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Spitz makes a better assumption for the signal PEF

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ABSTRACT

In real-world extraction of signal from data we are not given the needed signal prediction-error filter (PEF). Claerbout has taken S, the PEF of the signal, to be that of the data, $S \approx D$. Spitz takes it to be $S \approx D/N$. Where noises are highly predictable in time or space, Spitz gets significantly better results. Theoretically, a reason is that the essential character of a PEF is contained *where it is small.*

INTRODUCTION

Knowledge of signal spectrum and noise spectrum allows us to find filters for optimally separating data $\mathbf{d}$ into two components, signal $\mathbf{s}$ and noise $\mathbf{n}$ (Claerbout, 1999). Actually, it is the inverses of these spectra which are required. In Claerbout's textbook example (1999) he estimates these inverse spectra by estimating prediction-error filters (PEFs) from the data. He estimates both a signal PEF and a noise PEF from the same data $\mathbf{d}$. A PEF based on data $\mathbf{d}$ might be expected to be named the data PEF D, but Claerbout estimates two different PEFS from $\mathbf{d}$ and calls them the signal PEF S and the noise PEF N. They differ by being estimated with different number of adjustable coefficients, one matching a signal model (two plane waves) having three positions on the space axis, the other matching a noise model having one position on the space axis.

Meanwhile, using a different approach, Spitz (1999) concludes that the signal, noise, and data inverse spectra should be related by $D = SN$. The conclusion we reach in this paper is that Claerbout's estimate of S is more appropriately an estimate of the data PEF D. To find the most appropriate S and N we should use both the "variable templates" idea of Claerbout and the $D \approx SN$ idea of Spitz. Here we first provide a straightforward derivation of the Spitz insight and then we show some experimental results.

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BASIC THEORY

Signal spectrum plus the noise spectrum gives the data spectrum. Since a prediction-error filter tends to the inverse of a spectrum we have

$$\frac{1}{\overline{D}D} = \frac{1}{\overline{S}S} + \frac{1}{\overline{N}N} \tag{1}$$

$$\frac{1}{\overline{D}D} = \frac{\overline{S}S + \overline{N}N}{\overline{SN}SN} \tag{2}$$

or

$$\overline{D}D = \frac{\overline{SN}SN}{\overline{S}S + \overline{N}N} \tag{3}$$

Now we are ready for the Spitz approximation. Spitz builds his applications upon the assumption that we can estimate D and N from suitable chunks of raw data. His result may be obtained from (3) by ignoring its denominator getting $D \approx SN$ or

$$S \approx D/N \tag{4}$$

Ignoring the denominator in equation (3), is not so terrible an approximation as it might seem. Remember that PEFs are important *where they are small* because they are used as weighting functions. Where weighting functions are small, solutions are expected to be large. Although Claerbout's assumption $S \approx D$ might be somewhat valid for signal and data *spectra*, it is much less valid for their *PEFs*. In practice, signal unpolluted with noise is usually not available. Even a very good chunk of data tends to yield a poor estimate of the signal PEF S because the holes in the signal spectrum are easily intruded with noise.

Obviously the major difference between $S \approx D$ and $S \approx D/N$ is where the noise is large. Thus it is for "organized and predictable" noises (small N) where we expect to see the main difference.

Theoretically, we need not make the Spitz approximation. We could solve (1) for S by spectral factorization. Although the S obtained would be more theoretically satisfying, there would be some practical disadvantages. Getting the signal spectrum by subtracting that of the noise from that of the data leaves the danger of a negative result (which explodes the factorization). Thus, maintaining spectral positivity would require extra care. All these extra burdens are avoided by making the Spitz approximation. All the more so in applications with continuously varying estimates.

SIGNAL AND NOISE SEPARATION

We assume that the data vector **d** is composed of the signal and noise components **s** and **n**:

$$\mathbf{d} = \mathbf{s} + \mathbf{n} \, . \tag{5}$$

If both the signal and noise prediction-error filters S and N are known, then the signal can be extracted from the data by solving the following system by the least squares method:

$$0 \approx \mathbf{Nn} = \mathbf{N}(\mathbf{d}-\mathbf{s})\,; \tag{6}$$

$$0 \approx \epsilon\mathbf{Ss}\,, \tag{7}$$

where ϵ is a scalar scaling coefficient, reflecting the presumed signal-to-noise ration (Claerbout, 1999).

