Trace interpolation using spectral estimation

Jun Ji [1]

ABSTRACT

A scheme for missing-trace interpolation of linear events is proposed. For a two-dimensional dataset which contains linear events, a post-interpolation spectrum can be estimated from a portion of the original aliased spectrum. The restoration of missing trace data is accomplished by minimizing the energy after applying a filter which has an amplitude spectrum that is inverse to the estimated spectrum.

INTRODUCTION

Jon Claerbout (1991, p.175) has given us a direction for interpolation in his new book as follows: "A method for restoring missing data is to ensure that the restored data, after specified filtering, has minimum energy." He has also suggested choosing the filter to have an amplitude spectrum that is inverse to the spectrum we want for the interpolated data. Following his direction, we should predict the spectrum for the interpolated data. For a one-dimensional dataset that is regularly sampled and aliased, we cannot use the spectrum of the known data as a spectrum for the interpolated data because we cannot distinguish the unaliased spectrum from the aliased spectrum. Instead, we use the original spectrum, spectrally weighted. This approach gives low weighting to high frequency, where most of aliasing occurs. Such spectral weighting, however, is extremely subjective. On the other hand, for a two-dimensional dataset, we can estimate a spectrum which is supposed to be the result of interpolation when all events are linear.

SPECTRAL ESTIMATION

The phrase "spectral estimation" has been used for a long time to represent correct estimation of the power spectrum of a time series. Here, I use the phrase to mean estimation of the spectrum which is supposed to result from interpolation.

[1]**email:** Author has no known email address.

Let us consider a plane wave, which is a linear event in the time-space (t-x) domain, and its spectrum, which is also a linear event in the frequency-wavenumber (f-k) domain. If we sample densely enough to avoid aliasing in the time direction and sparsely along the space direction, the spectrum of the plane wave will show spatial aliasing as is then the case in a conventional seismic survey. Figure 1a shows a spectrum containing several linear events with spatial aliasing. Figure 1b shows the spectrum of such data from $-3\pi/\Delta x$ to $3\pi/\Delta x$ along the wavenumber axis. From Figure 1b, we can see that the aliasing is caused by a portion of the original spectrum shifted by $\pm 2\pi/\Delta x$, which is still located in the region $-\pi/\Delta x$ to $\pi/\Delta x$ along the wavenumber axis. If we oversample with $\Delta x = \Delta x_{orig}/2$, the spectrum which caused the aliasing effect will be shifted further, to $\pm 4\pi/\Delta x$ (Figure 1c). As a result, one of the aliased events will disappear where $-2\pi/\Delta x \leq k \leq 2\pi/\Delta x$. On the other hand, we can mimic a spectrum like Figure 1c by taking the aliased spectrum only from $-\pi/2\Delta t \leq f \leq \pi/2\Delta t$, as in Figure 1d.

Let us describe a linear event in the spectrum as a line segment

$$f = ak \tag{1}$$

where $a = df/dk$ is the inverse dip of a linear event, and f and k are defined on $-\pi/\Delta t \leq f \leq \pi/\Delta t$ and $-\pi/\Delta x \leq k \leq \pi/\Delta x$, respectively. Then, the aliased event can be described by a shifted line:

$$f = a(k \pm \frac{2\pi}{\Delta x}). \tag{2}$$

If we oversampled by $1/N$ times the given sampling interval in the space domain, the line will be

$$f = a(k \pm \frac{2N\pi}{\Delta x}) \tag{3}$$

where k is defined on $-N\pi/\Delta x \leq k \leq N\pi/\Delta x$. By pulling out N from the bracket on the right-hand side of equation (3) and changing variables according to $f' = f/N$ and $k' = k/N$, we find that equation (3) becomes

$$f' = a(k' \pm \frac{2\pi}{\Delta x}) \tag{4}$$

where f' and k' are defined on $-\pi/N\Delta t \leq f' \leq \pi/N\Delta t$ and $-\pi/\Delta x \leq k' \leq \pi/\Delta x$, respectively. Equation (4) tells us that, by taking a portion of the given spectrum, we can obtain a spectrum which corresponds to the spectrum after interpolation.

