

Figure 1: Schematic illustration of ray paths through a cell slowness model.

Scaling \mathbf{s} to find boundary point

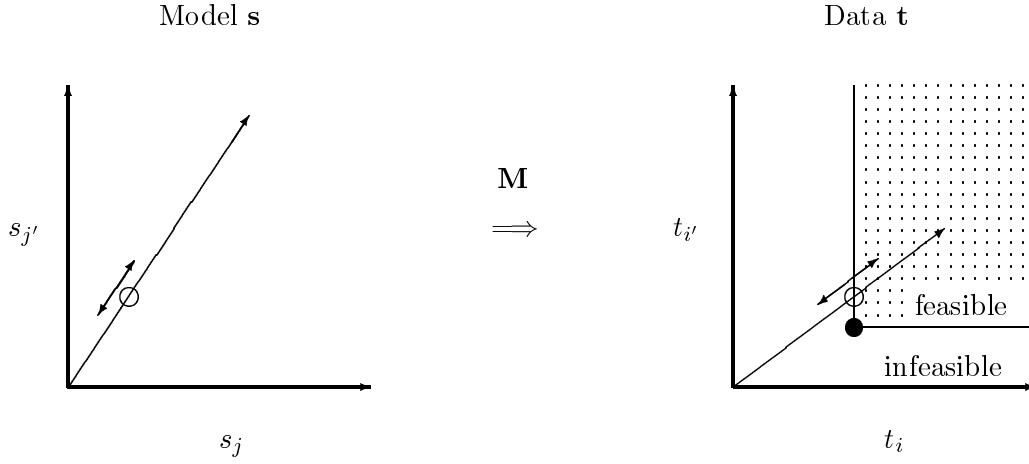


Figure 2: Scaling \mathbf{s} to find feasibility boundary point. Feasibility boundary in data space is explicitly determined by the data. Feasibility boundary in model space is implicitly determined through the traveltime calculation.

Mapping the feasibility boundary

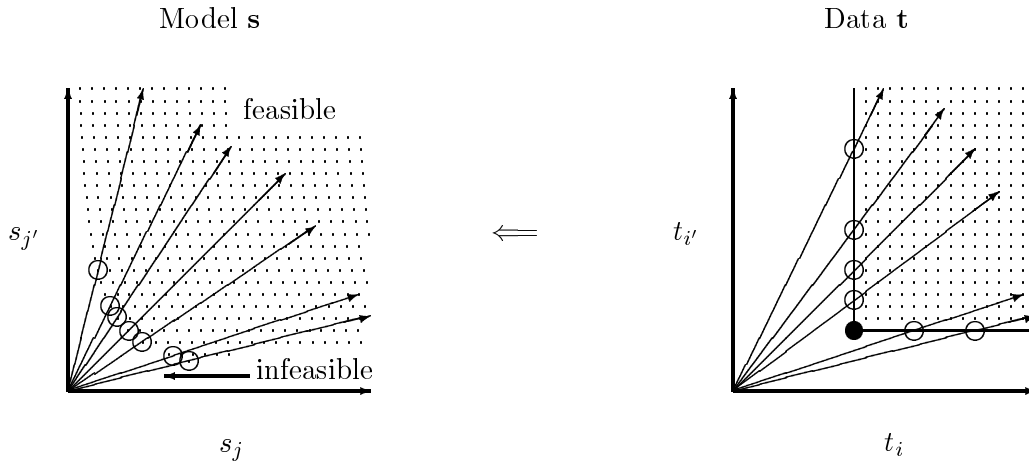


Figure 3: By scaling many slowness vectors \mathbf{s} , the location of the feasibility boundary in the model space can be mapped.

Modal analysis for string density

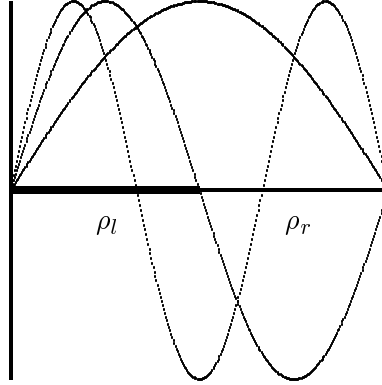


Figure 4: Density distribution of the string may be determined by analyzing the eigenfrequencies associated with its modes of vibration.