The formal solution of system (6-7) has the form of a *projection filter*:

$$\mathbf{s} = \left(\frac{\mathbf{N'N}}{\mathbf{N'N} + \epsilon^2\mathbf{S'S}}\right)\mathbf{d}\,. \tag{8}$$

Analogously, the signal vector is expressed as

$$\mathbf{n} = \mathbf{d}-\mathbf{s} = \left(\frac{\epsilon^2\mathbf{S'S}}{\mathbf{N'N} + \epsilon^2\mathbf{S'S}}\right)\mathbf{d}\,. \tag{9}$$

In 1-D or F-X setting, one can accomplish the division in formulas (8) and (9) directly by spectral factorization and inverse recursive filtering (Soubaras, 1995, 1994). A similar approach can be applied in the case of T-X or F-XY filtering with the help of the helix transform (Claerbout, 1998; Ozdemir et al., 1999) or by solving system (6-7) directly with an iterative method (Abma, 1995).

Claerbout's approach, implemented in the examples of *GEE* (Claerbout, 1999), is to estimate the signal and noise PEFs S and N from the data $\mathbf{d}$ by specifying different shape templates for these two filters. The filter estimates can be iteratively refined after the initial signal and noise separation. In some examples, such as those shown in this paper, the signal and noise templates are not easily separated. When the signal template behaves as an extension of the noise template so that the shape of S completely embeds the shape of N, our estimate of S serves as a predictor of both signal and noise. We might as well consider it as D, the prediction-error filter for the data.

Spitz (1999) argues that the data PEF D can be regarded as the convolution of the signal and noise PEFs S and N. This assertion suggests the following algorithm:

1. Estimate D and N.
2. Estimate S by deconvolving (polynomial division) D by N.
3. Solve the least-square system (6-7).

To avoid the division step, we suggest a simple modification of Spitz's algorithm, which results from multiplying both equations in system (6-7) by the noise filtering operator $\mathbf{N}$. The resulting system has the form

$$0 \approx \mathbf{N}^2\mathbf{n} = \mathbf{N}^2(\mathbf{d}-\mathbf{s})\,; \tag{10}$$

$$0 \approx \epsilon\mathbf{NSs} = \epsilon\mathbf{Ds}\,. \tag{11}$$

The modified algorithm is

1. Estimate D and N.
2. Convolve N with itself.
3. Solve the least-square system (10-11).

The formal least-squares solution of system (10-11) is

$$\mathbf{s} = \left(\frac{\mathbf{N'N'NN}}{\mathbf{N'N'NN} + \epsilon^2 \mathbf{D'D}} \right) \mathbf{d} = \left(\frac{\mathbf{N'N'NN}}{\mathbf{N'N'NN} + \epsilon^2 \mathbf{N'S'SN}} \right) \mathbf{d} \, . \tag{12}$$

Comparing (12) with (8), we can see that both the numerator and the denominator in the two expressions differ by the same multiplier $\mathbf{N'N}$. This multiplication should not effect the result of projection filtering.

Figure 1 shows a simple example of signal and noise separation taken from *GEE* (Claerbout, 1999). The signal consists of two crossing plane waves with random amplitudes, and the noise is spatially random. The data and noise T-X prediction-error filters were estimated from the same data by applying different filter templates. The template for D is

```
  a a
  a a
  a a
1 a a
a a a
a a a
a a a
```

where the `a` symbol represents adjustable coefficients. The data filter shape has three columns, which allows it to predict two plane waves with different slopes. The noise filter N has only one column. Its template is

```
1
a
a
a
```

The noise PEF can estimate the temporal spectrum but would fail to capture the signal predictability in the space direction. Figure 2 shows the result of applying the modified Spitz method according to equations (10-11). Comparing figures 1 and 2, we can see that using a modified system of equations brings a slightly modified result with more noise in the signal but more signal in the noise. It is as if ϵ has changed, and indeed this could be the principal effect of neglecting the denominator in equation (3).

To illustrate a significantly different result using the Spitz insight we examine the new situation shown in Figures 3 and 4. The wave with the positive slope is considered to be

