Short Note

Three-dimensional map de-migration/migration

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INTRODUCTION

3-D seismic wavefield can only be handled with 3-D processing, and more effectively with 3-D depth migration methods, because it contains signals from all possible directions.

In geological areas where lateral velocity variations are present, time migrated images or maps do not represent the proper position of seismic reflections (Hatton et al., 1981). Therefore, it is necessary, before applying a depth migration to the time migrated section to correct this problem. This is done using a principle of *de-migration*. Then depth migration is performed.

De-migration process moves time migrated sections to their original (observed) time, using the velocity model applied for time migration. In my previous work, *de-migration* is achieved via ray-tracing and uses GOCAD as a tool to represent the 3-D surfaces under investigation. This ray-tracing algorithm performing *de-migration* and depth migration has been tested on several examples and produces well positioned depth maps. In some cases, this method cannot carry out the *de-migration* of the initial time model. As a matter of fact, the algorithm fails when the dip of the layers is too important with respect to the velocity model used. This limitation drove us to investigate another way to perform *de-migration*.

3-D MAP DE-MIGRATION/MIGRATION

A few words about de-migration

Interpreters are much more comfortable interpreting time migrated sections than stacked sections. Time migrated sections give a much clearer structural picture of the subsurface than stacked sections but are inherently mispositioned due to the limitations of time migration.

The parameters that could be observed on the time migrated sections or maps are not reflecting the geological reality. As a matter of fact, time migration does not always produce properly migrated time sections. This is the reason why it is necessary

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to do (time) *de-migration*. This method allows us to retrieve all the information needed to perform depth migration. *De-migration* means cancelling the effect of time migration. The principle of *de-migration* corresponds to the inverse operation of time migration.

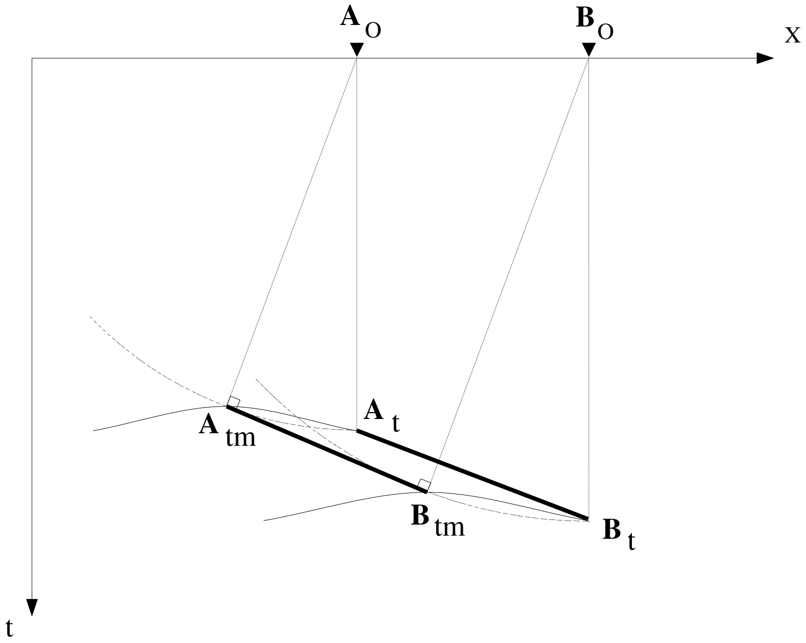

Figure 1: *De-migration* in terms of ray geometry. demigration [NR]

In figure (1), one can see the time migrated position $A_{tm}B_{tm}$ of a dipping reflector. *De-migration* is performed to retrieve the observed time position A_tB_t in the inverse way that was applied to do time migration.

In terms of ray geometry we can formulate *de-migration* as a "ray-tracing" from a time migrated position A_{tm} to the position A_0 on the free surface in order to retrieve the observed time t_0, this is the time that has been measured during seismic acquisition. The ray, as it is built, is perpendicular to the time migrated event locally depth converted at the A_{tm} position.

