEE**)

These books are all freely available on the web at http://sepwww.stanford.edu/sep/prof/.

0.2 WHY GEOPHYSICS USES FOURIER ANALYSIS

When earth material properties are constant in any of the cartesian variables (t,x,y,z) then it is useful to Fourier transform (FT) that variable.

In seismology, the earth does not change with time (the ocean does!) so for the earth, we can generally gain by Fourier transforming the time axis thereby converting time-dependent differential equations (hard) to algebraic equations (easier) in frequency (temporal frequency).

In seismology, the earth generally changes rather strongly with depth, so we cannot usefully Fourier transform the depth z axis and we are stuck with differential equations in z. On the other hand, we can model a layered earth where each layer has material properties that are constant in z. Then we get analytic solutions in layers and we need to patch them together.

Thirty years ago, computers were so weak that we always Fourier transformed the x and y coordinates. That meant that their analyses were limited to earth models in which velocity was horizontally layered. Today we still often Fourier transform t,x,y but not z, so we reduce the partial differential equations of physics to ordinary differential equations (ODEs). A big advantage of knowing FT theory is that it enables us to visualize physical behavior without us needing to use a computer.

The Fourier transform variables are called frequencies. For each axis (t,x,y,z) we have a corresponding frequency (ω,k_x,k_y,k_z). The k's are spatial frequencies, ω is the temporal frequency.

The frequency is inverse to the wavelength. Question: A seismic wave from the fast earth goes into the slow ocean. The temporal frequency stays the same. What happens to the spatial frequency (inverse spatial wavelength)?

In a layered earth, the horizonal spatial frequency is a constant function of depth. We will find this to be Snell's law.

In a spherical coordinate system or a cylindrical coordinate system, Fourier transforms are useless but they are closely related to "spherical harmonic functions" and Bessel transformations which play a role similar to FT.

After we develop some techniques for 2-D Fourier transform of surface seismic data, we'll see how to use wave theory to take these observations made on the earth's surface and "downward continue" them, to extrapolate them into the earth. This is a central tool in earth imaging.

Then we introduce the concepts of reflection coefficient and layered media. The main difference between wave theory found in physics books and geophysics books is due to the near omnipresent gravitational stratification of earth materials.

A succeeding chapter considers two-dimensional spectra of any function, how such functions can be modeled, what it means to deconvolve 2-D functions, and an all-purpose method of filling in missing data in a 2-D function based on its spectrum.

The final chapter returns us to one dimension for the "uncertainty principle," for the basic concepts of resolution and statistical fluctuation.