GROUND ROLL: SUPPRESSING IT WHILE MAPPING LAYER THICKNESS

- A new idea, a new direction!
- It might be wrong.
- Easy to code and test.
- Opens the door for routine production.
- Theory goes into 3-D, maybe. Your prize?

Ground roll is pesky stuff. Historically, the only way the industry could get rid of this stuff was frequency filtering to eliminate low frequencies and local receiver arrays to effect a crude low-velocity dip filter. Nowadays with denser data sampling, better low-velocity cutoff filters are possible.

But the best way to eliminate anything is to model it and then subtract it. Trouble with ground roll is the modeling quickly demands many more parameters than we care to estimate. And it's frequency dispersive. The modeling parameters can change pretty fast in both space and time. What we need is a simplified model that is rapidly adaptive in time and space. I'm going to try for a ground roll modeling and suppression program that has only two parameters,

v	an LMO velocity	H_2O. On land your choice.
$H(x)$ or $H(x, y)$	effective layer thickness	you will discover

My old book PVI applies a finite differencing stencil $(\partial_x - p\partial_t)$ to data $D(t, x)$. This gives two planes $\partial_x D(t, x)$ and $\partial_t D(t, x)$. I combined them with a scale factor p and minimized misfit, thereby finding a simple expression for stepout p. Mathematically, I appear to have a different p value at each point in time and space, but in practice we naturally expect to do smoothing to reduce noise. A delightful aspect of this approach is that it is easy to window the (t, x)-plane with with whatever size and shape 2-D smoothing windows we choose.

An alternate approach worth noticing is that I could have started with the 1-D scalar wave equation: $(\partial_x - p\partial_t)(\partial_x + p\partial_t)D(t, x) = 0$. That would simultaneously be fitting wave slopes with both $\pm p$. But the application I had in mind expected only one of the two slopes.

Begin from the scalar wave equation. We will specialize it to a thin plate of thickness H. In the marine environment H will be roughly the water depth. It is not exactly that because what is usually called "normal modes" have some penetration into the subsurface. We will call H the "effective layer thickness". Let us write the dispersion relation for scalar waves in an infinite medium, then express the vertical wave number k_z in terms of vertical wavelength, $k_z = 2\pi/\lambda_z$.

$$
\begin{aligned}
0 &= k_x^2 + k_z^2 - \omega^2/v^2 && (1)\\
0 &= k_x^2 + \left(\frac{2\pi}{\lambda_z}\right)^2 - \omega^2/v^2 && (2)\\
0 &= k_x^2 + \left(\frac{2\pi}{H(1/2+n)}\right)^2 - \omega^2/v^2 && (3)
\end{aligned}
$$

The denominator chooses odd harmonics (integer n) because the layer top and bottom, have opposite boundary conditions, one being free, the other rigid. In other words, your first harmonic $n = 0$ is a half wavelength, the next one $3/2$, etc. Now we specialize to the $n = 0$ case, the "fundamental mode". To avoid fractions, let us switch from layer thickness H to thinness $T = 4\pi/H$. Thus $0 = k_x^2 + T^2 - \omega^2/v^2$.

We wish to separate this wave equation into two parts, one for each of the two directions of propagation. Today's study is analogous to 40 years ago when I wished to separate up and down going waves in 2-D. In later years people always liked to do the wave separation by choosing a sign $k_z = \pm\sqrt{(\omega/v)^2 - k_x^2}$. But in earlier years I got a good first approximation without the complications of a square root, simply by identifying a small quantity and neglecting its square. I follow that approach now. We transform to new coordinates moving along with the wave of interest. That's when $k_x \approx \omega/v$. To get a feeling for this, look at the data in Figures 1 and 2. We are looking near $t = 0$ after linear moveout. After LMO the horizontal wavelength becomes long, $\tilde{k}_x = k_x - \omega/v$ is small. We will drop terms in $\tilde{k}_x^2$.