INTERPOLATION

The interpolation considered in this paper is first-order interpolation, achieved by interlacing the space axis. Higher-order interpolation can be obtained by applying the first-order interpolation scheme several times. Now we know the spectrum which is supposed to result after interpolation, our next step is to find a filter which has an amplitude spectrum that is inverse to the spectrum we estimate. The filter can be

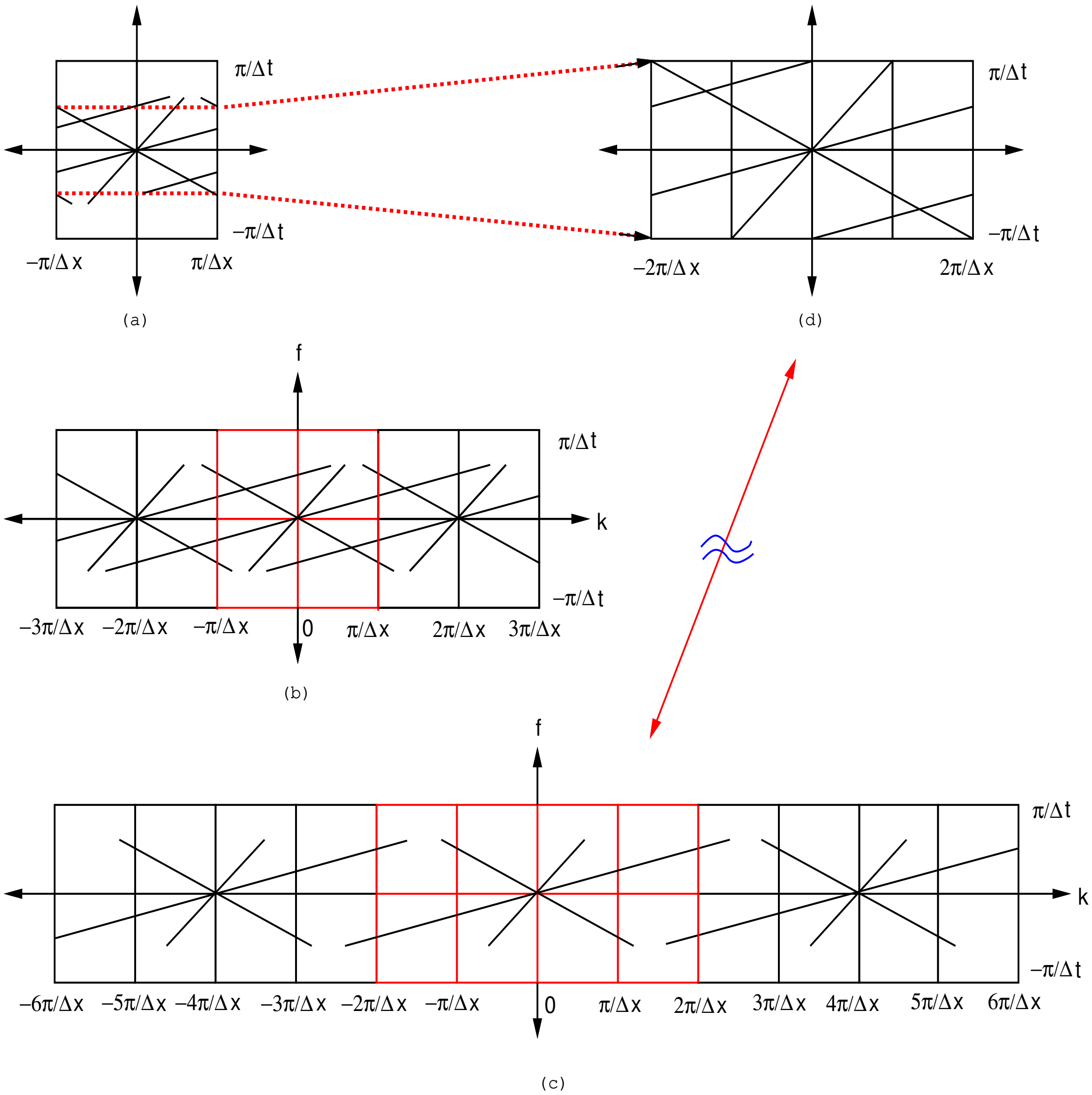

Figure 1: (a) The spatially aliased spectrum. (b) The spatially aliased spectrum with replication. (c) The oversampled spectrum which is the spectrum after interpolation. (d) The spectrum estimated from the aliased spectrum. $\boxed{\text{fig1}}$ [NR]

