Trace interpolation by simulating a prediction-error filter

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ABSTRACT

By simulating a prediction-error filter using a dip-spectrum, I devise an interpolation scheme. The procedure has three stages: (1) First of all, the dips of linear events in a given data set are estimated from the dip spectrum obtained from the slant stack. (2) Next, the filters in the (f, x) domain are invented by putting zeros along the dips picked in the (f, k) spectrum. (3) Finally, missing traces are found by minimizing the filtered output in the least square sense. The advantage of this approach lies in its application to both regularly and irregularly sampled traces.

INTRODUCTION

Seismic trace interpolation has long been used to overcome spatial aliasing and trace regridding problems. One of the most popular methods for seismic trace interpolation has been to search multitrace data for the locally most coherent dips and to interpolate amplitudes along those dip directions in the time-space domain (Larner et al, 1981). But this type of interpolation algorithm cannot correctly handle more than one dip at a point in the time-space domain.

Recently, Claerbout (1991) and Spitz (1991) have introduced interpolation schemes using a prediction error filter in the time-space domain and in the frequency-space domain, respectively. Both approaches can handle multi-dips at a position, but Spitz's method fails to find prediction error filters when the given data are completely aliased in space. To overcome this limit, Zhang et al.(1992) developed an algorithm which can interpolate in such situations. All these approaches use prediction-error filters to find dips. Although being efficient, they cannot be applied to irregularly sampled

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traces.

This paper describes an algorithm that combines the advantages of both approaches: the approach of linear interpolation and the approach of prediction-error filter. In the first section, I explain a new algorithm with three steps and the relation to prior algorithms. In the second section, the new method is tested with both synthetic data and real data, which have regular trace sampling interval. The last section describes a way to apply the new method for regridding of irregularly sampled traces and also shows the results of experiments with synthetic data.

THREE STEP INTERPOLATION

Many trace interpolation schemes function in two stages. The first stage finds dips of plane waves or finds filters which kill dips in a given data set. The second stage linearly interpolates missing traces along the dips found or finds missing traces in the least square sense by minimizing the filtered output.

In order to interpolate multi-dips which cross each other, the approach that finds missing traces by minimizing filtered output is preferable. In this approach, the filter's spectrum should be the inverse of the data spectrum desired after interpolation. For the filter which has an inverse spectrum, an approach, such as Claerbout's (1991) and Spitz's (1991) algorithm, uses a prediction-error filter (PEF) obtained from a low frequency portion of data (Ji, 1991). A shortcoming of the approach using the prediction error filter is that we cannot obtain the filter when the missing traces are randomly located among traces. However, an approach that interpolates linearly along the most coherent dip at a point presents no difficulty in such a situation.

In order to overcome the pitfalls of both approaches, I propose a three-step algorithm. The step of finding missing traces uses the approach of minimization of the filtered output in a least square sense because this approach correctly handles the amplitude when dips cross each other. The step of finding a prediction-error filter can be understood as finding zeros in the spectrum to minimize the filtered output. Therefore the spectrum of the prediction error filter resembles the inverse spectrum of a given data set. If we decompose the step of finding the prediction-error filter into two parts as a step for finding dips and a step for finding filters which kill dips found, the following three steps can be used as an interpolation scheme:

1. Pick several dominant dips from the dip spectrum.
2. Find a filter which kills the dips found.
3. Find missing traces that minimize the filtered output in the least square sense.

Dip picking

The most important step in the interpolation is the dip picking step, because if the dips picked are wrong, the remaining interpolation steps have no meaning. Many techniques, such as the slant stack method, the eigen-structure method (Biondi and Kostov, 1989) and the plane wave destructor method (Claerbout, 1991), can be used to find dips. Among these methods, I have chosen the slant stack method because it can be applied to any kind of missing traces problem such as randomly missing and irregularly sampled traces.

After $\tau - p$ transform, a dip spectrum is made by picking maximum absolute value on the τ axis for each ray parameter. The dip spectrum usually looks too spiky and needs some smoothing for picking dominant dips. The criteria for picking dips from the dip spectrum vary depending on the data quality and the purposes of interpolation. In the approach using PEF, the number of coefficients of the filter determines the number of dips to pick. In the dip spectrum, one can pick several dominant dips like the PEF approach, or any number of dips according to a given criterion. Thus the dip picking from the dip spectra is more flexible in deciding the dips to interpolate.

Simulating PEF using dips picked

In the case of randomly missing traces in regularly sampled data, a prediction-error filter cannot be found. This section explains an alternative way to find a filter whose spectrum is the inverse of the given data's spectrum.

If there exist several linear events in a data set, the zeros which were found by the prediction-error filter would locate along the dips in the spectrum. If we know the dips of the events, therefore, we can simulate the prediction-error filter by putting zeros along the dips in the spectrum.

Suppose we have N plane waves and each dip is p_j, $j = 1, ..., N$, respectively. For each frequency ω, then, N zeros along the wavenumber are determined as follows :

$$k_j = p_j \omega$$

and the corresponding zeros are

$$z_j = \exp(-ik_j).$$

From those zeros, the prediction-error filter for each ω is determined in the form of the Z transform like

$$F(z) = (1 - z/z_1)(1 - z/z_2)\dots(1 - z/z_N).$$

Interpolation

This step can be accomplished either by interpolating linearly along the picked dip or by finding missing traces which minimize the filtered output. A shortcoming of the former approach is that it cannot handle more than one dip at a point, which happen at the crossing of linear events. I choose the latter approach, which finds missing traces by minimizing the filtered output using the filter simulated from the previous section.

For minimization, I use Claerbout's conjugate-gradient algorithm (Claerbout, 1991) because other algorithms must formulate the problem in the form of known values, splitting unknown values, which is very difficult if unknown values are randomly spread among known values.

Summary of the algorithm

The algorithm proposed in this paper assumes that all events are linear. In many cases, the events in a seismic dataset are not linear, but are considered as quasi-linear across the space-grid lines. In other words, we can assume the seismic events are piecewise linear and the events on a small window in time-space domain can be considered as linear events. The algorithm for interpolating regularly sampled traces is applied to each small window and is summarized as follows:

```
for each subwindow
[   Slant stack d̂(τ,p) = Σ_x d(τ − px, x)
    find dip spectrum sptrm(p) = Max_τ|d̂(τ,p)|
    pick several dominant dips p_i from sptrm(p)
    D(f,x) = 1DFFT[d(t,x)]
    for each frequency
    [   construct filter P(f) from p_i
        compute D̂(f,x) with least squares
    d̂(t,x) = 1DFFT^{-1}[D̂(f,x)]
    merge subwindows
```

NUMERICAL EXAMPLES

Synthetic data

In this section I discuss the results obtained by applying the scheme to three synthetic data. Figure 1a and 1b show a synthetic data set which contains three linear events and its spectrum, respectively. From the slant stack of the data, the dip spectrum shown in Figure 1c was obtained by cuttting the noisy low amplitude portion followed by smoothing after picking the maximum absolute values along the τ for each ray parameter. The prediction-error filter is generated by putting zeros along the dips picked and its spectrum, shown in Figure 1d. The interpolated result, Figure 1e, shows quite well aligned linear events and the spectrum after interpolation, Figure 1f, shows no more aliasing.