$$
\begin{aligned}
0 &= k_x^2 + T^2 - \omega^2/v^2 && (4)\\
0 &= (\tilde{k}_x + \omega/v)^2 + T^2 - \omega^2/v^2 && (5)\\
0 &= \tilde{k}_x^2 + 2\tilde{k}_x\omega/v + T^2 && (6)\\
0 &= 2\tilde{k}_x\omega/v + T^2 && (7)\\
0 &= \tilde{k}_x\omega + \tilde{T}^2 && (8)
\end{aligned}
$$

Thus, the final equation for modeling ground roll has only one free parameter, a revised expression for layer thinness $\tilde{T}^2 = T^2 v/2$. We haven't completely removed velocity from the analysis, since earlier we used it for linear moveout of the data. Let us have a look at the convolution stencils we will place on $D(t, x)$.

$$
\begin{aligned}
\mathbf{0} &\approx \left(\frac{1}{\Delta t \Delta x} \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} + \frac{\tilde{T}^2}{4} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \right) * D(t,x) && (9)\\
\mathbf{0} &\approx \mathbf{a} + \tilde{T}^2 \; \mathbf{b} && (10)\\
\tilde{T}^2 &= -(\mathbf{a}\cdot\mathbf{b})/(\mathbf{b}\cdot\mathbf{b}) && (11)
\end{aligned}
$$

where the dot products in equation (11) would be taken over a small range around $t = 0$ and in arbitrarily shaped windows that move around the x-axis (or, hopefully, the (x, y)-plane).

That's it. You moveout your data with any chosen v and look only near $t = 0$ where you have approximately flattened your ground roll or normal mode of interest. Then you slap these templates on your data, convolve, and choose thinness $\tilde{T}$ to minimize energy.

What should we do first? I'm guessing. I don't understand this equation at all. What if we change the moveout velocity a little bit?

To build synthetics, start with a fat blob function on t at $x = 0$ (or the opposite) and propagate it out into the (t, x) plane. I hope it is stable.

Making money in 3-D

And how will we make this a 3-D process? That *is* an interesting question! Figure 4 shows a sample of 3-D ground roll. Do conical moveout $\tau = t - v^{-1}\sqrt{x^2 + y^2}$ on that and watch the spatial aliasing magically disappear. This next equation, does it help?

$$\frac{\partial}{\partial r} \ = \ \cos\theta \, \frac{\partial}{\partial x} + \sin\theta \, \frac{\partial}{\partial y} \tag{12}$$

Oh, you want the back scattered and side scattered ground roll too? If you've got the data density, I've got the equation.

