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Amplitude preservation for offset continuation: Confirmation for Kirchhoff data[1]

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ABSTRACT

Offset continuation (OC) is the operator that transforms common-offset seismic reflection data from one offset to another. Earlier papers by the first author presented a partial differential equation in midpoint and offset to achieve this transformation. The equation was derived from the kinematics of the continuation process. This derivation is equivalent to proposing the wave equation from knowledge of the eikonal equation. While such a method will produce a PDE with the correct traveltimes, it does not guarantee that the amplitude will be correctly propagated by the resulting second-order partial differential equation. The second author (with J. K. Cohen) proposed a dip moveout (DMO) operator for which a verification of amplitude preservation was proven for Kirchhoff data. It was observed that the solution of the OC partial differential equation produced the same DMO solution when specialized to continue data to zero offset. Synthesizing these two approaches, we present here a proof that the solution of the OC partial differential equation does propagate amplitude properly at all offsets, at least to the same order of accuracy as the Kirchhoff approximation. That is, the OC equation provides a solution with the correct traveltime and correct leading-order amplitude. "Correct amplitude" in this case means that the transformed amplitude exhibits the right geometrical spreading and reflection-surface-curvature effects for the new offset. The reflection coefficient of the original offset is preserved in this transformation. This result is more general than the earlier results in that it does not rely on the two-and-one-half dimensional assumption.

INTRODUCTION

Offset continuation (OC) is the operator that transforms common-offset seismic reflection data to data with a different offset. Following the classic results of Deregowski and Rocca (1981), Bolondi et al. (1982; 1984) described OC as a continuous process of gradual change of the offset by means of a partial differential equation. Because it is based on the small-offset small-dip approximation, Bolondi's equation failed at large offsets or steep reflector dips. Nevertheless, the OC concept inspired a flood of research on dip moveout (DMO) correction (Hale, 1991). Since one can view DMO as a particular case of OC (continuation to zero offset),

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the offset continuation theory can serve as a natural basis for DMO theory. Its immediate application is in interpolating data undersampled in the offset dimension.

Fomel (1994; 1995a) recently introduced a revised version of the OC differential equation and proved that it provides the correct kinematics of the continued wavefield for any offset and reflector dip under the assumption of constant effective velocity. Studying the laws of amplitude transformation shows that in 2.5-D media the amplitudes of continued seismic gathers transform according to the rules of geometric seismics, except for the reflection coefficient, which remains unchanged (Fomel, 1995a; Goldin and Fomel, 1995). The solution of the boundary problem on the OC equation for the DMO case (Fomel, 1995b) coincides in high-frequency asymptotics with the amplitude-preserving DMO, also known as *Born DMO* (Liner, 1991; Bleistein, 1990). However, for the purposes of verifying that the amplitude is correct for any offset, this derivation is incomplete.

In this paper, we perform a direct test on the amplitude properties of the OC equation. We describe the input common-offset data by the Kirchhoff modeling integral, which represents the high-frequency approximation of a reflected (scattered) wavefield, recorded at the surface at nonzero offset (Bleistein, 1984). For reflected waves, the Kirchhoff approximation is accurate up to the two orders in the high-frequency series (the ray series) for the differential operator applied to the solution, with the first order describing the phase function alone and the second order describing the amplitude. We prove that both orders of accuracy are satisfied when the offset continuation equation is applied to Kirchhoff data. Thus, this differential equation is the "right" equation to two orders, producing the correct amplitude as well as the correct phase for offset continuation. That is, the geometric spreading effects and curvature effects of the reflected data are properly transformed. The angularly dependent reflection coefficient of the original offset is preserved.

This proof relates the OC equation with "wave-equation" processing. It also provides additional confirmation of the fact that the true-amplitude OC and DMO operators do not depend on the reflector curvature and can properly transform reflections from arbitrarily shaped reflectors (Bleistein and Cohen, 1995). The latter result was specifically a 2.5-D result, whereas the result of this paper does not depend on the 2.5-D assumption. That is, the result presented here remains valid when the reflector has out-of-plane variation.

Our method of proof is indirect. We first write the Kirchhoff representation for the reflected wave in a form that can be easily matched to the solution of the OC differential equation. We then present the analogues of the eikonal and transport equations for the OC equation and show that the amplitude and phase of the Kirchhoff representation satisfy those two equations.