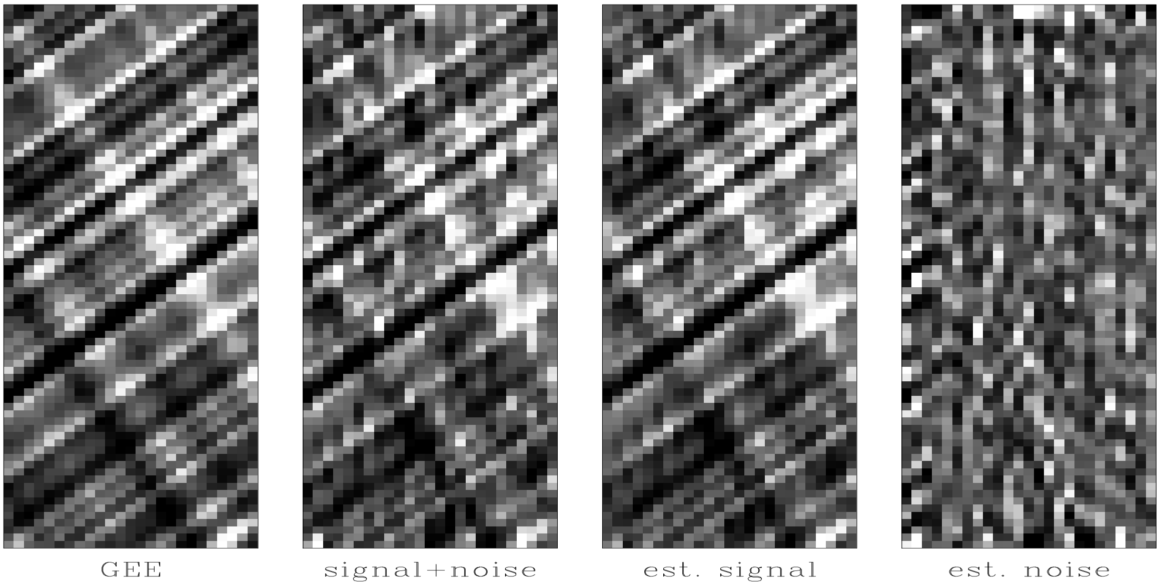

Figure 1: Signal and noise separation with the original GEE method. The input signal is on the left. Next is that signal with random noise added. Next are the estimated signal and the estimated noise. spitz-signoi90 [ER]

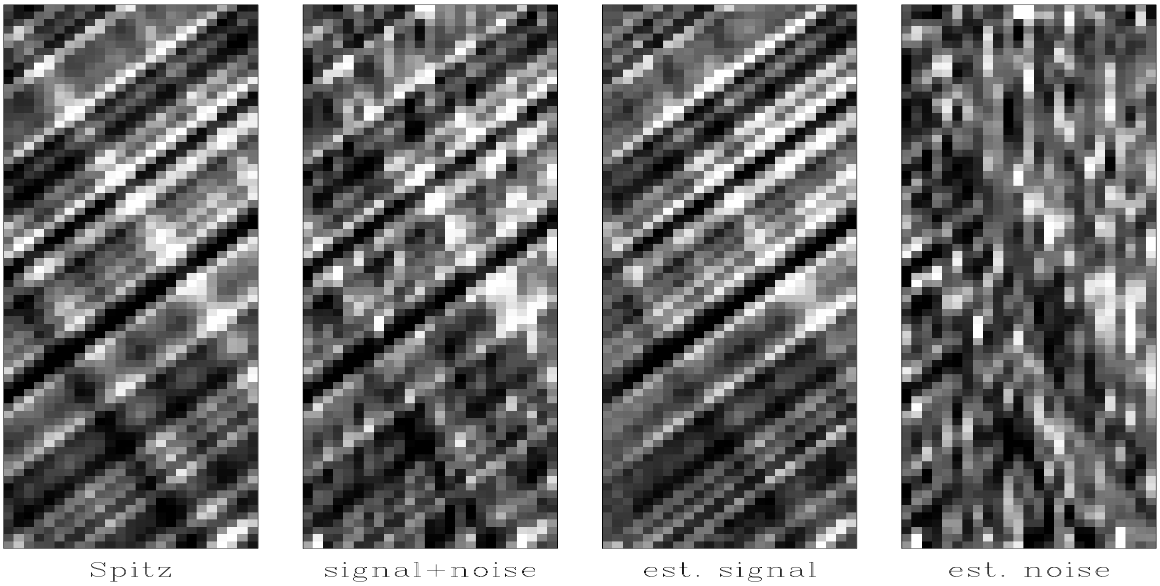

Figure 2: Signal and noise separation with the modified Spitz method. The input signal is on the left. Next is that signal with random noise added. Next are the estimated signal and the estimated noise. spitz-signoi [ER]

regular noise; the other wave is signal. The noise PEF N was estimated from the data by restricting the filter shape so that it could predict only positive slopes. The corresponding template is

```
  a
1 a
```

The data PEF template is

```
  a a
  a a
1 a a
a a a
a a a
```

Using the data PEF as a substitute for the signal PEF produces a poor result, shown in Figure 3. We see a part of the signal sneaking into the noise estimate. Using the modified Spitz method, we obtain a clean separation of the plane waves (Figure 4).

