## Short Note

# 3-D Eikonal equation for Time Migration

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## INTRODUCTION

In the short-note I wrote in the previous SEP report (Berlioux, 1993), I presented a new method to perform depth migration in 3-D, starting from time-migrated maps. Since time migration does not always produce properly migrated images or maps in areas where velocity variations are present (Hatton et al., 1981), the method I proposed consists of two parts: first undo the time migration by a principle of *de-migration*, and then perform the depth migration.

In my previous work these two parts were based on a ray-tracing algorithm, and the de-migration process corresponded to the inverse operation of Kirchhoff time migration. De-migration and time migration have to remain associated as inverse processes. I now investigate the method introduced in 1991 by V.J. Khare to perform time migration in order to produce a de-migration scheme corresponding to this process.

In this paper, I explain briefly how V.J. Khare derived the 2-D Eikonal equation for time migration and what modifications have to be introduced to handle the 3-D case, and I make a proposal to use this equation for the de-migration process.

## INTRODUCTION OF EIKONAL EQUATION FOR TIME-MIGRATION

In two dimensions, considering one-way time, the wave equation is

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial z^2} = \frac{1}{V^2(x,z)} \frac{\partial^2 p}{\partial t^2} \tag{1}$$

where $p(x,z,t)$ is the pressure wave field, and $V(x,z)$ is the velocity of the wave propagation in the medium.

We can make the following approximation of the wave field:

$$p(x,z,t) = \hat{p}(x,z)\, e^{i\omega(t-\tau(x,z))} \tag{2}$$

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expressing the pressure wave field in term of amplitude $\hat{p}(x, z)$ and phase $\omega\,(t \;-\; \tau\,(x, z)\,)$, where $\tau$ is the traveltime along the ray paths. Then replacing (2) in equation (1) and making the high frequency approximation, we obtain the 2-D Eikonal equation:

$$\left(\frac{\partial \tau}{\partial x}\right)^2 \;+\; \left(\frac{\partial \tau}{\partial z}\right)^2 \;=\; \frac{1}{V^2\,(x, z)} \tag{3}$$

Among other uses, this equation is useful to compute traveltimes of rays in a given earth model.

V.J. Khare introduced the following equation for the ray-theoretic interpretation of *all-angle* time migration (Khare, 1991):

$$\frac{\partial^2 p'}{\partial x'^2} \;+\; \frac{1}{\tilde{V}^2\,(x', \zeta)}\,\frac{\partial^2 p'}{\partial \zeta^2} \;=\; -\,\frac{2}{\tilde{V}^2(x', \zeta)}\,\frac{\partial^2 p'}{\partial t'\,\partial \zeta} \tag{4}$$

where $p'\,(x', \zeta, t')$ and $\tilde{V}\,(x', \zeta)$ are respectively the pressure wave field and the wave propagation velocity in the time-retarded coordinate system (Claerbout, 1976),

$$\begin{aligned} x' \;&=\; x \\ t' \;&=\; t \,+\, \zeta \end{aligned} \tag{5}$$

with the time-like vertical coordinate $\zeta$ (Lowenthal et al., 1985),

$$\zeta \;=\; \int_0^z \frac{dz}{V\,(x, z)} \tag{6}$$

If we make the following approximation of the wave field in the new coordinate system, equivalent to the equation (2):

$$p'\,(x', \zeta, t') \;=\; \hat{p}'\,(x', \zeta)\,e^{\,i\,\omega\,(t' \,-\, \tau'\,(x', \zeta))} \tag{7}$$

we can express the 2-D Eikonal equation for time migration giving the traveltime $\tau'$ in the frequency domain in the time-retarded coordinate system:

$$\left(\frac{\partial \tau'}{\partial x'}\right)^2 \;+\; \frac{1}{\tilde{V}^2\,(x', \zeta)}\left(\frac{\partial \tau'}{\partial \zeta}\right)^2 \;=\; \frac{2}{\tilde{V}^2\,(x', \zeta)}\,\frac{\partial \tau'}{\partial \zeta} \tag{8}$$