THE KIRCHHOFF MODELING APPROXIMATION

In this section, we introduce the Kirchhoff approximate integral representation of the upward propagating response to a single reflector, with separated source and receiver points. We then show how the amplitude of this integrand is related to the zero-offset amplitude at the source receiver point on the ray, making equal angles at the scattering point with the rays from the separated source and receiver. The Kirchhoff integral representation (Haddon and Buchen, 1981; Bleistein, 1984) describes the wavefield scattered from a single reflector. This representation is applicable in situations where the high-frequency assumption is valid (the wavelength is smaller than the characteristic dimensions of the model) and corresponds in accuracy to the WKBJ approximation for reflected waves, including phase shifts through buried foci. The general form of the Kirchhoff modeling integral is

$$U_S(\mathbf{r},\mathbf{s},\omega) = \int_\Sigma R(\mathbf{x};\mathbf{r},\mathbf{s}) \frac{\partial}{\partial n} \left[U_I(\mathbf{s},\mathbf{x},\omega)\, G(\mathbf{x},\mathbf{r},\omega)\right] d\Sigma \;, \tag{1}$$

where $\mathbf{s} = (s,0,0)$ and $\mathbf{r} = (r,0,0)$ stand for the source and the receiver location vectors at the surface of observation; $\mathbf{x}$ denotes a point on the reflector surface Σ; R is the reflection coefficient at Σ; n is the upward normal to the reflector at the point $\mathbf{x}$; and U_I and G are the incident wavefield and Green's function, respectively represented by their WKBJ approximation,

$$U_I(\mathbf{s},\mathbf{x},\omega) = F(\omega)\, A_s(\mathbf{s},\mathbf{x})\, e^{i\omega\tau_s(\mathbf{s},\mathbf{x})} \;, \tag{2}$$

$$G(\mathbf{x},\mathbf{r},\omega) = A_r(\mathbf{x},\mathbf{r})\, e^{i\omega\tau_r(\mathbf{x},\mathbf{r})} \;. \tag{3}$$

In this equation, $\tau_s(\mathbf{s},\mathbf{x})$ and $A_s(\mathbf{s},\mathbf{x})$ are the traveltime and the amplitude of the wave propagating from $\mathbf{s}$ to $\mathbf{x}$; $\tau_r(\mathbf{x},\mathbf{r})$ and $A_r(\mathbf{x},\mathbf{r})$ are the corresponding quantities for the wave propagating from $\mathbf{x}$ to $\mathbf{r}$; and $F(\omega)$ is the spectrum of the input signal, assumed to be the transform of a bandlimited impulsive source. In the time domain, the Kirchhoff modeling integral transforms to

$$u_S(\mathbf{r},\mathbf{s},t) = \int_\Sigma R(\mathbf{x};\mathbf{r},\mathbf{s}) \frac{\partial}{\partial n} \left[A_s(\mathbf{s},\mathbf{x})\, A_r(\mathbf{x},\mathbf{r})\, f\left(t - \tau_s(\mathbf{s},\mathbf{x}) - \tau_r(\mathbf{x},\mathbf{r})\right)\right] d\mathbf{x} \,, \tag{4}$$

with f denoting the inverse temporal transform of F. The reflection traveltime τ_{sr} corresponds physically to the diffraction from a point diffractor located at the point $\mathbf{x}$ on the surface Σ, and the amplitudes A_s and A_r are point diffractor amplitudes, as well.

The main goal of this paper is to test the compliance of representation (4) with the offset continuation differential equation. The OC equation contains the derivatives of the wavefield with respect to the parameters of observation ($\mathbf{s}, \mathbf{r}$, and t). According to the rules of classic calculus, these derivatives can be taken under the sign of integration in formula (4). Furthermore, since we do not assume that the true-amplitude OC operator affects the reflection coefficient R, the offset-dependence of this coefficient is outside the scope of consideration. Therefore, the only term to be considered as a trial solution to the OC equation is the kernel of the Kirchhoff integral, which is contained in the square brackets in equations (1) and (4) and has the form

$$k(\mathbf{s},\mathbf{r},\mathbf{x},t) = A_{sr}(\mathbf{s},\mathbf{r},\mathbf{x})\, f\left(t - \tau_{sr}(\mathbf{s},\mathbf{r},\mathbf{x})\right) \;, \tag{5}$$

where

$$\tau_{sr}(\mathbf{s},\mathbf{r},\mathbf{x}) = \tau_s(\mathbf{s},\mathbf{x}) + \tau_r(\mathbf{x},\mathbf{r}) \,, \tag{6}$$

$$A_{sr}(\mathbf{s},\mathbf{r},\mathbf{x}) = A_s(\mathbf{s},\mathbf{x})\, A_r(\mathbf{x},\mathbf{r}) \;. \tag{7}$$