For testing the applicability to noisy data, random noise with the amplitude of .2 times of signal amplitude was added to the synthetic data set shown in Figure 1a. Figure 2a and 2b show such a synthetic data and its spectrum. After interpolation, Figure 2e shows a good interpolation except that noises are also interpolated along the picked dips. Figure 2f shows the spectrum after interpolation and it shows clearly the effect to the noises. But this artifact does not cause a serious problem because the noise level is relatively low and noises are only locally interpolated.

Figure 3a shows a synthetic data set which contains several linear events with some background noise and 50 percent of the traces are randomly missing. For this case, again, the dip spectrum picks three dominant dips as in the interlacing case. Since there are fewer constraints in the region of more missing traces, the convergence of this model is a little bit slower than that of the interlacing case. The result of interpolation, Figure 3e, shows quite good interpolation along the given dips except that the noise region tends to line up along the dip. But the amplitude of the noise is relatively low and does not affect the spectrum too much.

Real data

In Figure 4a, a crosswell data set has been windowed, to provide a small test data set and subsampled using $2\Delta x$. The result of interpolation from the subsampled version is shown in Figure 4e. For comparison between the original and interpolated, Figure 5 shows the original data set before subsampling and the difference with the interpolated. Figure 5c and 5d are the spectrum of the original data and of the interpolated from the subsampled version. These show almost the same pattern except that a positive dip event is very weakly interpolated, but this is because the positive dip component in the spectrum of the subsampled data set is very weak.

REGRIDDING IRREGULARLY SAMPLED TRACES

For regridding irregularly sampled traces, the dip picking stage is the same as the case of regularly sampled traces because slant stack works in both cases. We cannot use the simple Z-transform technique, however, to simulate the prediction error filter for irregularly sampled case.

The filtering operation in a continuous signal is a convolution as follows:

$$y(t) = \int f(\tau)x(t-\tau)d\tau \tag{1}$$

where $y(t)$ is an filtered output, $x(t)$ is an input and $f(t)$ is a filter. In the case of a discrete signal, the integral operator can be approximated by summation operator as follows

$$y(t_i) = \sum_j f(\tau_j)x(t_i - \tau_j)\Delta\tau_j. \tag{2}$$

If discrete points are uniformly sampled, $d\tau_j$ is always same so that it can be dropped. But for irregularly sampled signal, the $d\tau_j$ must be considered in the convolution formula because it will vary with sampling intervals. If we write a filtering operation as a form of matrix multiplication, the matrix which corresponds to this operation

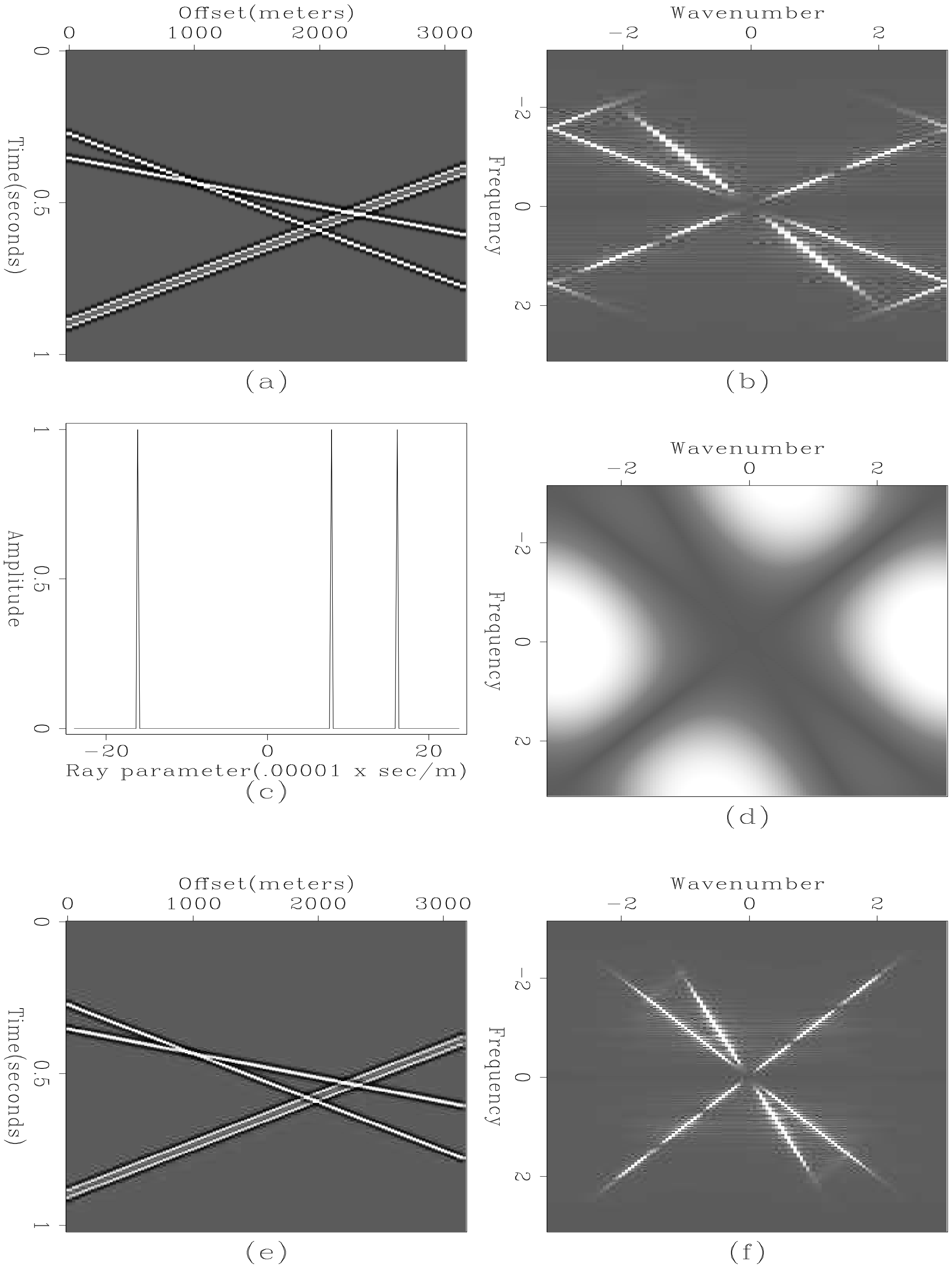


Figure 1: (a) A synthetic data set with three dipping events (b) the spectrum of the synthetic data set (c) The dips picked from slant stack (d) the spectrum of the prediction-error filters (e) The interlaced data set (f) the spectrum of the interlaced data set synlace [R]

