Do traveltimes in pulse-transmission experiments yield anisotropic group or phase velocities?

Joe Dellinger and Lev Vernik [1]

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INTRODUCTION

The elastic properties of layered rocks are often measured using the pulse through-transmission technique on sets of cylindrical cores cut at angles of 0, 90, and 45 degrees to the layering normal (e.g., Vernik and Nur, 1992; Lo et al., 1986; Jones and Wang, 1981). In this method transducers are attached to the flat ends of the three cores (see Figure 1), the first-break traveltimes of P, SV, and SH-waves down the axes are measured, and a set of transversely isotropic elastic constants are fit to the results. The usual assumption is that frequency dispersion, boundary reflections, and near-field effects can all be safely ignored, and that the traveltimes measure either vertical anisotropic group velocity (if the transducers are very small compared to their separation) or phase velocity (if the transducers are relatively wide compared to their separation) (Auld, 1973).

To discover whether typical experiments of this kind are more likely to measure group velocity, phase velocity, or something in between, we numerically model a laboratory pulse-transmission experiment of Vernik and Nur (1992). In their experiment the separation between source and receiver was more than three times greater than the transducer width. Although this configuration might seem closer to the group-velocity case than the phase-velocity one, Vernik and Nur assumed their recorded first-breaks represented phase velocities.

Our numerical results show Vernik and Nur were (almost) correct. Except for a slight underestimate in the P-wave velocity measurements at 45 degrees to the layering, they did record phase-velocity traveltimes in their experiment. Most experiments are clearly closer to the phase-velocity case than Vernik and Nur's; we therefore conclude that almost all pulse-transmission experiments of this kind should measure anisotropic phase, not group, velocities.

[1]**email:** jdellinger@trc.amoco.com, lev@pangea.stanford.edu, respectively

METHODOLOGY

Figure 1 shows the core size and P-wave transducer width for one of Vernik and Nur's (1992) early experimental configurations to scale. The cores were 40 mm tall and 26 mm wide; the P-wave transducers were 12 mm wide. The SV and SH transducers (not shown) were 20 mm wide, nearly as wide as the core itself. The central frequencies of the transducers were 1.0 MHz for P-waves and 0.7 MHz for S. For each core Vernik and Nur picked first-break P, SV, and SH traveltimes.

A complete numerical simulation of this experiment would take into account the complex three-dimensional boundary conditions of tilted-axis transverse isotropy interacting with a truncated cylindrical surface. We do not need to model the entire waveform, however; Vernik and Nur did not use the entire seismogram to make their traveltime measurements, but only the first breaks. Assuming the direct waves arrive ahead of waves that have interacted with the core boundaries, a homogeneous model should suffice.

Our numerical model uses a two-dimensional finite-difference algorithm, Fourier spectral in space and Chebychev spectral in time (Etgen and Dellinger, 1989; Tal-Ezer, 1987). This algorithm is accurate as long as the wavelet is not spatially aliased; the time step can be arbitrarily large. We use a scale of .5 mm per spatial gridpoint (well within the Nyquist criteria for the source wavelet) and .05 μs per time step. The transversely isotropic (TI) medium is homogeneous, so the density has been normalized out; in units of velocity squared, the normalized elastic constants ($W_{ij} = C_{ij}/\rho$) are $W_{11} = 20.16$, $W_{33} = 11.97$, $W_{55} = 4.00$, $W_{66} = 6.86$, and $W_{13} = 5.51$ $(\mathrm{km/s})^2 \equiv (\mathrm{mm}/\mu\mathrm{s})^2$. These constants are for a core cut perpendicular to the layering. Figure 2 shows the shape of the highly anisotropic qP, qSV, and SH wavefronts in this medium, a sample of Bakken Shale (Vernik and Nur, 1992).

The numerical model source time function is a Ricker wavelet with central frequency 0.9 MHz for both wavetypes. The wavefield excited by a finite-width transducer is constructed by convolving the wavefield produced by a single-gridpoint source with a horizontal bar of impulses, 25 adjacent gridpoints for the P transducer, 41 for the S. We defined the first-break time as the point when the recorded wavelet first attains an amplitude of 1 percent of its maximum; this seems to pick about the same point on the trace that we would choose by eye.

PHASE VERSUS GROUP

We begin by examining model results for two "canonical" experiments. The lower plot in Figure 3 shows how vertical group velocity (the velocity of vertically traveling energy) can be measured. The anisotropic wavefront radiates out from a point source at the bottom, and a point receiver at the top detects the part of the wavefront with vertically traveling energy. The distance between the two transducers is divided by the measured traveltime to find the vertical group velocity.

The upper plot in Figure 3 shows how vertical phase velocity (the velocity of a vertically traveling plane wave) can be measured. The source must be wide enough to launch a reasonable facsimile of a plane wave. If the source transducer were truly infinite, the horizontal position of the receiver would be irrelevant; in practice, the position of the receiver does not matter if it samples the flat central part of the wavefront away from the diffracting truncated edge. The *vertical* distance between source and receiver divided by the measured traveltime then gives the vertical phase velocity (Mignogna, 1990).