found in both the $t - x$ domain and the $f - x$ domain. In this paper, I choose the $f - x$ domain for its simplicity.

The procedure for an interpolation can be summarized as follows:

1. I pad zeros in the time domain with the same number of original time samples, and Fourier transform over time to (ω, x).

2. For each frequency from zero to half Nyquist, I find a minimum phase wavelet whose inverse spectrum is the spectrum of the signal along the x axis. Here, I used Kolmogoroff spectral factorization for finding minimum phase filters.

3. For each frequency, I find missing data by minimizing the filtered output in the least-squares sense using a conjugate-gradient algorithm. The filter applied is the wavelet which comes from the half frequency.

Another approach for finding filters is to obtain a prediction-error filter for each frequency, because the prediction-error filter has a spectrum that is inverse to the input (Claerbout, 1991). If you want to limit the length of filter, the prediction-error filter is more attractive than spectral factorization.

RELATION TO CLAERBOUT'S AND SPITZ'S SCHEMES

Recently Claerbout (1991) and Spitz (1991) showed comparable results in trace interpolation in the $t-x$ and the $f-x$ domains, respectively. Claerbout gives several schemes in his book, however, in order to compare methods, I consider only the two-step linearized scheme. Both Claerbout and Spitz used prediction-error filters to interpolate traces. Since a prediction-error filter itself has a spectrum that is inverse to the input, the input spectra of their prediction-error filters will help us to understand how both methods estimate the spectrum we want for the interpolated data.

Claerbout found a two-dimensional prediction-error filter with subsampling in the time direction. According to the stretching theorem of the Fourier transform, subsampling in the time is stretching in the spectrum in the frequency. This means that his decimated input spectrum for a prediction-error filter is a spectrum stretched by a factor of two. If the original spectrum has no energy beyond the half the temporal Nyquist frequency, the spectrum stretched by a factor of two is equivalent to taking the original spectrum only from zero to half temporal Nyquist frequency and considering it as a spectrum for the full frequency range.

Spitz computed one-dimensional prediction-error filters along the x direction for each frequency from zero to half the Nyquist frequency and applied each filter to a frequency whose magnitude is twice the original frequency. This approach also implies that he considered the spectrum from zero to half the temporal Nyquist frequency as a whole spectrum after first-order interpolation.

NUMERICAL EXAMPLES

To compare the proposed interpolation scheme with Spitz's method, I applied the scheme to a data set which was similar to that used by Spitz (1991, Spitz). Figure 2 shows synthetic data which is spatially aliased linear events with random noises, and the result of first-order interpolation. The result is comparable to that of Spitz. Figure 3 shows the spectra of data before and after interpolation. The spectrum after interpolation fits well to our guess which can be intuitively conjectured from the original spectrum.

For a more realistic test, I have applied the interpolation scheme to a real dataset which has aliased events. The dataset, a 24-fold CMP stack of NMO-corrected Western offshore Texas data, has been windowed to provide a smaller test dataset. Figure 4 illustrates the performance of the interpolation scheme proposed. It shows that the proposed algorithm may accommodate curvatures and lateral amplitude variation.

CONCLUSION

I present a method for missing-trace interpolation of linear events. For a two-dimensional dataset which contains linear events, a post-interpolation spectrum can be estimated from a portion of the original aliased spectrum. The restoration of missing trace data is accomplished by minimizing the energy after applying a filter which has an amplitude spectrum that is inverse to the estimated spectrum.