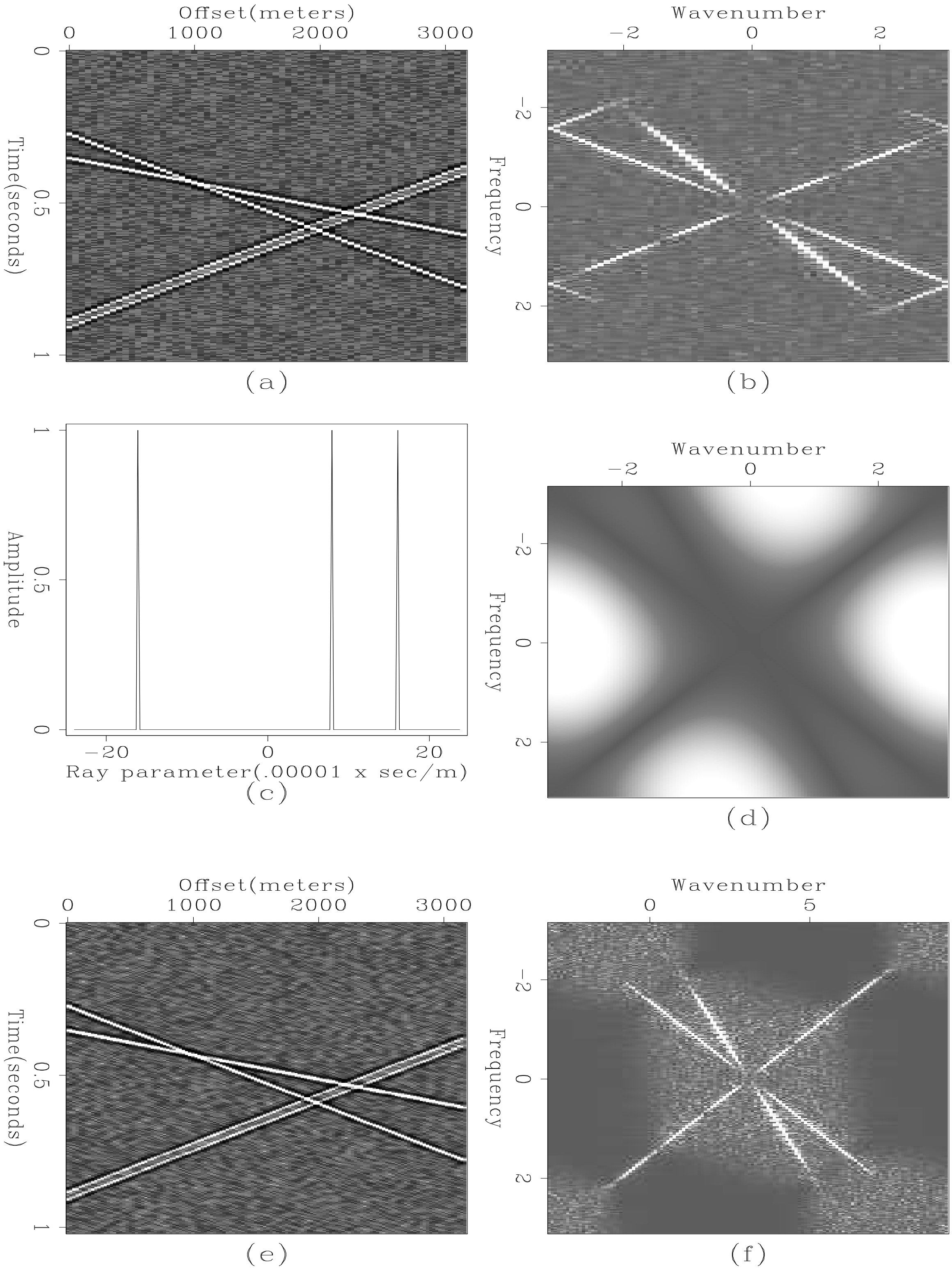


Figure 2: (a) A synthetic data set with three dipping events and additional noise (b) the spectrum of the synthetic data set (c) The dips picked from slant stack (d) the spectrum of the prediction-error filters (e) The interlaced data set (f) the spectrum of the interlaced data set synlacewn [R]

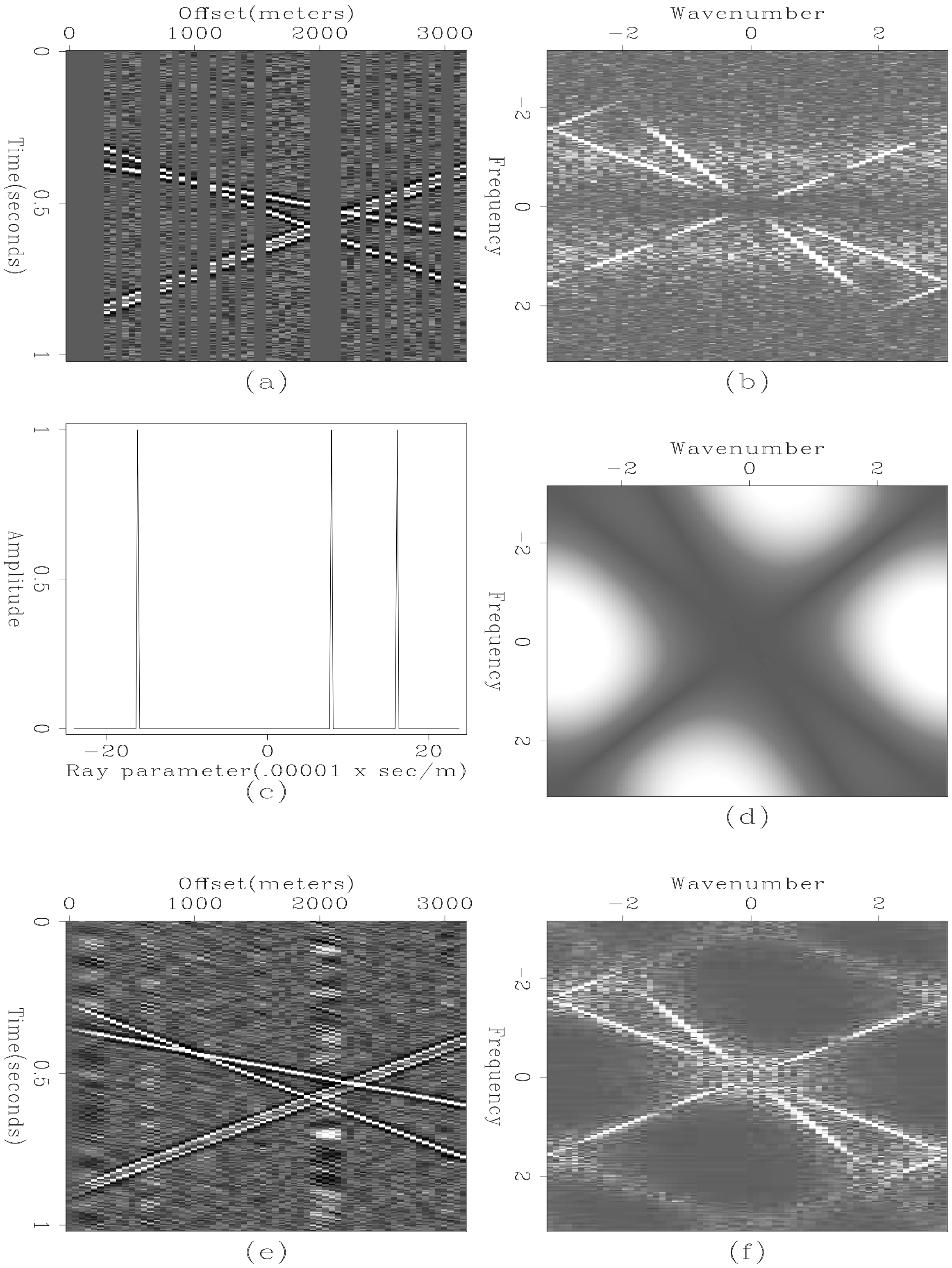


Figure 3: (a) A synthetic data set with randomly missing traces (50 percent of total traces are missing) (b) the spectrum of the synthetic data set (c) The dips picked from slant stack (d) the spectrum of the filter simulated (e) The interpolated data set (f) the spectrum of the interpolated data set synmisswn [R]