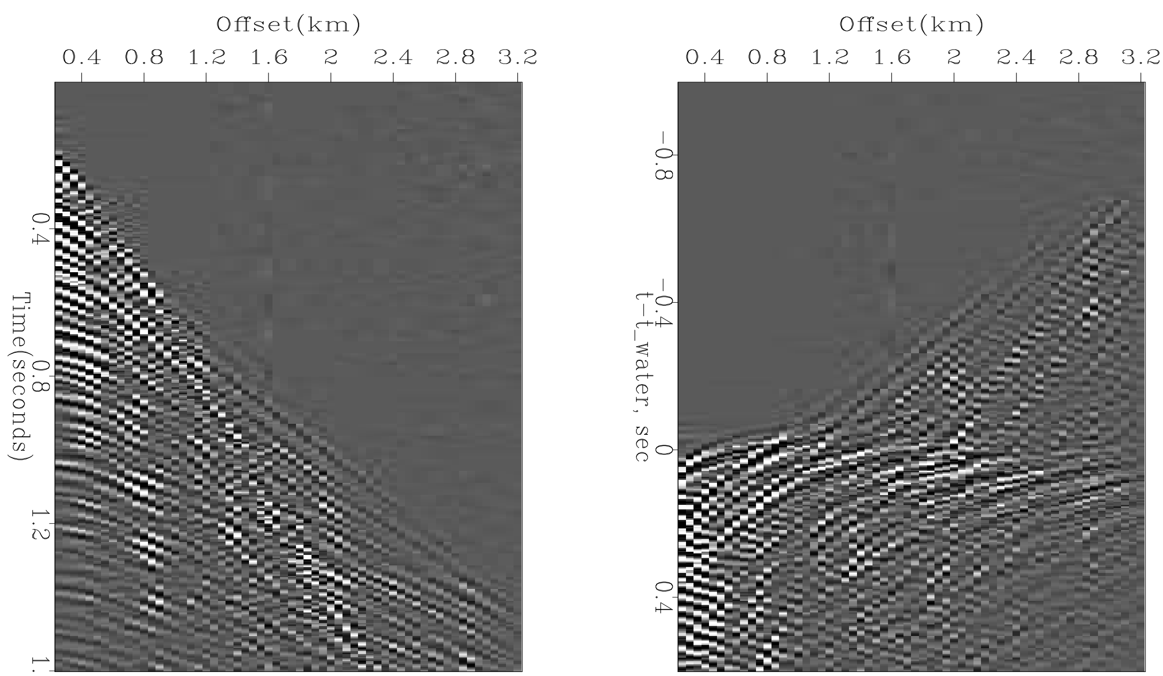

Figure 1: Shallow water marine, moved out at water velocity, shows fundamental mode with group velocity a bit slower than water velocity.

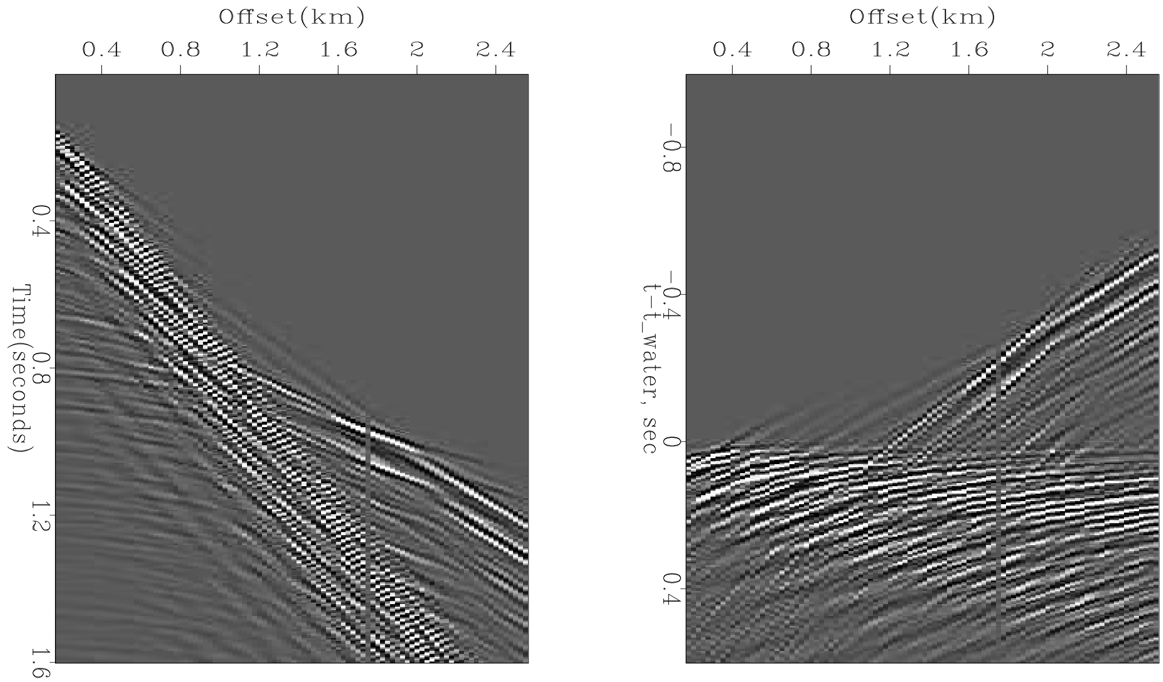

Figure 2: Shallow water marine, moved out at water velocity, shows fundamental and one higher mode.