Using equations (5) and (6), we obtain in the physical coordinate system the 2-D Eikonal equation for time migration:

$$\left(\frac{\partial \tau}{\partial x} \;+\; \mathcal{A}\,(x, z)\,\frac{\partial \tau}{\partial z}\right)^2 \;+\; \left(\frac{\partial \tau}{\partial z}\right)^2 \;=\; \frac{1}{V^2\,(x, z)} \tag{9}$$

with

$$\mathcal{A}(x,z) \;=\; V(x,z) \int_0^z \frac{V_x(x,z)}{V^2(x,z)}\, dz \;=\; -V(x,z) \frac{\partial \zeta}{\partial x} \tag{10}$$

## USING 3-D EIKONAL EQUATION FOR DE-MIGRATION

### Presentation of the 3-D case

In the 3-D case, the wave equation corresponding to the ray-theoretic interpretation of *all-angle* time migration is

$$\frac{\partial^2 p'}{\partial x'^2} + \frac{\partial^2 p'}{\partial y'^2} + \frac{1}{\tilde{V}^2(x',y',\zeta)} \frac{\partial^2 p'}{\partial \zeta^2} \;=\; -\frac{2}{\tilde{V}^2(x',y',\zeta)} \frac{\partial^2 p'}{\partial t'\, \partial \zeta} \tag{11}$$

in the 3-D time-retarded coordinate system

$$\begin{aligned} x' &= x \\ y' &= y \\ t' &= t + \zeta \end{aligned} \tag{12}$$

with the new time-like vertical coordinate $\zeta$,

$$\zeta \;=\; \int_0^z \frac{dz}{V(x,y,z)} \tag{13}$$

Following the same scheme as for the 2-D case and using equations (12) and (13), we finally obtain the 3-D Eikonal equation for time migration:

$$\left(\frac{\partial \tau}{\partial x} + \mathcal{A}(x,y,z) \frac{\partial \tau}{\partial z}\right)^2 + \left(\frac{\partial \tau}{\partial y} + \mathcal{B}(x,y,z) \frac{\partial \tau}{\partial z}\right)^2 + \left(\frac{\partial \tau}{\partial z}\right)^2 \;=\; \frac{1}{V^2(x,y,z)} \tag{14}$$

with

$$\mathcal{A}(x,y,z) \;=\; -V(x,y,z) \frac{\partial \zeta}{\partial x} \tag{15}$$

and

$$\mathcal{B}(x,y,z) \;=\; -V(x,y,z) \frac{\partial \zeta}{\partial y} \tag{16}$$

### Using the Eikonal equation for de-migration: a proposal

It is obvious that this equation in 3-D cannot be solved analytically for any kind of velocity model. Since the Eikonal equation relates the gradient of the traveltime $\tau$

to the velocity structure, several methods have been formulated to study the classical Eikonal equation (3) for a velocity structure sampled at discrete points in a 3-D space (3-D grid) and using finite-difference algorithms: M. Reshef and D. Kosloff (1986) integrate a finite-difference approximation of the Eikonal equation with a Runge-Kutta method; J. Vidale (1988) also uses a finite-difference algorithm but solves directly for traveltimes using a planar or circular wavefront extrapolation; and J. Van Trier and W. Symes (1990) derive a conservation law from the Eikonal equation depicting the first arrival time field.

Each differential term in Eikonal equation (14) can be approximated with finite difference to give the traveltime at a particular point of a grid with respect to the previously calculated traveltimes. To apply a finite-difference scheme to this equation, we need a 3-D grid of velocities, and since I use Gocad to handle 3-D surfaces and structures, I present in this report a method to compute a 3-D grid of velocities from a Gocad model.

The aim of this project is to find how we can obtain a de-migration process from this Eikonal equation for time-migration. Nevertheless, to find an "inverse" process to the time migration method presented here, it is necessary to understand clearly how it is performed. I will therefore try to operate the time migration on several synthetic models of zero-offset data.

## CONCLUSIONS

Until now, I did not try to implement any of the methods proposed in the papers mentioned above. Most of them call for finite-difference algorithms, and it appears to me necessary to find a way to produce 3-D grids easily and rapidly from a 3-D model.

The Eikonal equation for time migration introduced by V.J. Khare and the theory that comes with it seem to represent an interesting way to obtain a de-migration process accurate enough to handle most of the complicated models.

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