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Approximate Methods for Time-Reversal Processing of Large Seismic Reflection Data Sets

Speaker: James G. Berryman

University of California, LLNL

Collaborators:

Seismic Migration: Marie Clapp / Robert Clapp / Sergey Fomel

T/R: Liliana Borcea / George C. Papanicolaou / Chrysoula Tsogka

Outline

- What and Why of Time-Reversal Acoustics
  - Acoustic wave scattering
  - Seismic reflection data transfer matrix
  - Singular value decomposition
  - Applications to reflection seismic data
- Multiple Signal Classification (MUSIC)
- Examples
- Conclusions

Acoustic Wave Scattering (1)

We assume that the problems of interest are well-approximated by the inhomogeneous Helmholtz equation:

$$\left[\Delta + k_0^2 n^2(x)\right] u(x) = s(x),$$

where $u(x)$ is the wave amplitude, $s(x)$ is a localized source function, $k_0 = \omega/c_0 = 2\pi f/\lambda$, with f being frequency, λ wavelength, and c_0 the assumed homogeneous background wave speed, while $n(x)$ is the acoustic index of refraction such that

$$n(x) = \frac{c_0}{c(x)}.$$

Thus, $n^2(x) = 1$ in the background and $a(x) = n^2(x) - 1$ measures the change in wave speed at the scatterers.

Acoustic Wave Scattering (2)

Pertinent fundamental solutions for this problem satisfy:

$$\left[\Delta + k_0^2\right] G_0(x, x') = -\delta(x - x'),$$

and

$$\left[\Delta + k_0^2 n^2(x)\right] G(x, x') = -\delta(x - x')$$

for the homogeneous and inhomogeneous media, respectively.

The solution for the homogeneous medium is well-known in 3D to be

$$G_0(x, x') = \frac{e^{ik_0|x-x'|}}{4\pi|x-x'|}.$$

Acoustic Wave Scattering (3)

The fundamental solution for the inhomogeneous problem can be written in terms of that for the homogeneous one in the usual way as:

$$G(x, x') = G_0(x, x') + k_0^2 \int a(y) G_0(x, y) G(y, x') d^3y.$$

Note that the right hand side depends also on G. For T/R imaging, the regions of nonzero $a(x)$ are assumed to be finite in number, denoted by N, in compact domains Ω_n, small compared to the wavelength λ. Then, there will be some position y_n (usually) inside each domain, characterizing the location of each of the N scatterers.

Acoustic Wave Scattering (4)

There is also a constant

$$q_n = k_0^2 \int_{\Omega_n} a(y) d^3 y$$

that characterizes the strength of the scatterer. Then,

$$G(x, x') \simeq G_0(x, x') + \sum_{n=1}^{N} q_n G_0(x, y_n) G(y_n, x').$$

Furthermore, if the scatterers are sufficiently far apart, and the scattering strengths q_n are not too large, then $G(y_n, x')$ on the far right can be replaced by $G_0(y_n, x')$, and we finally have

$$G(x, x') \simeq G_0(x, x') + \sum_{n=1}^{N} q_n G_0(x, y_n) G_0(y_n, x').$$

No Multiples Allowed

The approximation just made is equivalent to assuming that multiple scattering is negligible. Or, for realistic applications to seismic data — where multiples clearly occur, we must assume that the multiples have been eliminated (which is very commonly done for seismic data) before we proceed to the next step in the analysis. The technical term for this approximation is the "first Born approximation," implying only one iteration of the original integral equation.

Seismic Reflection Data Transfer Matrix (1)

In the absence of clutter (such as known scatterers with high albedo), we have the elements of the seismic reflection data transfer matrix K in the form

$$K_{l'l}(\omega) = \sum_{n=1}^{N} q_n G_{nl'} G_{nl} = \sum_{n=1}^{N} \sigma_n \frac{(q_n/|q_n|) G_{nl'} G_{nl}}{\sum_{r=1}^{L} |G_{nr}|^2},$$

where G_{nl} is a component of the vector of propagators

$$\mathcal{G}_n = \begin{pmatrix} G_{n,1} \\ G_{n,2} \\ \vdots \\ G_{n,L} \end{pmatrix}.$$

Main (assumed localized for now) scattering targets are labelled by n, while array elements are labelled by l.

Transfer Matrix (2)

The full transfer matrix can therefore be written as

$$K(\omega) = \sum_{n=1}^{N} q_n \mathcal{G}_n \mathcal{G}_n^T ,$$

for N reflectors. So, when $N = 1$, the vector $\mathcal{G}_1$ is clearly the unique (within a phase factor and normalization factor) singular vector of the rank one matrix $K(\omega)$. Thus, we expect these vectors $\mathcal{G}_n$ associated with particular scattering points $1 \leq n \leq N$ to play a very important role in the analysis, and sometimes to be reasonable approximations for the singular vectors of the seismic reflection data transfer matrix.