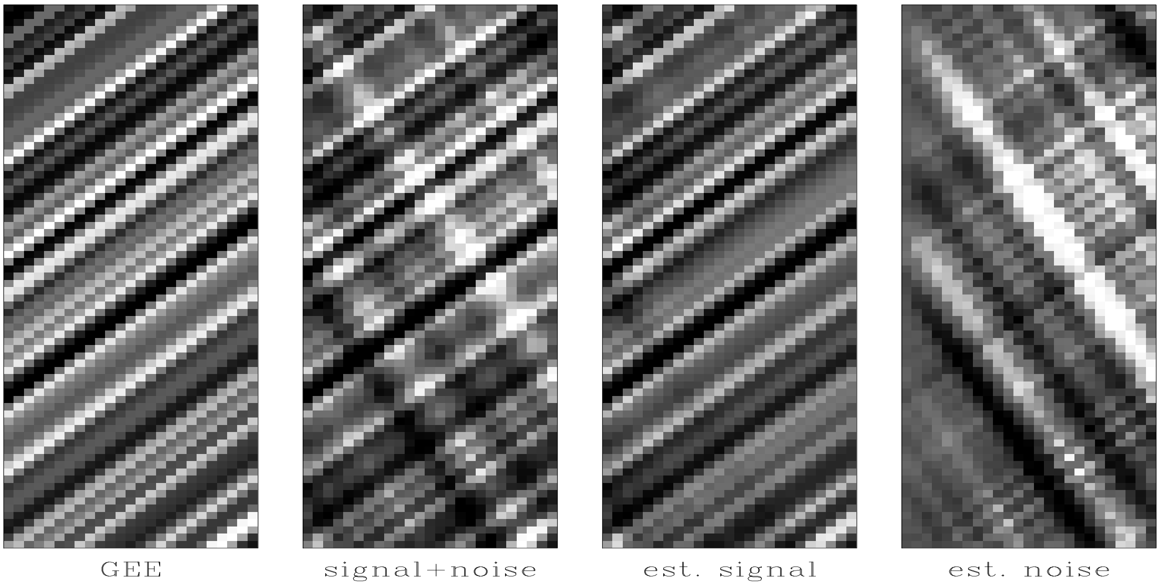

Figure 3: Plane wave separation with the GEE method. The input signal is on the left. Next is that signal with noise added. Next are the estimated signal and the estimated noise. spitz-planes90 [ER]

Clapp and Brown (1999, 2000) and Brown et al. (1999) show applications of the least-squares signal-noise separation to multiple and ground-roll elimination.

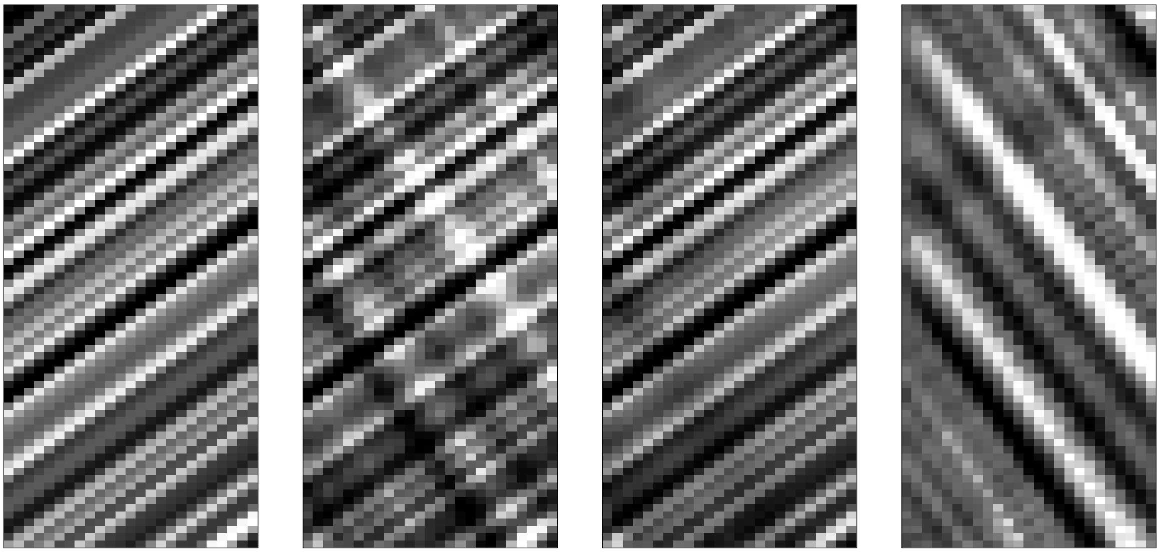

Figure 4: Plane wave separation with the modified Spitz method. The input signal is on the left. Next is that signal with noise added. Next are the estimated signal and the estimated noise. spitz-planes [ER]

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