ACKNOWLEDGEMENTS

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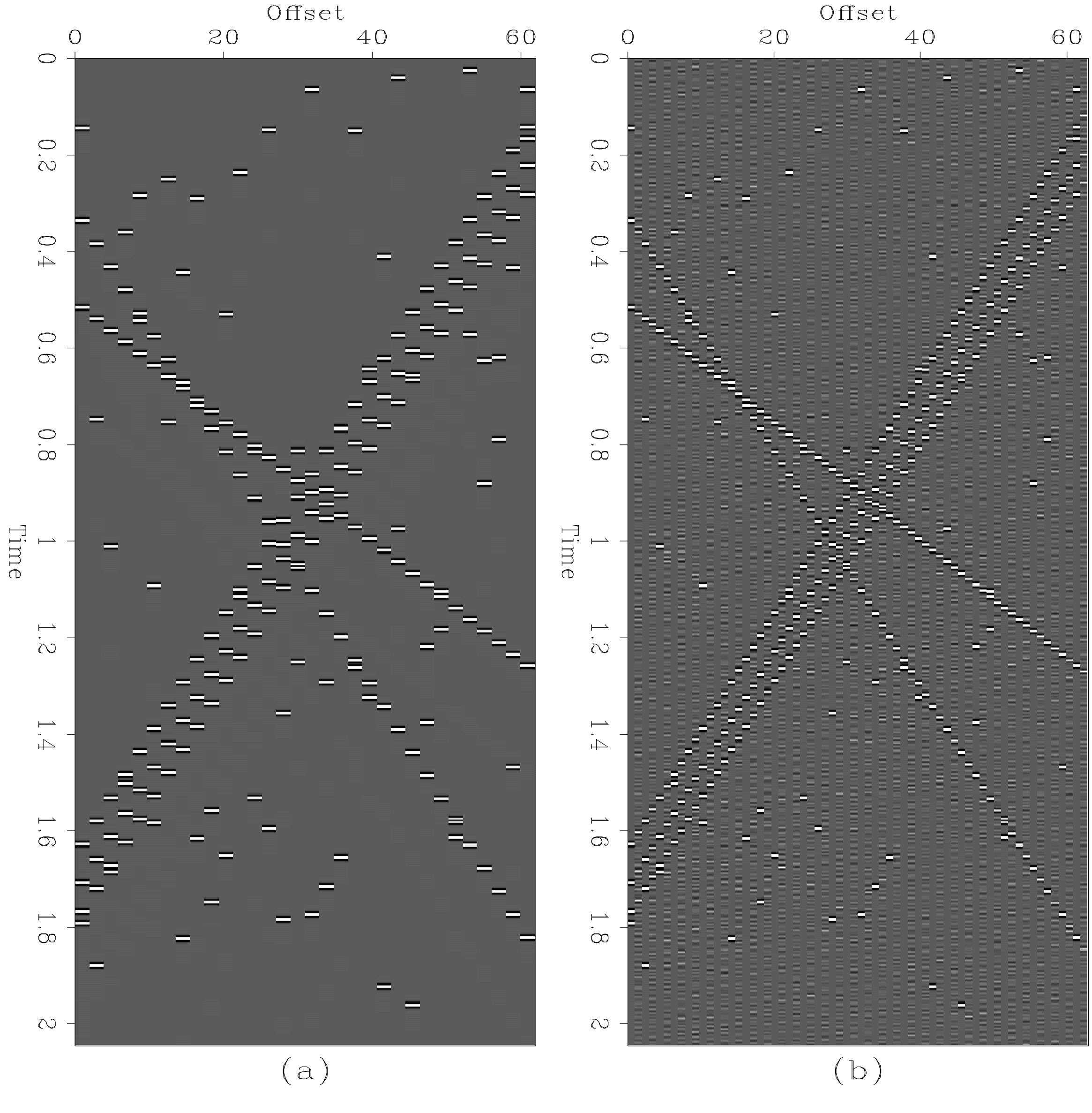

Figure 2: (a)Input data made of three aliased linear events (b) First order interpolated data. Note that the parallel events have been correctly interpolated. fig2 [NR]