The upper plot in Figure 3 was constructed from the lower plot in the same figure by summing copies of the latter shifted over the range -20 to $+20$ mm; the flat part of the wavefront in the upper plot thus corresponds to the highest point on the wavefront in the lower plot. This implies that we could have measured the vertical phase velocity directly from the lower plot by positioning the receiver where the wavefront *first encountered the upper surface* (around the -11 mm position) and dividing by the *vertical* distance from source to receiver (40 mm). This fundamental relationship is used by Sahay et al. (1992) to design experiments that use movable transducers to unambiguously measure phase velocities.

Note that if we instead divided by the the true distance between the source and the offset receiver, $\sqrt{40^2 + 11^2}$ mm, we would be finding the (nonvertical) group velocity associated with the vertical phase velocity. The three kinds of velocities in Figure 3 satisfy an inequality: the associated group velocity is greater than or equal to the vertical phase velocity, which in turn is greater than or equal to the vertical group velocity.

MODEL RESULTS

P-waves

Figure 4 shows snapshots of qP-waves propagating in our numerical model of Vernik and Nur's (1992) experiment. In the upper plot the layers are vertical, propagation is along a symmetry axis of the medium, and so the wave energy is not deflected by the layering and the flat part of the wavefront impacts the receiver transducer centrally.

In the lower plot the layers extend from the lower right to the upper left at a 45-degree angle to the core axis. The qP-wave travels faster along the layers than across them, so the flat part of the wavefront containing the main focus of energy is deflected to the left. [See Green (1973) for photographs of wavefronts doing this.] In this example the accumulated lateral displacement from bottom to top happens to be almost the same as the transducer width. Not surprisingly, Vernik and Nur noticed the direct qP-waves were distinctly weaker in their 45-degree cores than in the others.

Figure 5 shows how this grazing miss affects first-break traveltimes. The left plot shows the model output for a multioffset survey over the shale-core model; the horizontal axis shows source-receiver offset. The earliest first break occurs at 10.29 μs at an offset of -12 mm. The first break at zero offset, corresponding to the actual geometry in Vernik and Nur's (1992) experiment, occurs .5 percent later at 10.34 μs.

This error is smaller than the 1 percent P-wave measurement uncertainty Vernik and Nur found in their experiment (Vernik and Nur, 1993) but is still detectable.

The right part of Figure 5 shows the results of re-running the numerical experiment with zero offset but a range of transducer sizes. With a point source and point receiver, the first break defines the vertical group-velocity arrival time, here 10.63 μs. As the transducer size is increased towards 12 mm the first break moves rapidly earlier to within .5 percent of the true phase-velocity arrival time; for further increases the first-break time continues to converge on the exact phase-velocity arrival time, but much more slowly.

As we saw in Figure 3, when *any* of the leading flat part of the wavefront hits the receiver it theoretically initiates a first break *at the phase-velocity arrival time*. In this example the lateral offset of the qP-wave energy is slightly less than 12 mm. Wider transducers than 12 mm continue to record slightly earlier times because we pick the first break when the amplitude exceeds 1 percent of its peak value, not at the "theoretical" first break. For the same reason, the "infinite-transducer, zero-offset" and "12 mm transducer, −12 mm offset" first breaks (at either end of the upper dotted line in Figure 5) are also not quite at the same time.

S-waves

The upper plot in Figure 6 shows numerical model results for the 45-degree SH-wave case. Although the lateral drift is nearly the same as for the 45-degree qP case shown in Figure 4, the receiver transducer solidly encounters the key flat part of the wavefront because of the larger S-wave transducer size.

qSV-waves in TI media can display complex anisotropic effects. Wavefront cusps are always associated with precursive energy, making the definition of "theoretical first break" problematical. These are point-source, not plane-source phenomena, however; in practice they should only be significant near the edges of the "flat part" of the wavefront (if they occur at all). qSV-wavefronts in TI media do almost always display near-symmetry about 45 degrees, so we expect little lateral displacement to occur; the lower plot in Figure 6 shows that the displacement in our example is only 2 mm (to the right).

DISCUSSION

Detecting measurement errors

Vestrum (1994) notes that the distinction between anisotropic group and phase velocity is not always significant; even in a strongly anisotropic medium there may happen to be near-symmetry about 45 degrees. If lateral displacement of the direct arrivals may be a problem, a consistency check can be made by determining the elastic constant C_{13} from the 45-degree P-wave and qSV-wave measurements independently. Because the phase-velocity arrival time is the earliest possible, a "partial miss" must always result in a delay and thus an underestimate of the phase velocity. (Similarly, insufficiently narrow transducers cause group velocities to be overestimated.) An erroneously slow 45-degree qP phase velocity results in finding a C_{13} that is too low, while an erroneously slow 45-degree qSV phase velocity results in a C_{13} that is too high; thus any errors in interpreting the traveltimes as group or phase must cause the two values of C_{13} to diverge.