IGNORE THIS ANCIENT WISDOM AT YOUR PERIL

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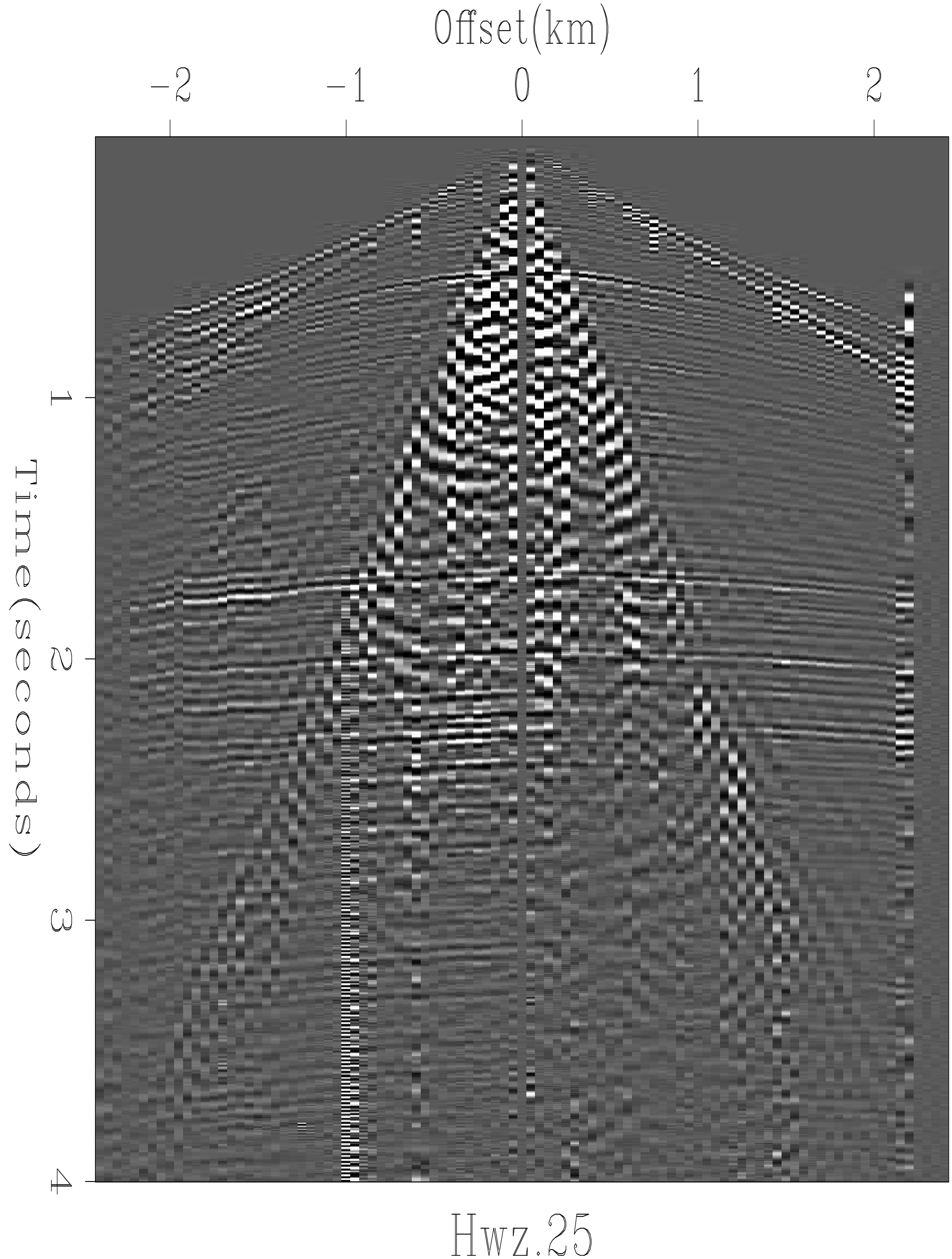

Figure 3: Classic Alberta land data. Notice 2-3 modes. Notice spatial alias. Will be fun to see this after linear moveout.