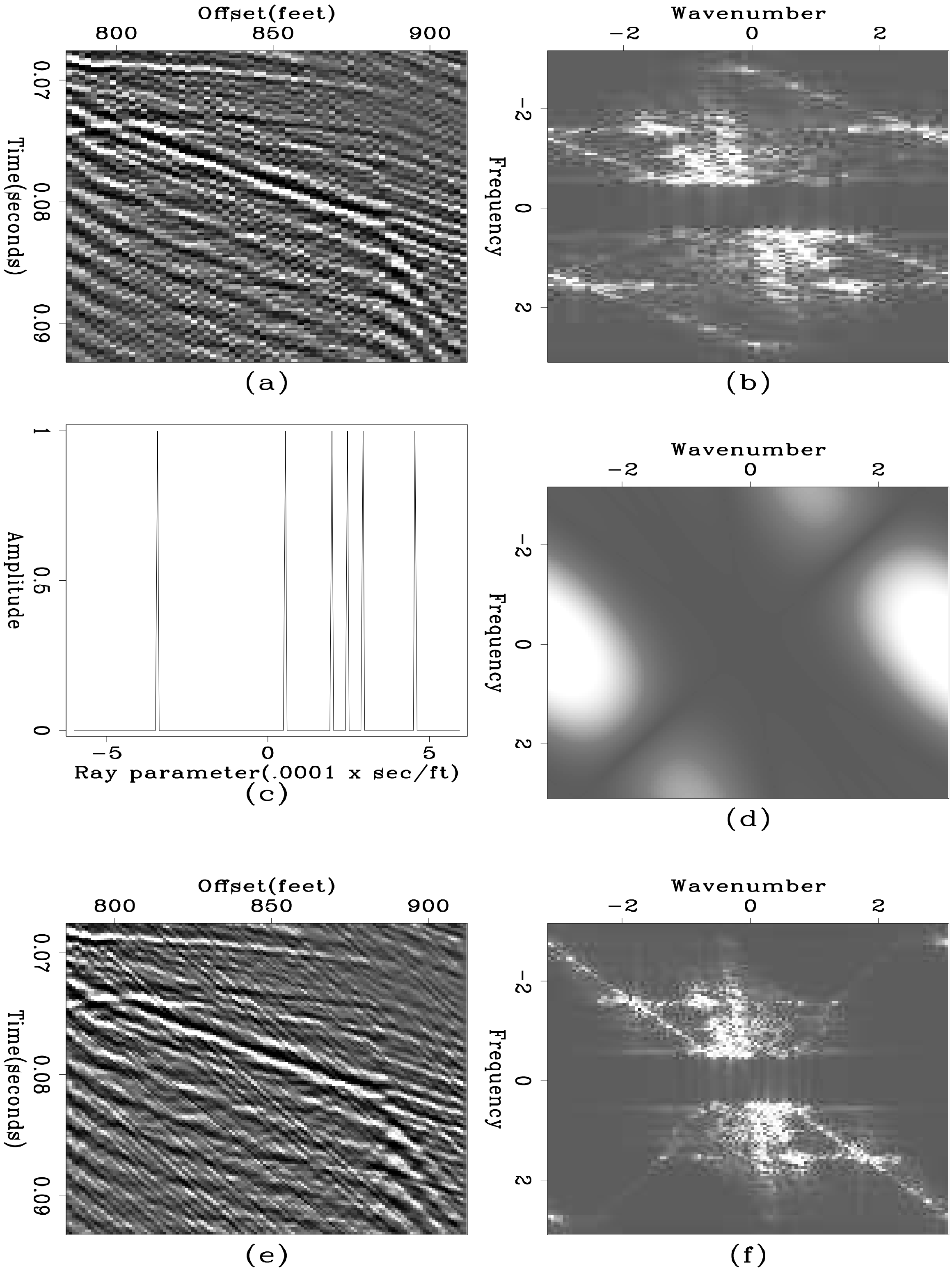


Figure 4: (a) A real crosswell data set which is subsampled with 2 feet sampling intervalfrom Figure 6a where sampling interval is 1 foot. (b) the spectrum of the crosswell data set (c) the dips picked from slant stack (d) the spectrum of the prediction-error filters (e) the interlaced data set (sampling interval is 1 foot) (f) the spectrum of the interlaced data set. cwlace [R]