Scaling ρ to find boundary point

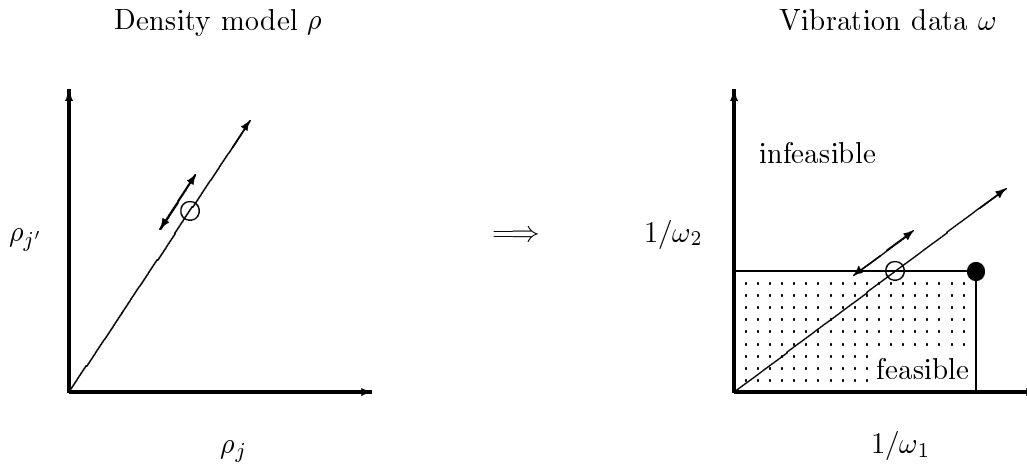


Figure 5: Feasible part of the density space is determined implicitly by the explicit boundaries defined by the frequency data.

Mapping the feasibility boundary

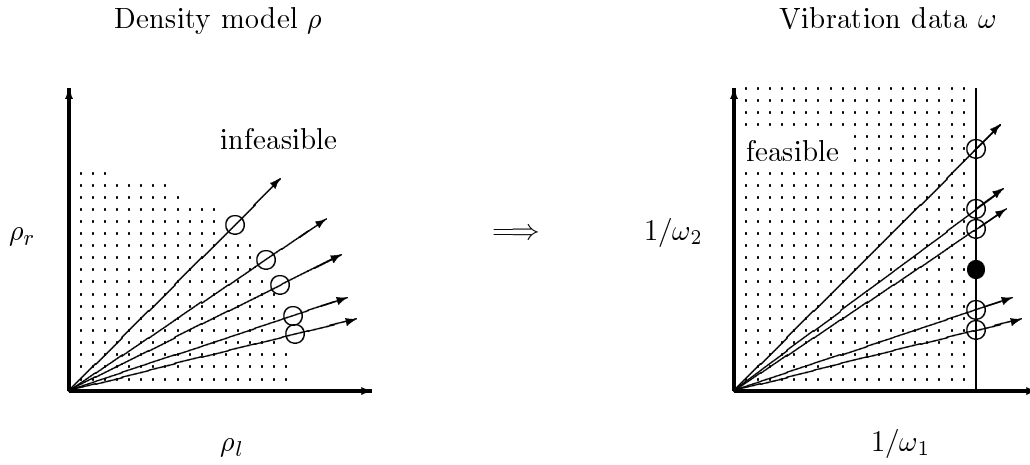


Figure 6: By scaling many density distributions, the location of the feasibility boundary can be mapped.

