

Lecture Notes on  
Nonlinear Inversion and Tomography:  
I. Borehole Seismic Tomography

*From a Series of Lectures by*

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