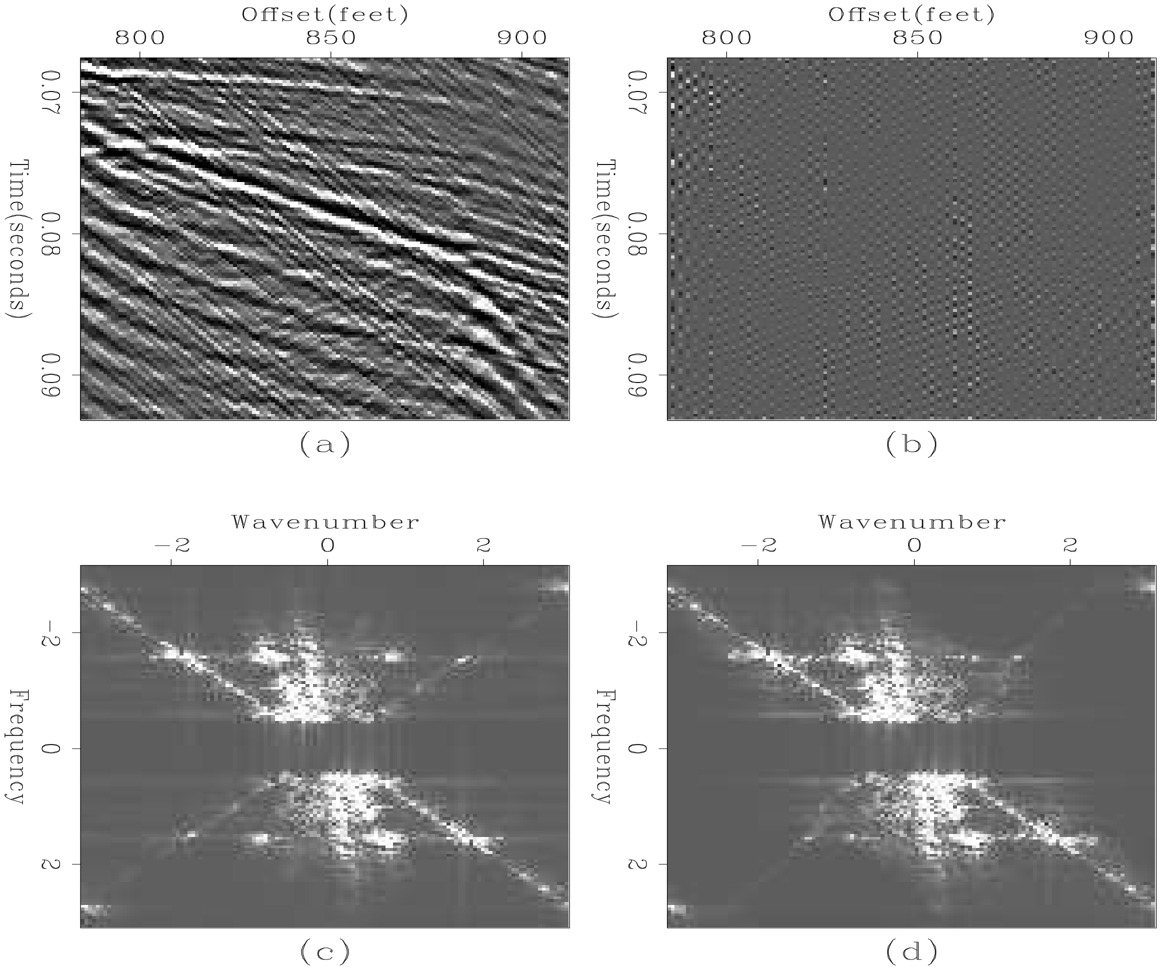


Figure 5: (a) A real crosswell data set (b) the difference between the original data and interpolated data set (= Figure 6a - Figure 5e) (c) the spectrum of the original data (d) the spectrum of the interpolated data set from the subsampled version. $\boxed{\text{cwlacediff}}$ [R]

will have a structure as shown below.

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix} = \begin{bmatrix} f_{01}\Delta\tau_r & & & & \\ f_{11}\Delta\tau_1 & f_{02}\Delta\tau_r & & & \\ f_{21}\Delta\tau_1 & f_{12}\Delta\tau_2 & f_{03}\Delta\tau_r & & \\ f_{31}\Delta\tau_1 & f_{22}\Delta\tau_2 & f_{13}\Delta\tau_3 & f_{04}\Delta\tau_r & \\ f_{41}\Delta\tau_1 & f_{32}\Delta\tau_2 & f_{23}\Delta\tau_3 & f_{14}\Delta\tau_4 & f_{05}\Delta\tau_r \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$$

where $\Delta\tau_i$ represent offset interval between i th trace and $i+1$ th trace and $\Delta\tau_r$ represent the interval of regularly sampled traces which is used to generate the prediction error filter. The operation described above look like a nonstationary filtering operation. But, the filter itself does not change and the change during the convolution is the sampling point on the filter. Each row of the matrix represents the same filter but the different sampling points as explained in Figure 6. Figure 6 schematically shows this filtering operation. At each position of the filter shifted to convolve with the irregularly sampled data, all coefficients of the filter should be recalculated from the spectrum of the filter. Here, I make a spectrum by FFT for the prediction error filter simulated as in the first section, and sample at points where the filter meets the irregularly sampled data points during the convolution. Figure 7 shows an example of the real and imaginary parts of the matrix for filtering on irregularly sampled grid for a frequency.

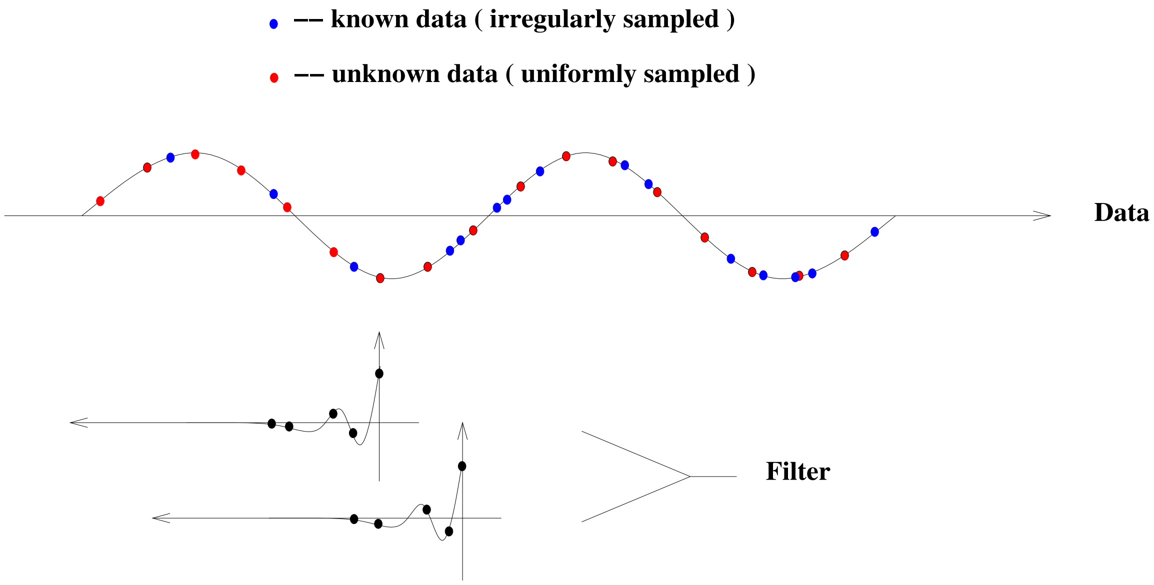


Figure 6: The convolution of an irregularly sampled data with a filter which is continuous. (irrgrim) [NR]

Synthetic data example

For testing of this algorithm, a synthetic data set containing four linear events without noises, shown in Figure 8c, is prepared. This synthetic data set is irregularly sampled along the offset axis, while the offsets, shown in Figure 8a, do not increase

