Purpose. Maps of discontinuity attributes suppress the sedimentary layers of a three-dimensional seismic image and delineate fault planes, channel sands, salt boundaries, and other sedimentary layer boundaries. Good discontinuity maps help interpret seismic image volumes rapidly and reliably.

Literature. Interpreters of seismic images derive structural and stratigraphic models of the subsurface geology . Bahorich and Farmer (**?**) introduced an image enhancement technique that delineates discontinuities, such as faults, in seismic image volumes. Subsequently, various other authors (Bednar, 1998; Marfurt et al., 1998) published similar tools.

My investigation. In this presentation, I compare three increasingly sophisticated seismic discontinuity attribute families. The techniques comprise standard edge enhancements (e.g., gradient magnitude), misfit of the best-fitting plane wave (correlation), and prediction error (removal of the linearly predictable image component). Each technique is tested at a simple synthetic example that simulates a normal fault (Figure 1) and at two genuine seismic image volumes (e.g., Figure 2).

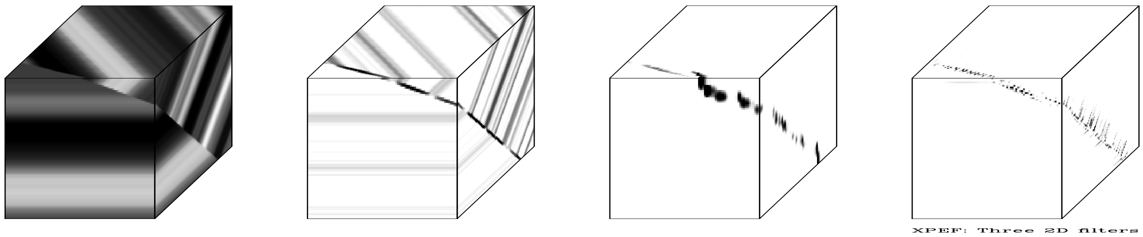

Figure 1: Synthetic fault image and its discontinuity maps. From left to right: The original seismic image, the magnitude of the image gradient, the misfit of the best-fitting local plane wave, and the prediction-error. $\boxed{\text{zeroFoltFour}}$ [ER]

Gradient Magnitude. The magnitude of the local image gradient $d_{grd} = |\nabla g(\mathbf{x})|$ of the seismic image volume g is a traditional edge enhancement. As expected, the gradient-magnitude map fails to detect seismic discontinuities, since the operator does not distinguish between amplitude edges due to faults and due to sedimentary layers.

Plane-wave misfit. The assumption that the seismic image of undisturbed sedimentary layers locally resembles a plane wave leads to an alternative discontinuity formulation. A simple misfit measure between the image and its local best-fitting plane waves yields a discontinuity attribute map.

To compute the discontinuity map, I split the original image volume into small overlapping image regions that are generally well approximated by local plane waves. For each image region g, I estimate the best-fitting plane wave $f = f(\mathbf{p}\cdot\mathbf{x})$, where f is the one-dimensional waveform of the plane wave and $\mathbf{p}$ is the corresponding plane-wave normal. To measure the misfit between the plane-wave image and the original image, I compute the normalized correlation coefficient between the individual corresponding traces. To represent the correlation coefficients in a discontinuity image region of equal size as the input image, I set all samples of the output trace to the one corresponding coefficient:

$$trace(d_{cor}) = Cor_z\left[trace(f), trace(g)\right] = const.$$

The coefficient is zero, if the original image and the plane wave correlate perfectly; the coefficient is nonzero, if the plane wave and the original image differ. Finally, I carefully interpolate the overlapping image regions to the original image volume.

In general, the technique reliably delineates a wide range of faults and discontinuities. I found the correlation scheme superior to other alternative misfit measures of the best-fitting plane wave. However, the scheme's resolution is limited to the size of the local image regions. If an image region contains a fault, the plane-wave assumption is violated, the local image region is nonstationary, and the entire local region yields a significant residual.

The plane-wave correlation method seems similar to commercially available seismic discontinuity attributes. There is some uncertainty, however, since some publications only outline the fundamental approach and do not state any technical details.

