# Mathematical Example of the Dispersive Effect



showing how the effect arises in our equations. to help us construct a simple mathematical example Now we will use our newfound physical understanding

shear strain under a compressional load (in this of reciprocity) others that produce a change in in volume under an applied shear stress, and (because anisotropic. case pore pressure). The matrix must also be locally New coefficients must be added to produce a change

## Mathematical Example (2)



$$\begin{pmatrix} e_{11} \\ e_{22} \\ e_{33} \\ -\zeta \\ e_{23} \\ e_{31} \\ e_{23} \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} & S_{13} & -\beta^{(1)} \\ S_{12} & S_{11} & S_{13} & -\beta^{(1)} \\ S_{13} & S_{13} & S_{33} & -\beta^{(3)} \\ -\beta^{(1)} & -\beta^{(1)} & -\beta^{(3)} & \gamma & -\omega \\ e_{23} \\ e_{23} \\ e_{31} \end{pmatrix} \begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ -\rho_f \\ \sigma_{23} \\ \sigma_{31} \\ \sigma_{31} \\ \sigma_{12} \\ \sigma_{12} \end{pmatrix}$$

#### Mathematical Example (3)



is then given by pressure  $p_f$  by setting  $\zeta = 0$ , solving for  $p_f$ , which eliminate both the liquid increment  $\zeta$  and the liquid So now we use the standard (Gassmann) trick to

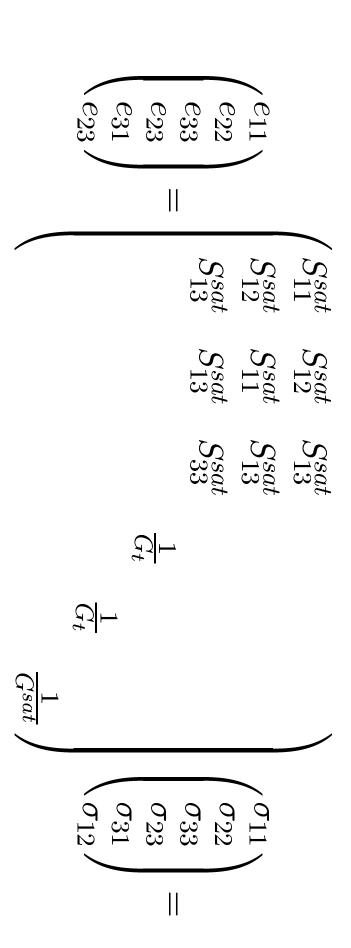
$$p_f = -\frac{\beta}{\gamma} \left( \sigma_{11} + \sigma_{22} + \sigma_{33} \right) - \frac{\omega}{\gamma} \sigma_{12},$$

equations. and substituting this back into the previous set of

The result is:

# Mathematical Example (4a)





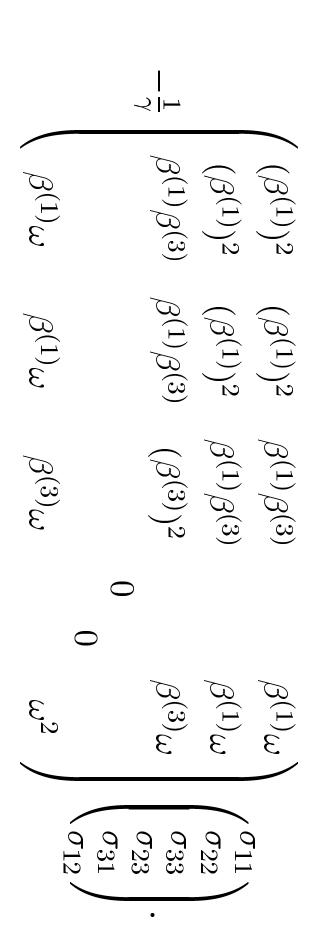
# Mathematical Example (4b)



$$\begin{pmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{11} & S_{13} \\ S_{13} & S_{13} & S_{33} \\ \frac{1}{G_t} & & & & & & & & & & & & \\ & & & \frac{1}{G_{dr}} & & & & & & & & & & \\ & & & & \frac{1}{G_{dr}} & & & & & & & & & & \\ \end{pmatrix} \begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{31} \\ \sigma_{12} \end{pmatrix}$$

## Mathematical Example (4c)





#### Mathematical Example (5)



modulus for the saturated system now contains a term The important result we obtain shows that the shear

$$G^{sat} = \frac{G_{dr}}{1 - G_{dr}\omega^2/\gamma},$$

depending on the mechanical properties of the liquid through the coefficients  $\omega$  and  $\gamma$ .

than the drained modulus for nonvanishing  $\omega$ . Note that the saturated shear modulus is always greater