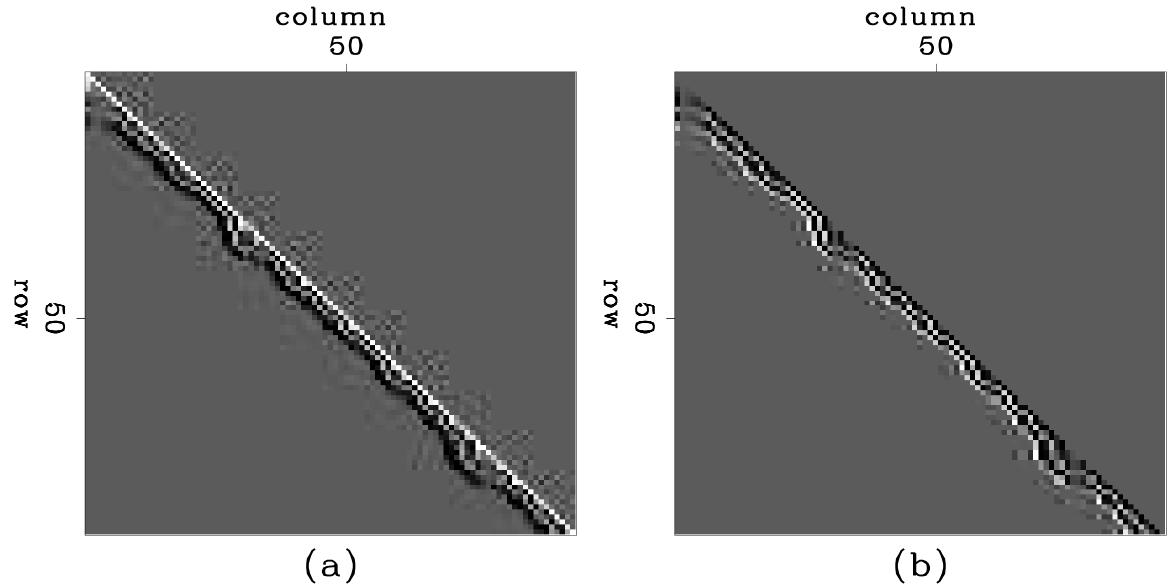


Figure 7: (a) Real component of the filter for the frequency of an half of the Nyquist. (b) Imaginary component of the filter for the frequency of an half of the Nyquist. nsflt [NR]

linearly with respect to the trace number. For a clearer understanding of the geometry of Figure 8c, I include Figure 8b, which is the gap between traces and it shows a random characteristic of the trace offsets. Figure 9 shows the result of regridding with uniform sampling interval of the data shown in Figure 8c. Even though the result produced some noises which did not exist in the original data set, all linear events show good line up with dips picked, and the spectrum in Figure 9 confirms a reasonable interpolation.

For a more realistic example, noises have been added to the data set, Figure 8c, and it is shown in Figure 10c. The offsets of traces in this data set are the same as the data set without noise. Figure 11 shows the result after regridding from Figure 10c, and also good line up along dips picked.

CONCLUSIONS

I presented a trace interpolation scheme using dip-spectra. The proposed interpolation scheme can be summarized by three steps. The first step, which is the most important step, picks dips from the dip spectrum. The second step simulates filter using dips picked. And the last step finds missing traces by minimizing filtered output. This scheme is applicable to many kinds of missing trace interpolation problems such as the interlacing of regularly sampled traces, randomly missing traces on regularly sampled traces and regridding irregularly sampled traces.

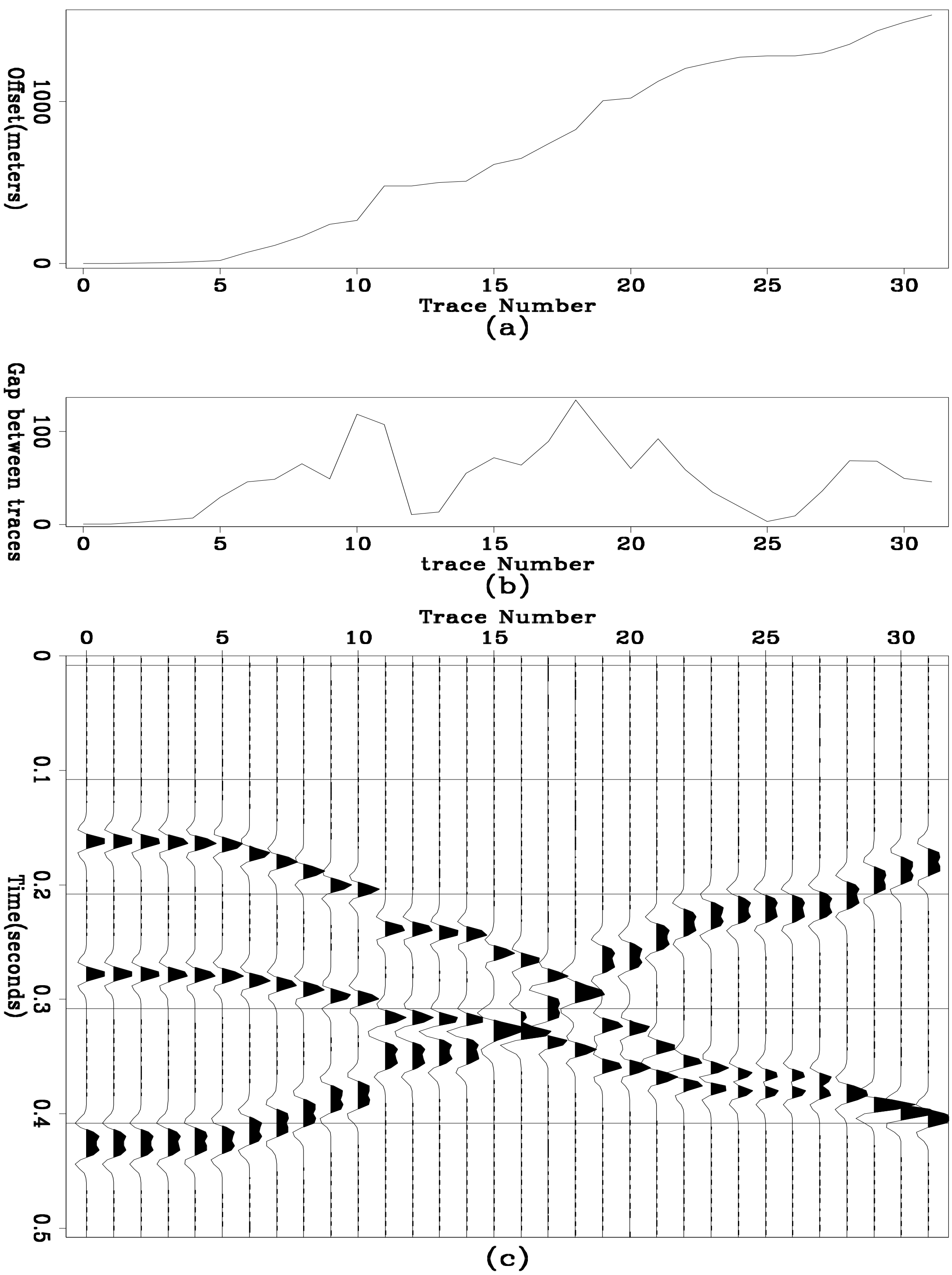
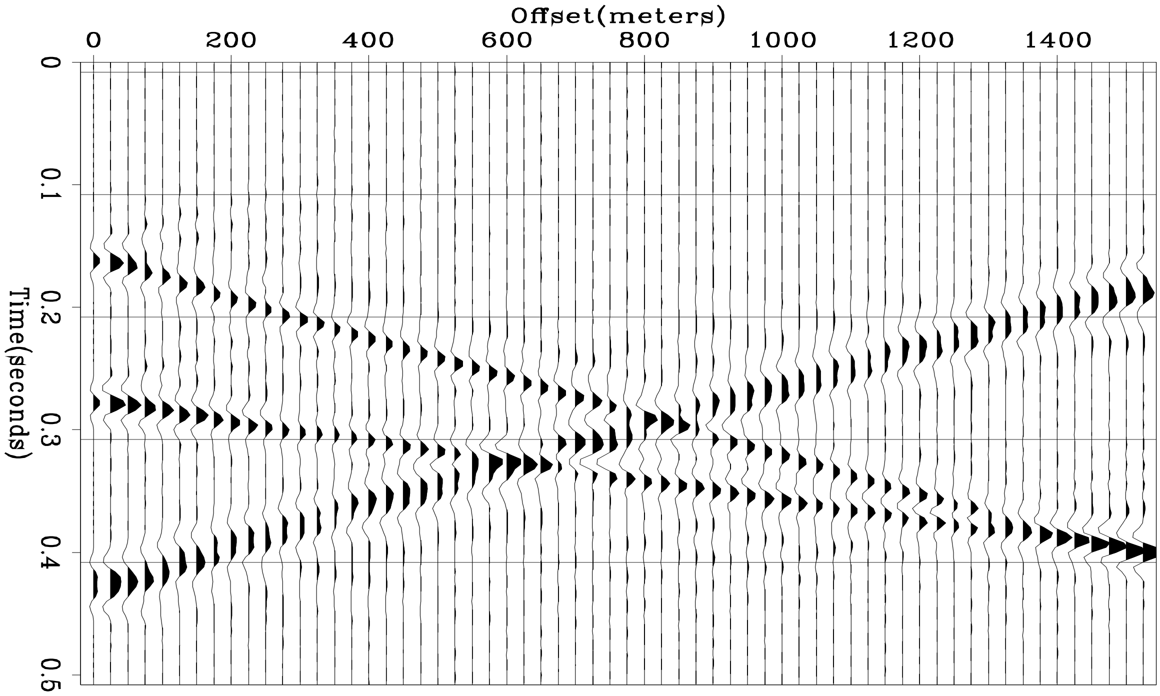