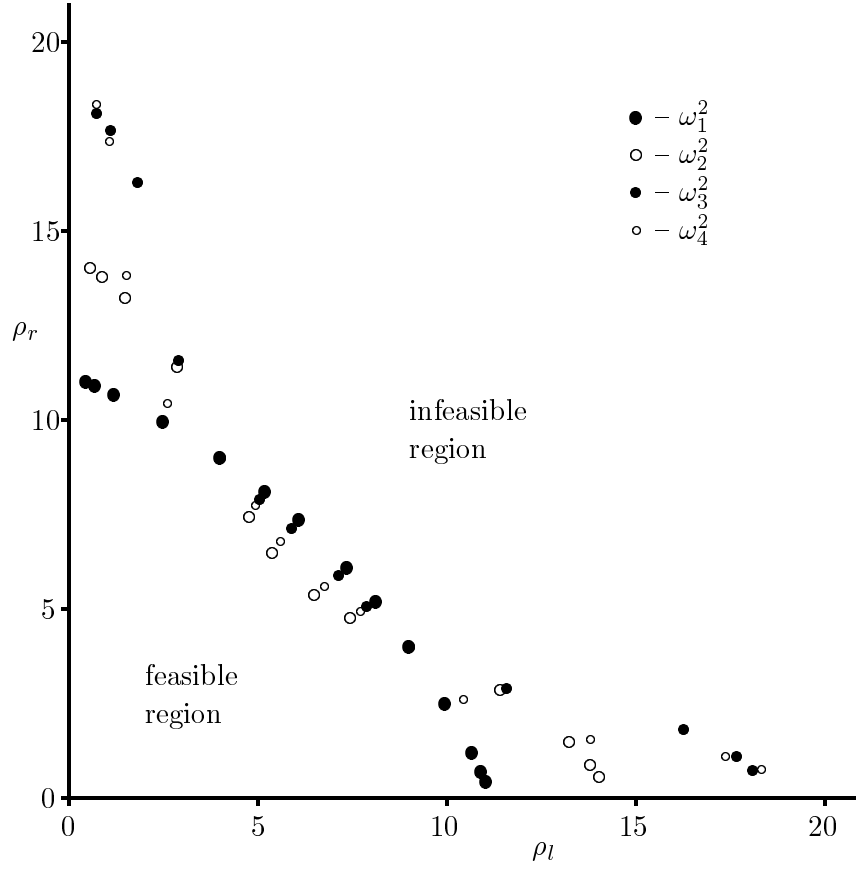


Figure 7: Using the four measurements of eigenfrequencies $\omega_1^2, \dots, \omega_4^2$ and exact calculations of the eigenfrequencies for various density ratios in TABLE 1, we obtain this plot through scaling of the densities for fixed density ratios. The region near the origin is feasible, while the exterior region is infeasible.

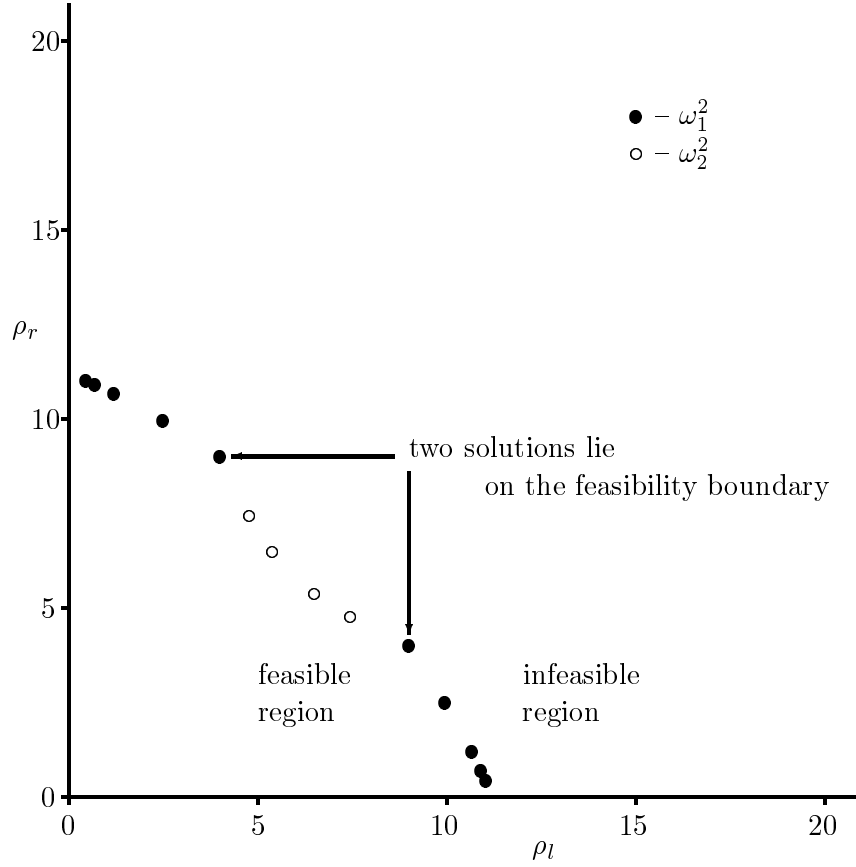


Figure 8: Using the data from Fig. 4, we find the boundary of the feasible region is determined in this problem by the data for ω_1^2 for large density ratios, and by ω_2^2 for density ratios closer to unity. These two sets of data cross at the two solutions to the inverse problem, which are seen to be two points in the model plane: (4,9) and (9,4). It is not possible to choose between these two solutions using frequency data alone.

