The Kuster-Toksoz Approximation in Elasticity

Low Frequency Scattering Coefficients

$$B_0(K_m, K_i, G_m) = \frac{K_m - K_i}{3K_i + 4G_m}$$

$$B_2(G_m, G_i, K_m) = \frac{20G_m(G_i - G_m)/3}{6G_i(K_m + 2G_m) + G_m(9K_m + 8G_m)}$$

Host medium uses subscript m, inclusions i.
Inclusion is a spherical scatterer imbedded in the host for the present argument.

Derivation of the Kuster-Toksoz Formulas

The composite (scattering) medium is imbedded in a fixed host material $m = h$, where h is any one of the constituents in the composite. Then, the composite inclusion of type-$*$, when imbedded in the h-matrix, should produce the correct amount of scattering at infinity if the single-scattering coefficients satisfy

$$B_0(K_h, K^*, G_h) = \sum_{i=1}^{n} f_i B_0(K_h, K_i, G_h),$$

$$B_2(G_h, G^*, K_h) = \sum_{i=1}^{n} f_i B_2(G_h, G_i, K_h).$$

Kuster-Toksoz Formulas

The final results we obtain using the Kuster-Toksoz approach are

$$\frac{1}{K^*+4G_h/3} = \sum_{i=1}^{n} \frac{f_i}{K_i+4G_h/3} = \left\langle \frac{1}{K(x)+4G_h/3} \right\rangle,$$

and

$$\frac{1}{G^*+F_h} = \sum_{i=1}^{n} \frac{f_i}{G_i+F_h} = \left\langle \frac{1}{G(x)+F_h} \right\rangle,$$

where $F \equiv G(9K+8G)/6(K+2G)$. Note that for spherical inclusions these equations are not coupled. If the host material is chosen to be one having either the largest or smallest constants, then Kuster-Toksoz gives the same results as the Hashin-Shtrikman bounds.