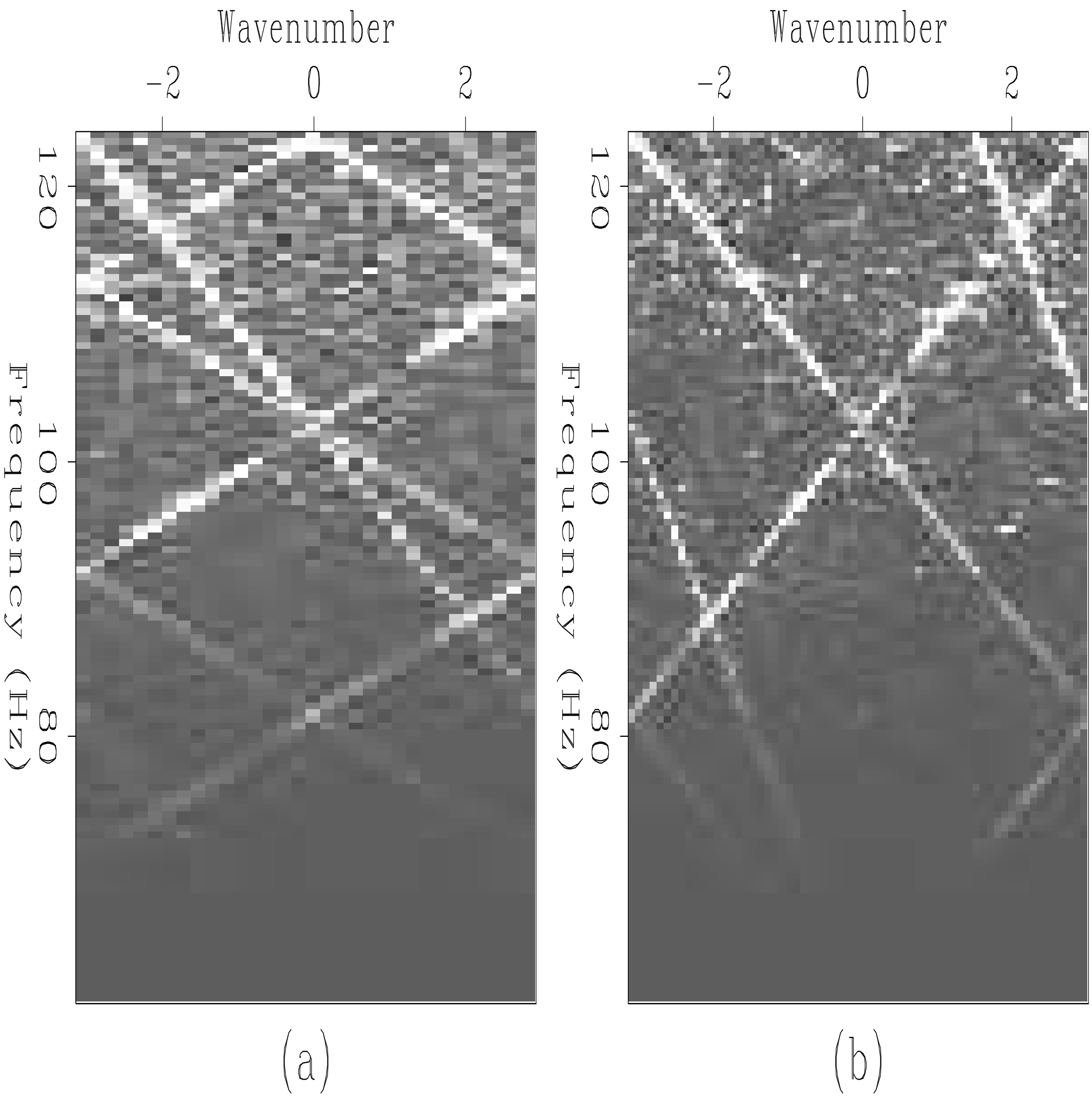

Figure 3: Amplitude f-k spectra. The horizontal axes represent normalized wavenumber, from -0.5 to 0.5 cycles. (a)f-k amplitude spectrum of the input shown in Figure 2(a). The unit scale of the normalized wavenumber is 1/32 cycles. (b) f-k amplitude spectrum of the interpolated data set shown in Figure 2(b). The unit scale of the normalized wavenumber is 1/64 cycles. fig3 [NR]