Singular Value Decomposition of K

The singular-vectors for $K(\omega)$ are approximately given by the time-reversed propagator vectors G^*_{il}, since

$$\sum_{l=1}^{L} K_{l'l}(\omega) G^*_{nl} = \sum_{n'=1}^{N} q_{n'} G_{n'l'} \sum_{l=1}^{L} G_{n'l} G^*_{nl} \simeq \sigma_n \frac{q_n}{|q_n|} G_{nl'}.$$

The real singular-value (square root of the eigenvalue of $K^* K$) is

$$\sigma_n = |q_n| \sum_{r=1}^{L} |G_{nr}|^2,$$

and where, for simplicity, we assumed that the localized and relatively small scatterers are well-separated so that

$$\mathcal{G}^T_{n'} \mathcal{G}^*_n = \sum_{l=1}^{L} G_{n'l} G^*_{nl} \simeq \left(\sum_{l=1}^{L} |G_{n'l}|^2 \right) \delta_{nn'}.$$

This statement is exactly right only for a single scatterer.

MUSIC: Theme & Variations (1)

MUSIC can be viewed as a method for determining whether or not each vector in a set of vectors is fully or only partially in the range of an operator. MUSIC stands for MUltiple SIgnal Classification.

If $T = KK^*$ is the operator of interest,
V_i is a known eigenvector of T, and
H_r is a vector from the test set
(i.e., a vector of Green's function propagators from test point r to all the members of the array).

MUSIC: Theme & Variations (2)

Next, we consider the "noise space" operator

$$n = \mathcal{I} - \sum_{i=1}^{N} V_i^* V_i^T = \mathcal{I} - \mathcal{R},$$

where $\mathcal{R}$ is the resolution operator (projecting onto the range space of the operators T and K).

We want to determine whether the test vector H_r is orthogonal to the noise space (and therefore in the reflector set). To do this we simply consider

$$H_r^T n H_r^* \simeq 0.$$

Think of this as a "fitting goal" for a reflection point in the model space. Also, define the square of the

direction cosine:

$$cos^2(V_i, H_r) = |V_i^T \cdot H_r^*|^2 / |H_r|^2.$$

MUSIC: Theme & Variations (3)

Assuming that r is a parameter (or vector) ranging over locations in space, then there are several related functionals we can plot in order to "image" the MUSIC classification of vector character, including:

$$cosec^2(\tilde{V}, H_r) = \frac{1}{1-\sum_{i=1}^{N} cos^2(V_i, H_r)}$$

and

$$cotan^2(\tilde{V}, H_r) = \frac{\sum_{i=1}^{N} cos^2(V_i, H_r)}{1-\sum_{i=1}^{N} cos^2(V_i, H_r)},$$

where $\tilde{V}$ is the set of vectors $V_1, \ldots, V_N$.

MUSIC: Theme & Variations (4)

Another variation on the MUSIC classification scheme is to consider a subset of the eigenvectors, and plot the incomplete versions (for $N' \leq N$) of the previous choices

$$cosec^2(\hat{V}, H_r) = \frac{1}{1 - \sum_{i=1}^{N'} cos^2(V_i, H_r)}$$

and

$$cotan^2(\hat{V}, H_r) = \frac{\sum_{i=1}^{N'} cos^2(V_i, H_r)}{1 - \sum_{i=1}^{N'} cos^2(V_i, H_r)},$$

where $\hat{V}$ is the set of vectors $V_1, \ldots, V_{N'}$.

MUSIC: Theme & Variations (5)

In particular, this scheme can be used in an iterative scheme for just a few eigenvectors at a time. Viewing eigenvectors as measurements, we see that using fewer eigenvectors will produce poorer resolution as less information is available to constrain the images.

Key point: It is **not necessary** to know the eigenvectors. It is sufficient to know a set of orthogonal vectors in the operator's range. What is important is to have information about the resolution operator $\mathcal{R}$, so the noise space operator can be determined by

$$n = \mathcal{I} - \mathcal{R}.$$

MUSIC: Theme & Variations (6)

The Stanford collaboration including Sergey Fomel & Marie and Robert Clapp has shown how to compute estimates of the resolution operators from a set of vectors coming from an iterative (Krylov subspace) scheme. If the orthonormal vectors coming from such a scheme (CG, LSQR, etc.) are θ_i, then

$$\mathcal{R} = \sum_i \theta_i^* \theta_i^T,$$

and, for example, the cosecant version of MUSIC is

$$cosec^2(\tilde{\theta}, H_r) = \frac{1}{1 - \sum_{i=1}^{N} cos^2(\theta_i, H_r)}.$$

This alternative is very advantageous for large data sets, as for example will always be present in 3D seismic surveys.

EXAMPLES

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CONCLUSIONS

- Treating the term "time-reversal data processing" as synonymous with "SVD of the transfer matrix" in acoustic/seismic reflection analysis, we have an array of possible imaging methods based on MUSIC and MUSIC-like methods, including resolution computation.
- The simplest method of all in this class involves direct iterative computation of the **diagonal resolution**.
- This method is entirely practical, and somewhat easier to implement than the full-blown T/R methods. But, for various applications (i.e., heterogeneous or cluttered environments), it can be important to use more sophisticated methods.
- Note: the examples were actually computed in the time domain.

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