In a 3-D medium with a constant velocity v, the traveltimes and amplitudes have the simple explicit expressions

$$\tau_s(\mathbf{s},\mathbf{x}) = \frac{\rho_s(\mathbf{s},\mathbf{x})}{v}\;, \;\; \tau_r(\mathbf{x},\mathbf{r}) = \frac{\rho_r(\mathbf{x},\mathbf{r})}{v}\;, \tag{8}$$

$$A_s(\mathbf{s},\mathbf{x}) = \frac{1}{4\pi\,\rho_s(\mathbf{s},\mathbf{x})}\;, \;\; A_r(\mathbf{x},\mathbf{r}) = \frac{1}{4\pi\,\rho_r(\mathbf{x},\mathbf{r})}\;, \tag{9}$$

where ρ_s and ρ_r are the lengths of the incident and reflected rays, respectively (Figure 1). If the reflector surface Σ is explicitly defined by some function $z = z(\mathbf{x})$, then

$$\rho_s(\mathbf{s},\mathbf{x}) = \sqrt{(\mathbf{x}-\mathbf{s})^2 + z^2(\mathbf{x})}\;, \;\; \rho_r(\mathbf{x},\mathbf{r}) = \sqrt{(\mathbf{r}-\mathbf{x})^2 + z^2(\mathbf{x})}\;. \tag{10}$$

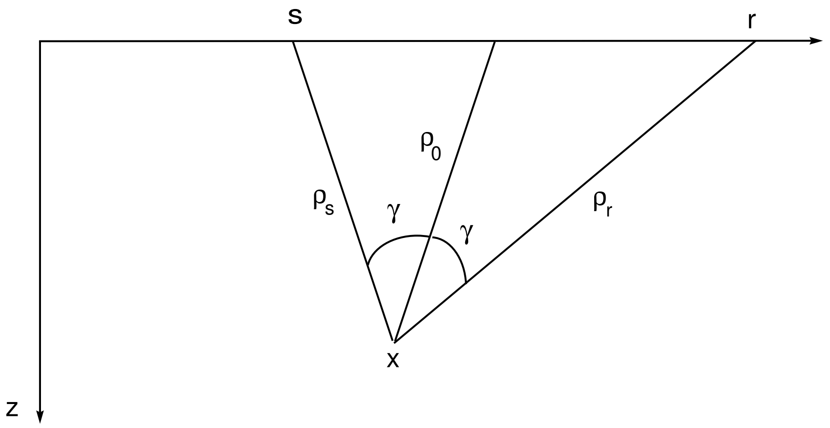

Figure 1: Geometry of diffraction in a constant velocity medium: view in the reflection plane. norm-cwpgen [NR]

We then introduce a particular zero-offset amplitude, namely the amplitude along the zero offset ray that bisects the angle between the incident and reflected ray in this plane, as shown in Figure 1. We denote the square of this amplitude as A_0. That is,

$$A_0 = \frac{1}{(4\pi\rho_0)^2}\;. \tag{11}$$

As follows from formulas (7) and (9), the amplitude transformation in DMO (continuation to zero offset) is characterized by the dimensionless ratio

$$\frac{A_{sr}}{A_0} = \frac{\rho_0^2}{\rho_s\,\rho_r}\;, \tag{12}$$

where ρ_0 is the length of the zero-offset ray (Figure 1).

As follows from the simple trigonometry of the triangles, formed by the incident and reflected rays (the law of cosines),

$$\sqrt{\rho_s^2+\rho_0^2-2\,\rho_s\,\rho_0\,\cos\gamma}+\sqrt{\rho_r^2+\rho_0^2-2\,\rho_r\,\rho_0\,\cos\gamma}= \\ =\sqrt{\rho_s^2+\rho_r^2-2\,\rho_s\,\rho_r\,\cos 2\gamma}\;, \tag{13}$$

where γ is the reflection angle, as shown in the figure. After straightforward algebraic transformations of equation (13), we arrive at the explicit relationship between the ray lengths:

$$\frac{(\rho_s+\rho_r)\,\rho_0}{2\,\rho_s\,\rho_r}=\cos\gamma\;. \tag{14}$$

Substituting (14) into (12) yields

$$\frac{A_{sr}}{A_0}=\frac{\tau_0}{\tau_{sr}}\cos\gamma\;, \tag{15}$$

where τ_0 is the zero-offset two-way traveltime ($\tau_0=2\,\rho_0/v$).