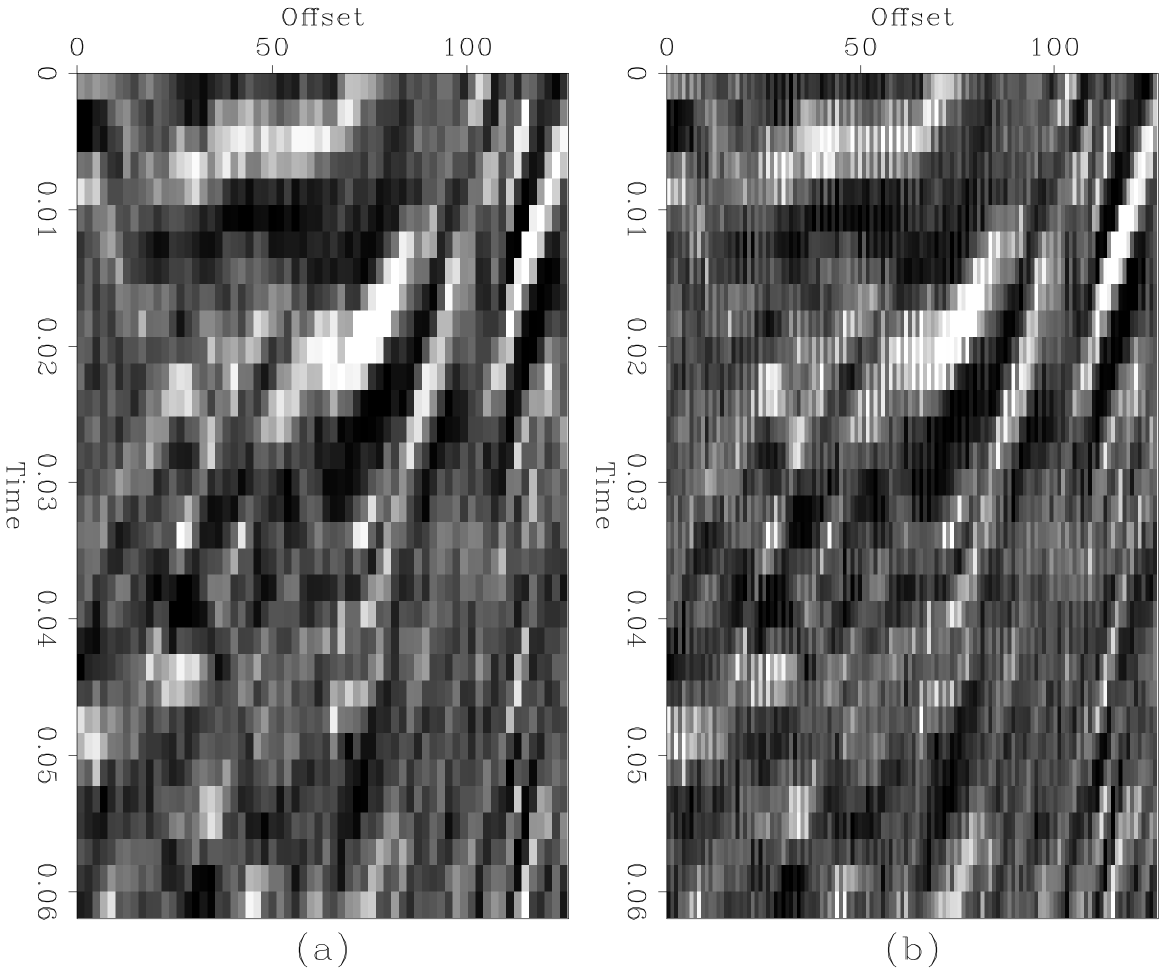

Figure 4: (a)Input data which has windowed from a CMP stack. (b) First order interpolated data. fig4 [NR]