Figure 8: (a) Offset with respect to the trace number. (b) Offset difference between the nearest traces. (c) Synthetic plane waves with irregularly sampled traces and the zero traces are regularly sampled trace which is to interpolated. irrdat [R]



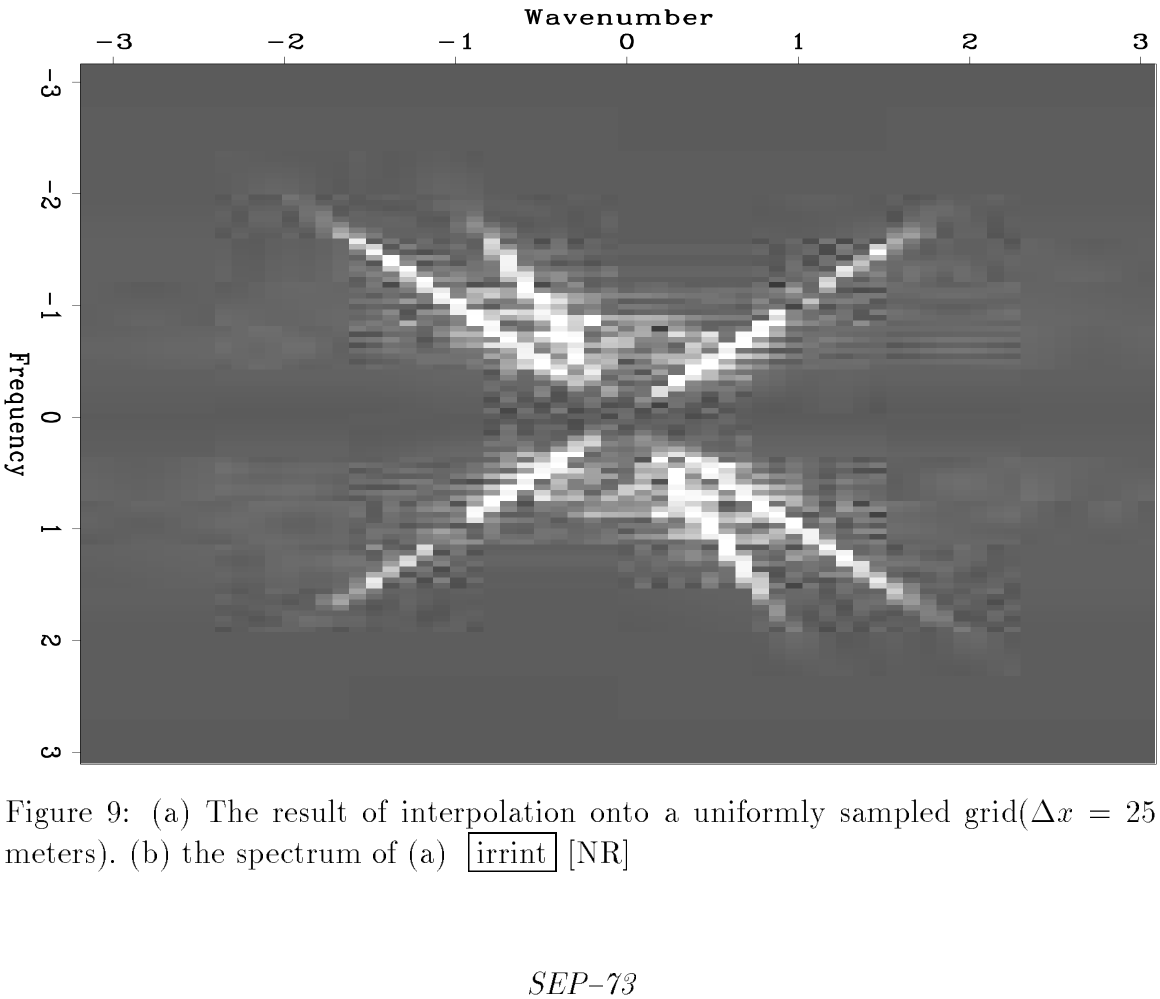


Figure 9: (a) The result of interpolation onto a uniformly sampled grid(Δx = 25 meters). (b) the spectrum of (a) irrint [NR]