Feasibility analysis for free oscillations of the Earth

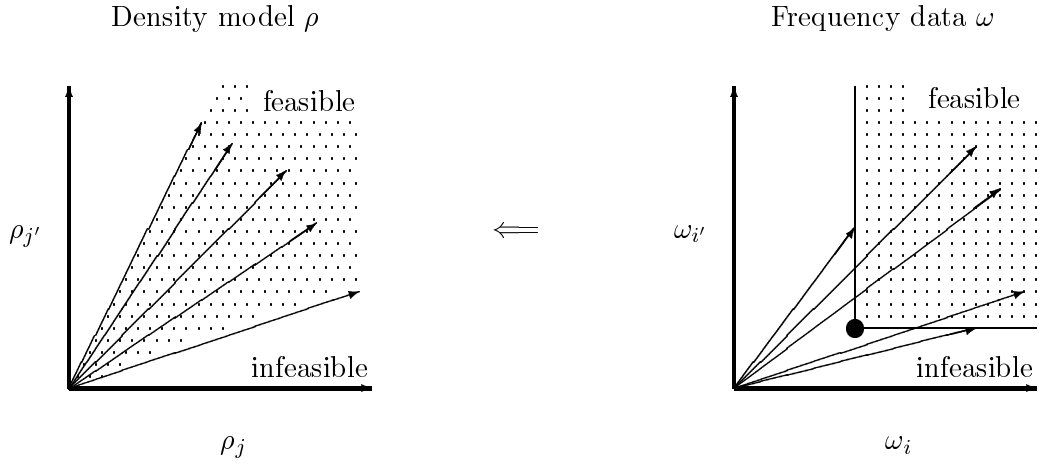


Figure 9: For free oscillations, scaling the density ρ by a positive constant does not change either the trial eigenfunctions or the frequencies, showing that any point along a ray in density space is equally good for satisfying the frequency data. Thus, the feasible region forms a cone in the density space.

Constraints to fix the scale of the density

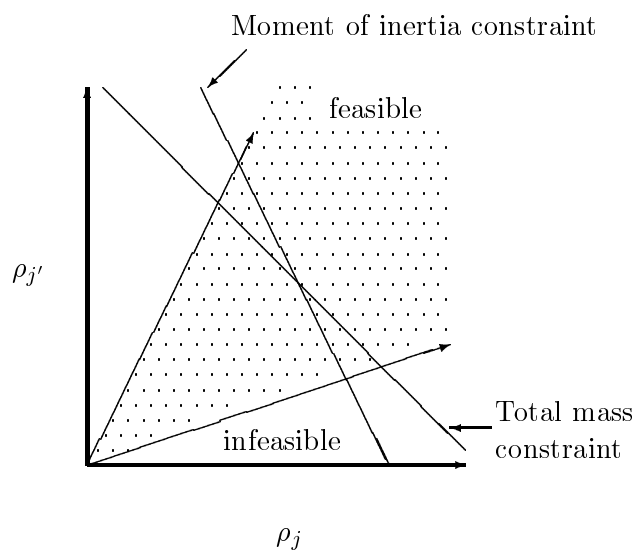


Figure 10: The scale of the density is determined by the total mass and/or the moment of inertia of the Earth.

Free oscillations of the Earth without traveltime tomography

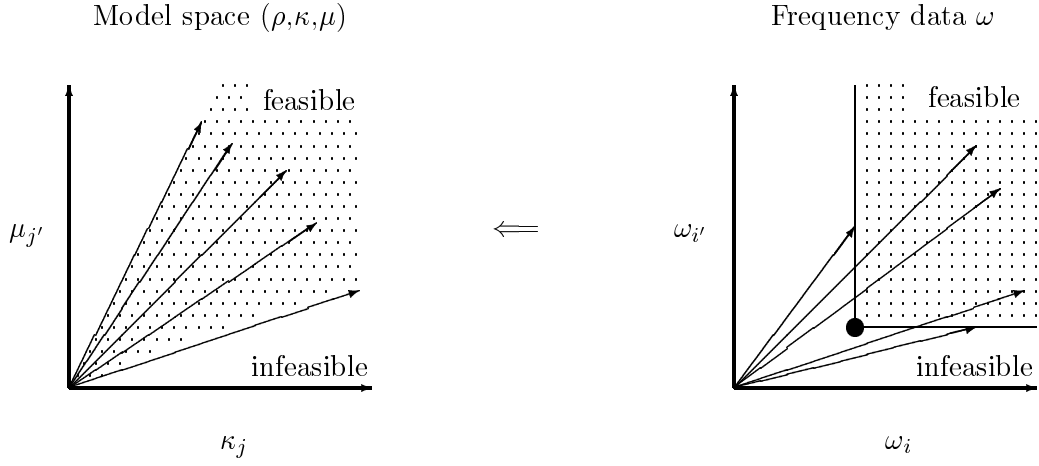


Figure 11: Scaling the triple (ρ, κ, μ) by a positive constant does not change either the trial eigenfunctions or the frequencies, showing again that any point along a ray in the model space is equally good for satisfying the frequency data. The overall scale is again determined by the total mass and/or the moment of inertia of the Earth.