Eaton (1992) examines Vernik and Nur's (1992) results using a variation of this method: the measured qP 45-degree velocity is used to predict a qSV 45-degree velocity that is then compared to the actual measurement. Eaton finds that the predicted qSV velocities tend to be a few percent too high, exactly as we should expect if the input qP phase-velocity estimates were around half a percent too low.

In the case of qP and qSV-waves in transverse isotropy the total lateral displacement at 45 degrees can be expressed directly in terms of the core height H and the (possibly inaccurately) measured phase velocities:

$$D = H\frac{(V_P(90)^2 - V_P(0)^2)}{2V_X(45)^2}\,\frac{(V_{SV}(90)^2 + V_{SV}(0)^2) - 2V_X(45)^2}{(V_P(90)^2 + V_P(0)^2) + (V_{SV}(90)^2 + V_{SV}(0)^2) - 4V_X(45)^2}, \tag{1}$$

where D is the total lateral displacement and V_X is either V_P or V_{SV}, depending on the wavetype being considered. D is positive if the offset is in the direction of the layers, the expected case for qP-waves. The corresponding equation for SH-waves is:

$$D = H\frac{(V_{SH}(90)^2 - V_{SH}(0)^2)}{2V_{SH}(45)^2}. \tag{2}$$

Note these equations always *overestimate* the total offset of the qP and SH-wavefronts if lateral displacement has caused the measured 45-degree phase velocities to be erroneously slow. Equations 1 and 2 are also valid for orthorhombic anisotropy measured in symmetry planes, as in Cheadle et al. (1991).

Physical versus numerical models

If a first break could be detected as soon as any of the leading flat part of the wavefront met the receiver our conclusions would be equally valid in either two or three dimensions. In practice the shape of the following wavelet must matter to some extent, and this will be different in a two-dimensional model than in a three-dimensional one. Our numerical model also does not attempt to include reflections at the boundary between the transducer and the core sample. As a check against reality we reversed Eaton's (1992) method and used Vernik and Nur's (1992) 45-degree qSV measurements (presumably unaffected by lateral-displacement errors) to predict the corresponding true 45-degree qP phase velocities. Although there was a large scatter in the discrepancies, reflecting the difficulty Vernik and Nur had measuring 45-degree qSV velocities, the average error for the 9 core-sample sets was found to be .61 percent, in good agreement with our simple model's predictions.

CONCLUSIONS

We expect that the Bakken Shale example investigated here represents about the most anisotropic geological specimen (with the largest P and SH lateral displacements) likely to be encountered in the laboratory. If this is true then the traveltimes measured in similar laboratory core-sample experiments should also represent phase velocities, assuming the critical ratio of core-sample height to transducer width is no greater than it was in Vernik and Nur's (1992) experiment (i.e., ≤ 3). Our results also suggest that *group* velocities are very difficult to measure directly; the ratio of core-sample height to transducer width needs to be at least 20, preferably even larger.

The usefulness of 45-degree qSV measurements as an independent source of information should not be overlooked, but even if they are unavailable equations (1) and (2) can be used to estimate the total lateral displacement. If the offset is predicted to be more than about half the transducer width caution should be exercised when interpreting the measured traveltimes. In any case, published elastic constants should always include information about what measurements were used and what magnitudes of error to expect in them (Seriff and Sriram, 1991).

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Figure Captions

Figure 1: Shale cores cut at 0, 90, and 45 degrees (perpendicular, parallel, and at a 45-degree angle to the layering respectively). The disks at the top and bottom of each core show the relative width of the P-wave transducers.

Figure 2: The shapes of qP (outer curve), qSV (inner solid curve), and SH (dotted) wavefronts for the medium used in our numerical model. The 90, 45, and 0-degree labels show the direction of vertical for the corresponding shale-core orientations.

Figure 3: Snapshots demonstrating two "ideal" core-sample experiments. The positions and sizes of the source and receiver transducers are indicated respectively by thick horizontal lines at the bottom and top of the model. Top: a wide-source experiment for measuring vertical phase velocity. Bottom: a point-source to point-receiver experiment for measuring vertical group velocity.

Figure 4: Snapshots showing the behavior of qP-waves in our 90 (top) and 45 (bottom) degree core-sample simulations. The top snapshot is taken at 7.5 μs, the bottom 10. μs. The dashed lines show the width of Vernik and Nur's cores; the thick solid lines at the bottom and top show the size and positions of the P-wave source and receiver transducers.

Figure 5: How the recorded signal in the 45-degree P-wave case depends on experimental geometry. In the left subplot the transducer width is fixed at 12 mm while the transducer offset is varied; in the right subplot the transducer offset is fixed at 0 mm while the transducer width is varied. Short horizontal lines mark first-break times.

Figure 6: Snapshots showing the behavior of SH (top) and qSV (bottom) waves in our 45-degree simulation. Both snapshots are taken at 17.5 μs.