What we have done is rewrite the finite-offset amplitude in the Kirchhoff integral in terms of a particular zero-offset amplitude. That zero-offset amplitude would arise as the geometric spreading effect if there were a reflector whose dip was such that the finite-offset pair would be specular at the scattering point. Of course, the zero-offset ray would also be specular in this case.

THE OFFSET CONTINUATION EQUATION

In this section, we introduce the offset continuation partial differential equation. We then develop its WKBJ, or ray theoretic, solution for phase and leading-order amplitude. We explain how we verify that the traveltime and amplitude of the integrand of the Kirchhoff representation (4) satisfy the "eikonal" and "transport" equations of the OC partial differential equation. To do so, we make use of relationship (15), derived from the Kirchhoff integral.

The offset continuation differential equation derived in earlier papers (Fomel, 1994, 1995a) is

$$h\left(\frac{\partial^2 P}{\partial y^2}-\frac{\partial^2 P}{\partial h^2}\right)=t_n\,\frac{\partial^2 P}{\partial t_n\,\partial h}\;. \tag{16}$$

In this equation, h is the half-offset ($h=l/2$), y is the midpoint ($\mathbf{y}=(\mathbf{s}+\mathbf{r})/2$) [hence, $y=(r+s)/2$], and t_n is the NMO-corrected traveltime

$$t_n=\sqrt{t^2-\frac{l^2}{v^2}}\;. \tag{17}$$

Equation (16) describes the process of seismogram transformation in the time-midpoint-offset domain. One can obtain the high-frequency asymptotics of its solution by standard methods, as follows. We introduce a trial asymptotic solution of the form

$$P\left(y,h,t_n\right) = A_n(y,h)\, f\left(t_n - \tau_n(y,h)\right) \; . \tag{18}$$

It is important to remember the assumption that f is a "rapidly varying function," for example, a bandlimited delta function. We substitute this solution into equation (16) and collect the terms in order of derivatives of f. This is the direct counterpart of collecting terms in powers of frequency when applying WKBJ in the frequency domain. From the leading asymptotic order (the second derivative of the function f), we obtain the eikonal equation describing the kinematics of the OC transformation:

$$h\left[\left(\frac{\partial \tau_n}{\partial y}\right)^2 - \left(\frac{\partial \tau_n}{\partial h}\right)^2\right] = -\,\tau_n\,\frac{\partial \tau_n}{\partial h} \; . \tag{19}$$

In this equation, we have replaced a multiplier of t_n by τ_n on the right side of the equation. This is consistent with our assumption that f is a bandlimited delta function or some equivalent impulsive source. Analogously, collecting the terms containing the first derivative of f leads to the transport equation describing the transformation of the amplitudes:

$$\left(\tau_n - 2h\,\frac{\partial \tau_n}{\partial h}\right)\frac{\partial A_n}{\partial h} + 2h\frac{\partial \tau_n}{\partial y}\frac{\partial A_n}{\partial y} + h\,A_n\left(\frac{\partial^2 \tau_n}{\partial y^2} - \frac{\partial^2 \tau_n}{\partial h^2}\right) = 0\,. \tag{20}$$

We then rewrite the eikonal equation (19) in the time-source-receiver coordinate system, as follows:

$$\left(\tau_{sr}^2 + \frac{l^2}{v^2}\right)\left(\frac{\partial \tau_{sr}}{\partial r} - \frac{\partial \tau_{sr}}{\partial s}\right) = 2\,l\,\tau_{sr}\left(\frac{1}{v^2} - \frac{\partial \tau_{sr}}{\partial r}\frac{\partial \tau_{sr}}{\partial s}\right) \; , \tag{21}$$

which makes it easy (using Mathematica) to verify that the explicit expression for the phase of the Kirchhoff integral kernel (6) satisfies the eikonal equation for any scattering point[1] $\mathbf{x} = (x_1, x_2, z)$. Here, τ_{sr} is related to τ_n as t is related to t_n in equation (17).

