You can see the sunset twice in one day

Where the sun sets over the ocean, the horizon line casts a shadow moving up the sides of mountains, buildings, trees, and even people. As the sinking sun is increasingly occluded by the horizon it becomes more of a point source. Thus the rising shadow line should be quite sharp. Think of a man seated at the ocean's edge with his eyes at an altitude of one meter. I will calculate that it requires 8 seconds for the shadow to reach from the water's edge up to his eyes. Three seconds later at 11 seconds the shadow reaches two meters, the eyes of a woman standing there. When the air is clear, the wink-out of the top edge of the sun can be quick compared to three seconds. On a clear day the sun seems to blink off in a sort of a brief moment. We may think of this as the sharp shadow washing upward across their faces.

Taking sunset at $t = 0$, a beam of light skims the earth (a circle of radius $R = 6371$ km). After touching the earth at $x = 0$ the earth drops away from the beam to altitude $h(t)$ at distance $x(t)$. We are aiming to tabulate $h(t)$.

$$R^2 + x^2 = (R+h)^2 \qquad \text{pythagoras} \tag{1}$$

$$x^2 = 2Rh \qquad \text{ignoring } h^2 \tag{2}$$

$$x = \sqrt{2R}\sqrt{h} = 3550\sqrt{h} \qquad \text{in meters} \tag{3}$$

$$\alpha = \frac{\text{earth circumference}}{\text{duration of day}} \tag{4}$$

$$x = \alpha t = \frac{\pi(2R)}{24 \cdot 60 \cdot 60}\, t \qquad \alpha \text{ is the speed of the earth surface} \tag{5}$$

$$h = x^2/(2R) = (\alpha t)^2/(2R) = (\,(\alpha/\sqrt{2R})\; t)^2 \tag{6}$$

$$h = \left(\frac{\pi\sqrt{2R}}{24 \cdot 60 \cdot 60}\, t\right)^2 \tag{7}$$

$$h = (0.13\; t)^2 \qquad \text{and} \qquad t = 8\sqrt{h} \tag{8}$$

In Python: import math, then: 3.14 * math.sqrt(2*6371000)/(24*60*60) = 0.13

To make an appealing video we need to chose an altitude, an altitude for a video of a face or of a whole body. The upward velocity $v = dh/dt$ is

$$v = dh/dt = .13^2 \cdot 2t = .13^2 \cdot 2 \cdot 8\sqrt{h} = .27\sqrt{h} \tag{9}$$

$$h = (v/.27)^2 = (4v)^2 \qquad \text{or alternately} \qquad v = \sqrt{h}/4 \tag{10}$$

Try making a selfie during sunset. If you get a successful Youtube, I can link it here[1].

I read that taking the elevator up the Burj Khalifa[2], you can see the sunset twice, first at the ground and later at the top. That's what led us here.

—Jon Claerbout 9/11/2019

[1] http://sep.stanford.edu/sep/jon/sunset.pdf

[2] https://gizmodo.com/did-you-know-that-the-burj-khalifa-is-so-tall-that-you-5917230

Observations

Bill Symes wrote me (11/3/19) saying:

I was out rowing around in West Sound this evening, with a line of hills to the SW, far enough away (maybe 2 miles) that the horizon was fairly sharp. I watched the sun until its top limb just grazed the hill line and "winked out", then immediately stood up. The top limb was visible again for perhaps 2 s. Estimating the height-of-eye difference at 1 m, and allowing for my reaction time and the time taken to stand up (carefully—it's just a rowboat), the experience was consistent with the 3 s/m estimate.

You could certainly say that I saw two sunsets.

Shadow of sunset cast by a mountain

My memory of seeing the sun "wink out" at sunset includes the presence of a cloud. This is a little like Bill seeing the wink-out over some hills. In this case the atmospheric physics is quite different because the ray does not graze the ocean. ...

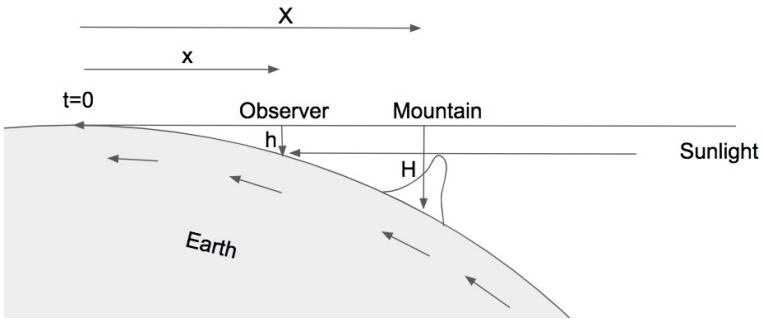

Figure 1: Sunset is earlier because of the mountain.

Drone goes up instead of elevator

I have been contacted by Peter C Lukens who attempted to keep sunset perpetually in sight by capturing video while the drone rises. He presents his results at https://tinyurl.com/y4putncd

Shadow sharpness

My curiosity is how sharp the shadow might be that rises up a nearby telephone pole. The sun does become a point-source as it sets, so the horizon should cast a sharp shadow. That, of course, depends on how sharp the horizon actually is. OK if it's the ocean, but then there is the issue of the long atmospheric path. Alternately, we could think of the shadow of a nearby mountain or hill should cast a sharp shadow. Having seen the sun suddenly blocked by a distant cloud I had the feeling that the sun "blinked off" which implies a sharp shadow.