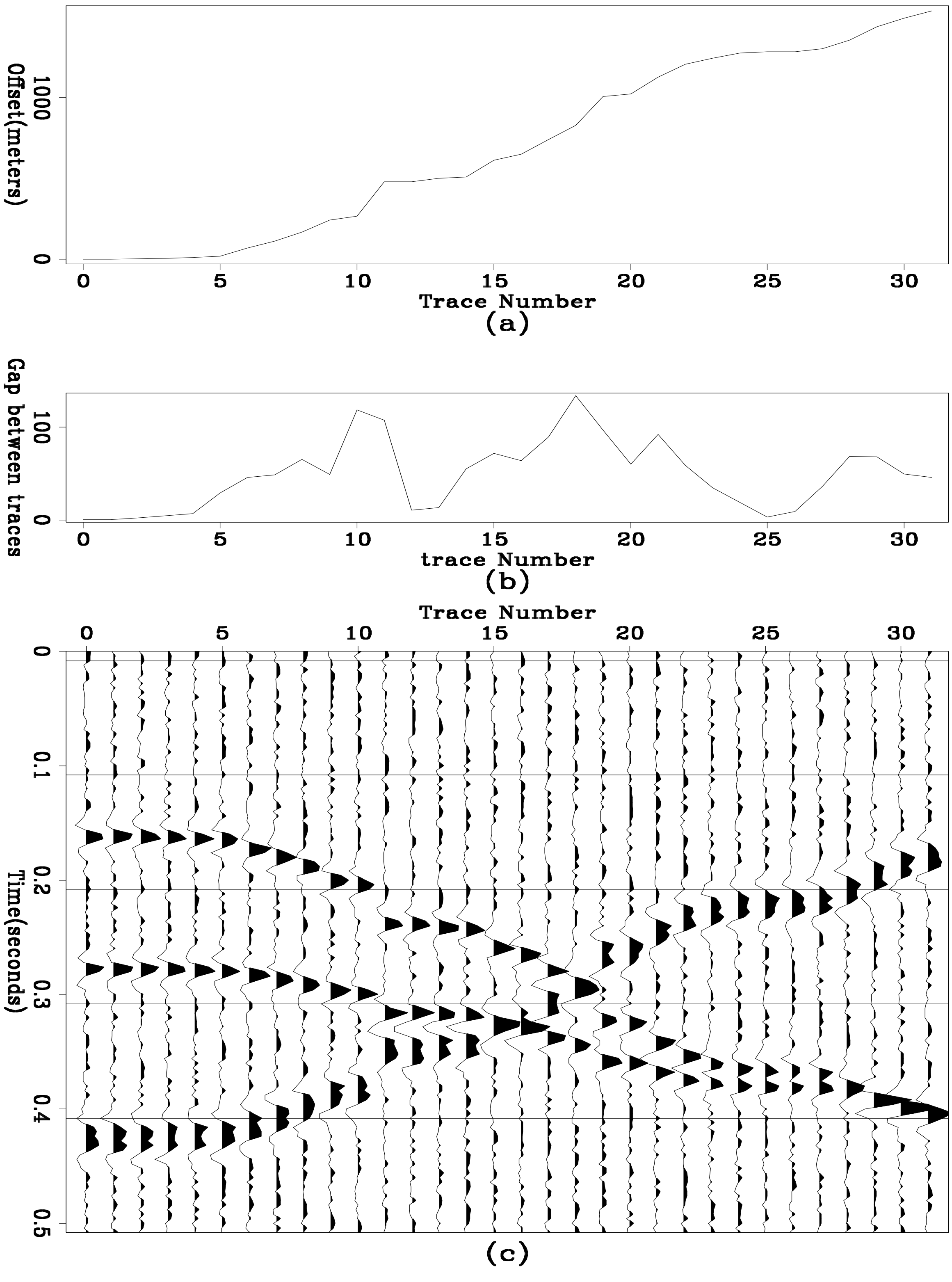


Figure 10: (a) Offset with respect to the trace number. (b) Offset difference between the nearest traces. (c) Synthetic plane waves with irregularly sampled traces and the zero traces are regularly sampled trace which is to interpolated. irrdatwn [R]

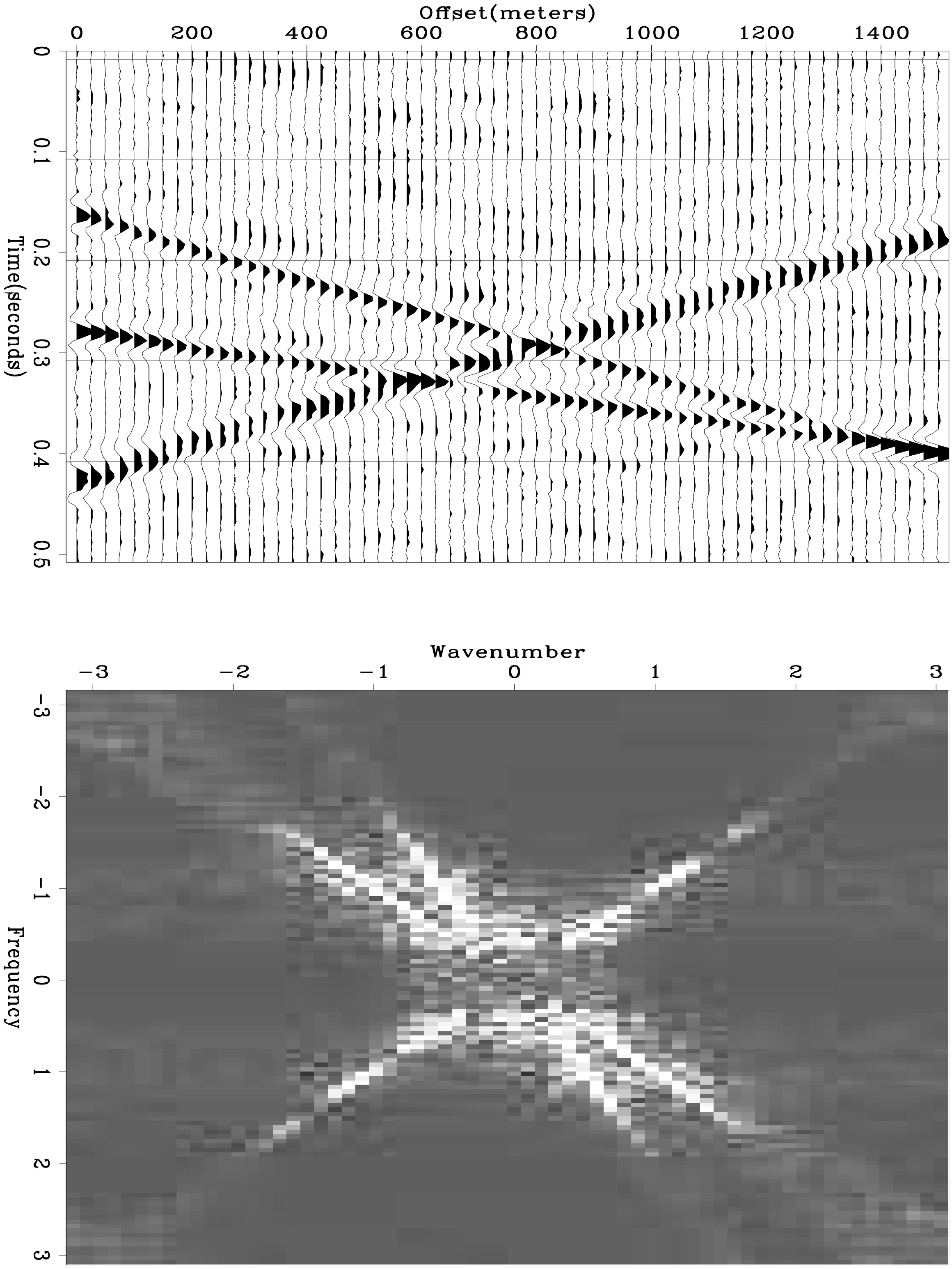


Figure 11: (a) The result of interpolation onto a uniformly sampled grid(Δx = 25 meters). (b) the spectrum of (a) $\boxed{\text{irrwnint}}$ [NR]

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