The general solution of the amplitude equation (20) has the form (Fomel, 1995a)

$$A_n = A_0\,\frac{\tau_0 \cos\gamma}{\tau_n}\left(\frac{1+\rho_0\,K}{\cos^2\gamma + \rho_0\,K}\right)^{1/2} \; , \tag{22}$$

where K is the reflector curvature at the reflection point. Since the kernel (5) of the Kirchhoff integral (4) corresponds kinematically to the reflection from a point diffractor, we can obtain the solution of the amplitude equation for this case by formally setting the curvature K to infinity (setting the radius of curvature to zero). The infinite curvature transforms formula (22) to the relationship

$$\frac{A_n}{A_0} = \frac{\tau_0}{\tau_n}\cos\gamma \; . \tag{23}$$

[1]Note that the scattering point $\mathbf{x}$ plays the role of a set of parameters in the partial differential equation for τ_{sr}. To pass from a two-dimensional in-plane traveltime to a three-dimensional traveltime, one need only replace z^2 with $x_2^2 + z^2$. The role of $x = x_1$ remains unchanged in the solution.

Again, we exploit the assumption that the signal f has the form of the delta function. In this case, the amplitudes before and after the NMO correction are connected according to the known properties of the delta function, as follows:

$$A_{sr}\,\delta\left(t-\tau_{sr}(\mathbf{s},\mathbf{r},\mathbf{x})\right)=\left|\frac{\partial t_n}{\partial t}\right|_{t=\tau_{sr}} A_{sr}\,\delta\left(t_n-\tau_n(\mathbf{s},\mathbf{r},\mathbf{x})\right)=A_n\,\delta\left(t_n-\tau_n(\mathbf{s},\mathbf{r},\mathbf{x})\right)\;, \tag{24}$$

with

$$A_n=\frac{\tau_{sr}}{\tau_n}\,A_{sr}\;. \tag{25}$$

Combining equations (25) and (23) yields

$$\frac{A_{sr}}{A_0}=\frac{\tau_0}{\tau_{sr}}\,\cos\gamma\;, \tag{26}$$

which coincides exactly with the previously found formula (15). As with the solution of the eikonal equation, we pass from an in-plane solution in two dimensions to a solution for a scattering point in three dimensions by replacing z^2 with $x_2^2+z^2$.

It is apparent that the OC differential equation (16) and the Kirchhoff representation have the same effect on reflection data, because the amplitude and phase of each matches those of the other. Thus, we see that the amplitude and phase of the Kirchhoff representation for arbitrary offset is the point diffractor WKBJ solution of the offset continuation differential equation. Hence, the Kirchhoff approximation is a solution of the OC differential equation when we hold the reflection coefficient constant. This means that the solution of the OC differential equation has all the features of amplitude preservation, as does the Kirchhoff representation, including geometrical spreading, curvature effects, and phase shift effects. Furthermore, in the Kirchhoff representation and the solution of the OC partial differential equation by WKBJ, we have not used the 2.5-D assumption. Therefore the preservation of amplitude is not restricted to cylindrical surfaces as it is in Bleistein's and Cohen's (1995) true-amplitude proof for DMO. This is what we sought to confirm.

DISCUSSION

We have proved that the offset continuation equation correctly transforms common-offset seismic data modeled by the Kirchhoff integral approximation. The kinematic and dynamic equivalence of the OC equation has been proved previously by different methods (Fomel, 1995a,b). However, connecting this equation with Kirchhoff modeling opens new insights into the theoretical basis of DMO and offset continuation:

1. The Kirchhoff integral can serve as a link between the wave-equation theory, conventionally used in seismic data processing, and the kinematically derived OC equation. Though the analysis in this paper follows the constant-velocity model, this link can be extended in principle to handle the case of a variable background velocity.

2. The OC equation operates on the kernel of the Kirchhoff integral, which is independent of the local dip and curvature of the reflector. This proves that the true-amplitude OC and DMO operators can properly transform reflections from curved reflectors. Moreover, this result does not imply any special orientation of the reflector curvature matrix. Therefore, it does not require a commonly made 2.5-D assumption (Bleistein and Cohen, 1995; Fomel, 1995a). Implicitly, this fact proves the amplitude preservation property of the three-dimensional azimuth moveout (AMO) operator (Biondi and Chemingui, 1994; Fomel and Biondi, 1995), based on cascading the true-amplitude DMO and inverse DMO operators.

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