Plane-wave estimation. The previous discontinuity attribute led me to develop an innovative plane-wave estimation that should be useful for other applications. The formulation directly and efficiently leads to an analytic plane-wave estimate. The approach is related to Symes' (1994) differential semblance optimization and Claerbout's (1994) dip-estimation scheme without picking.

For a given plane-wave g, the unknown plane-wave normal $\mathbf{p}$ satisfies the crossproduct $\mathbf{p}\times\nabla g = 0$. The constraint of a nonzero normal $\mathbf{p}$ is easily and without loss of generality implemented as $\mathbf{p} = (p_x, p_y, 1)$. The crossproduct is then simply

$$\begin{pmatrix} 0 & \mathbf{g}_z \\ \mathbf{g}_z & 0 \\ \mathbf{g}_y & -\mathbf{g}_x \end{pmatrix} \begin{pmatrix} p_x \\ p_y \end{pmatrix} = \begin{pmatrix} \mathbf{g}_y \\ \mathbf{g}_x \\ 0 \end{pmatrix},$$

where $\mathbf{g}_x$, $\mathbf{g}_y$, and $\mathbf{g}_z$ are vectors of the partial derivatives of g. The corresponding normal equations can be solved analytically for the two unknown normal vector components $\mathbf{p}_x$ and $\mathbf{p}_y$. A carefully normalized stack of the image along planes orthogonal to the estimated normal vector yields an estimate of the waveform $f = f(\mathbf{p}\cdot\mathbf{x})$. **Prediction error.** The third discontinuity attribute is based on a prediction-error operator that directly estimates and removes the local plane-wave component of a given seismic image. At each sample, the prediction-error filter A removes the image component that is predictable as a causal linear combination of its neighboring samples. The prediction error is zero everywhere, if the image samples are all predictable by the filter. Three two-dimensional prediction-error filters in orthogonal planes (A_x, A_y, A_z) generate three zero images if and only if the input image is a plane-wave volume. Given an image that is not a perfect plane wave, the three output images are nonzero.

I use the combination of such three prediction-error filters and their adjoints to estimate the plane-wave misfit. To compute a discontinuity map, I again split the original image into local overlapping regions. I estimate the coefficients of the three two-dimensional prediction-error filters for each isolated region. The three prediction-error images indicate the misfit of the local plane wave. To unite the output of the three filters to a single image, I additionally apply the adjoint of the three filters:

$$d_{pef} = \left[A_x, A_y, A_z\right]\left[A_x, A_y, A_z\right]^T g.$$

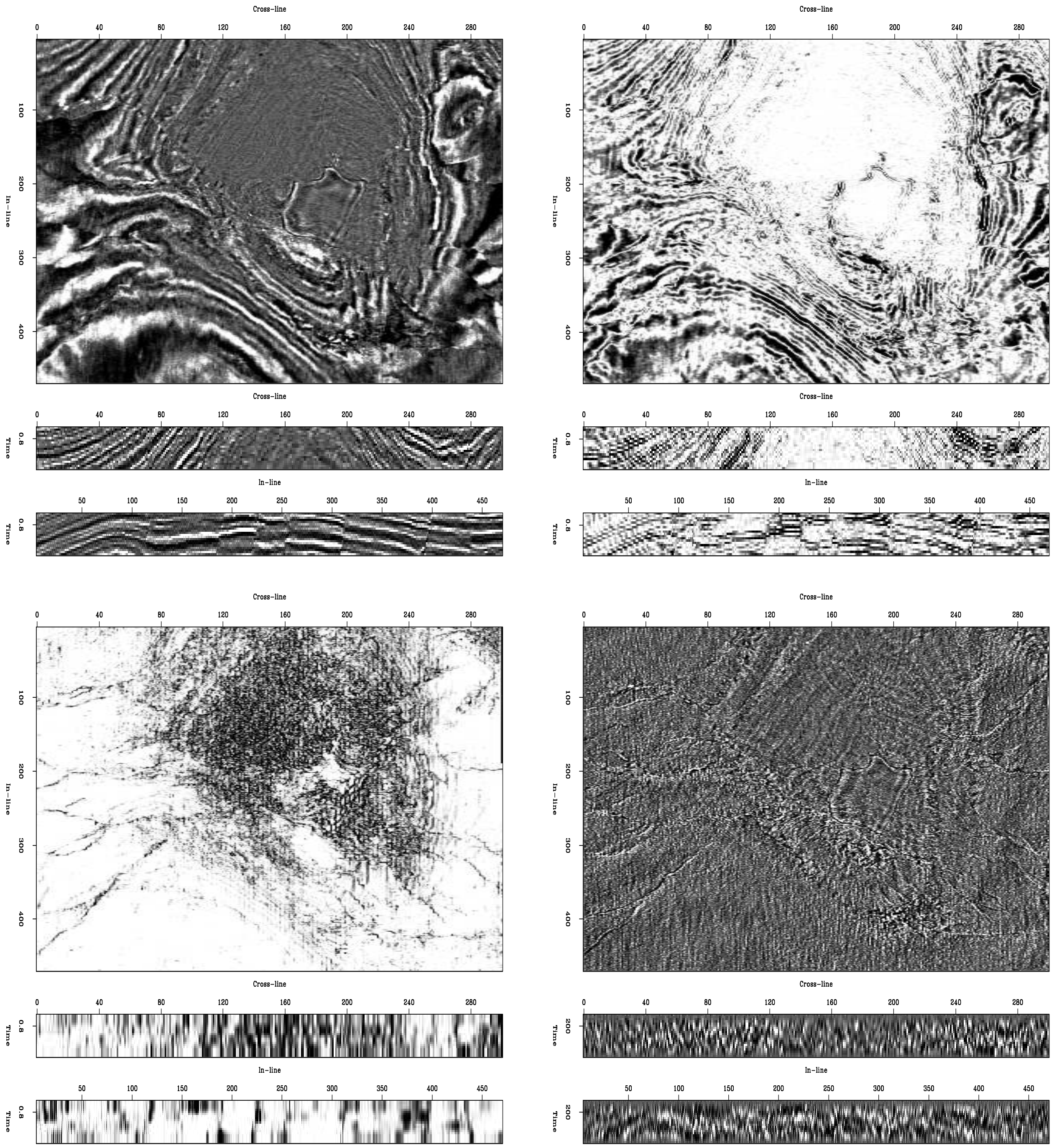

Figure 2: Gulf salt dome and its discontinuity maps. In clockwise direction: The original seismic time slice, the magnitude of the image gradient, the misfit of the best-fitting local plane wave, and the prediction-error. gulfFoltTotFour [ER]

Finally, I merge the local overlapping image regions to the original image volume.

Overall, the prediction error lacks the reliability of the plane-wave misfit approach, but when successful, the prediction-error maps are very detailed. The approach successfully maps sharp image discontinuities. Contrary to my initial expectation, smooth fault zones are, however, predicted and, consequently, removed. The discontinuity maps of such smooth images resemble white noise. Why? By its very nature, a smooth image discontinuity that is a few pixels wide is predictable at that local scale and the locally-adapted prediction-error filter tends to remove the discontinuity. Furthermore, the resolution of the prediction-error scheme is limited to the size of the process' local image region. The local filters predict and remove superpositions of plane waves but not the adjacent yet separate plane waves of a fault. In summary, the prediction error of a smooth discontinuity is smaller than I initially expected and the error of a fault's adjacent plane-wave volumes is larger and not focused at the faults but spreads in the entire local image region.

Summary. The successful discontinuity measures of this publication – plane-wave misfit and prediction error – suppress image regions that locally resemble plane-wave (sedimentary) layers. Image regions that significantly violate the plane-wave assumption are indicated by large residuals. The residual is not isolated at a region's fault but spreads across the entire local image region that was the base of the plane-wave or prediction-error-filter estimation. A flexible segmentation of the seismic image into amorphous stationary image regions (rather than the easily programmed rectangular regions used here) might lead to the automatic and reproducible interpretation and analysis of seismic images.

Reproducible research in Java. The results and the underlying software is available at the World Wide Web site of the Stanford Exploration Project [1].

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[1] http://sepwww.stanford.edu/sep/matt