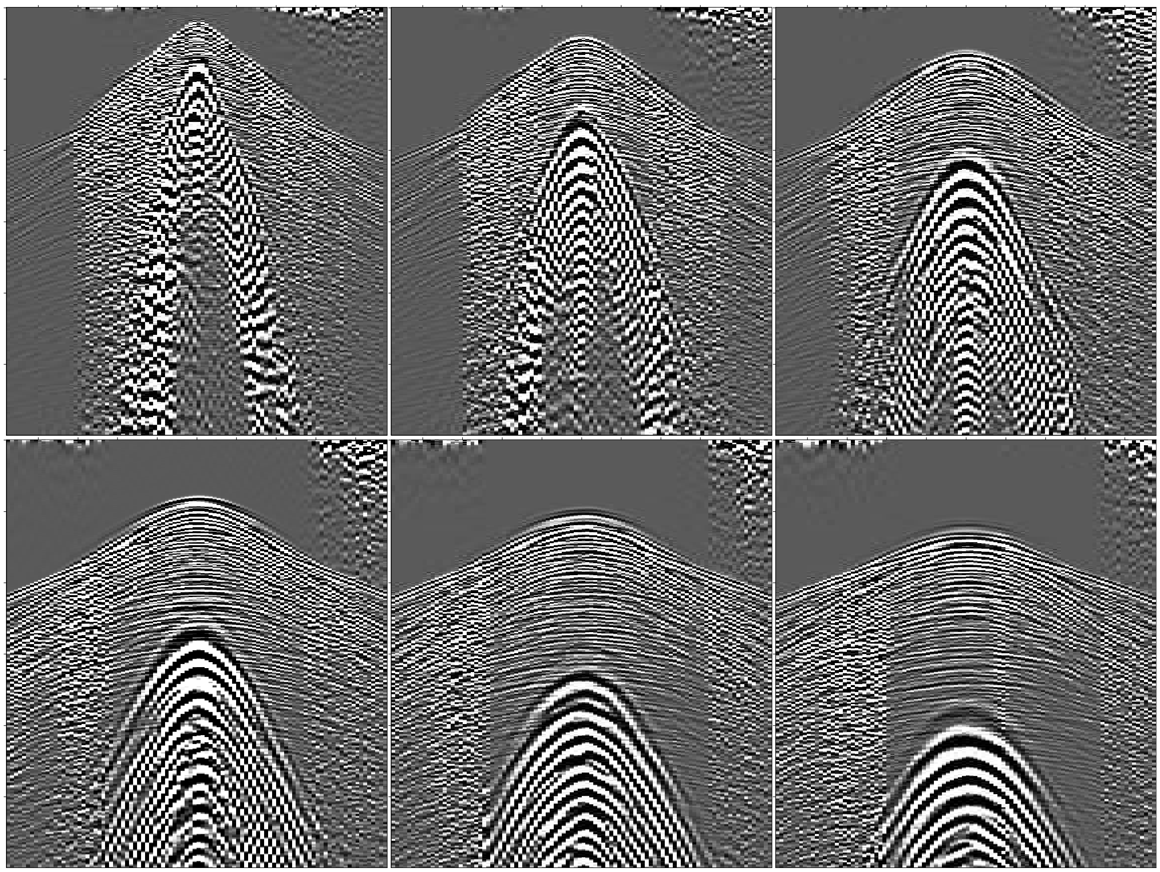

Figure 4: One shot, 6 parallel receiver lines. Ancient data from SA. Ground roll travel time from the shot is the cone $t = \sqrt{x^2 + y^2}/v$. Slices of it, what you see here, are hyperbolas. After using the moveout equation $\tau = t - v^{-1}\sqrt{x^2 + y^2}$, the apparent hyperbolas would be flattened near $\tau = 0$. Linear moveout would not flatten these hyperbolas, but conical moveout does. Do we know a radially dependent finite differencing star that estimates the effective layer thinness $T(x, y)$?