3-D *de-migration* equations are then expressed as Kirchhoff time migration inverse formulation:

$$
\begin{aligned}
x_0 &= x_m + t_m \tfrac{V_{rms}^2}{4} \left(\tfrac{\partial t}{\partial x}\right)_m \\
y_0 &= y_m + t_m \tfrac{V_{rms}^2}{4} \left(\tfrac{\partial t}{\partial y}\right)_m \\
t_0 &= \frac{t_m}{\sqrt{1 - \frac{V_{rms}^2}{4}\left[\left(\frac{\partial t}{\partial x}\right)_m^2 + \left(\frac{\partial t}{\partial y}\right)_m^2\right]}}
\end{aligned}
\tag{1}
$$

3-D de-migration/migration via GOCAD

The whole process of *de-migration* and depth migration is carried out using GOCAD which is a software for handling 3-D surfaces. The initial data are the time migrated surfaces and a velocity model that will be used during the depth migration. We assume that there exists or that we can obtain a V_{rms} velocity at each point of the triangulated time migrated surface.

Provided with the initial data, one can perform the first step: the (time) *de-migration*, in order to obtain the observed time surfaces. In the second step, we have to do the "exact" inverse work of zero offset acquisition, that is to say, map migration is a process that uses ray-tracing to compute the path of a ray between the *de-migrated* surface, equivalent to a free surface, and the future depth migrated surface. From each atom of the *de-migrated* surface a ray is sent with an imposed initial direction of shooting. This ray is computed taking into account the refractions at each interface cut by the ray (and previously depth migrated) and the consumption of the time t_0 given by *de-migration*.

To check the accuracy of the method described above, I have used several examples of synthetic data: horizontal layers, dipping layers, multi-layered time models and time syncline (that produces a triplication as a time *de-migrated* surface). All these examples have produced good results. In particular, I had the opportunity to observe the inverse effect of time migration produced by *de-migration* in the case of a dipping reflector model: the generated surface is moved downdip, is less steep and elongated (Yilmaz, 1987).

LIMITATIONS AND FUTURE WORK

Limitation of the ray-tracing method

From the equation of *de-migration* (1), it is noticeable that the observed time t_0 exists only if:

$$
\left(\frac{\partial t}{\partial x}\right)_m^2 + \left(\frac{\partial t}{\partial y}\right)_m^2 < \frac{4}{V_{rms}^2}
\tag{2}
$$

The equation (2) above expresses the degree of limitation of the method with respect to the V_{rms} velocity. This means that if the dip of an initial time migrated surface overflows a certain value imposed by the value of the V_{rms} velocity, the *de-migration* process will fail.

In practice, I observed that for a dipping plane with a V_{rms} velocity equal to 2000m/s, this limitation problem occurs for a dip angle equal to 45 degrees.

Introducing Eikonal equation and future work

In two dimensions, considering one-way time, the wave equation is:

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial z^2} = \frac{1}{V^2}\frac{\partial^2 p}{\partial t^2} \tag{3}$$

where $p(x, z, t)$ is the pressure wave field and $V(x, z)$ is the velocity of the wave propagation in the medium.

In 1991, Vijay Khare from Exxon proposed to use the following formula as the wave equation corresponding to the ray-theoretic interpretation of all-angle time migration he wants to introduce (Khare, 1991):

$$\frac{\partial^2 p'}{\partial x'^2} + \frac{1}{\tilde{V}^2(x', \zeta)}\frac{\partial^2 p'}{\partial \zeta^2} = -\frac{2}{\tilde{V}^2(x', \zeta)}\frac{\partial^2 p'}{\partial t' \partial \zeta} \tag{4}$$

where $\zeta(x, z)$ is the time-like vertical coordinate.

As we can express the familiar Eikonal equation for the travel time $\tau(x, z)$ in high frequency domain, V. Khare proposes from equation (4) the following paraxial approximation of Eikonal equation for time migration:

$$\left(\frac{\partial \tau'}{\partial x'}\right)^2 + \frac{1}{\tilde{V}^2(x', \zeta)}\left(\frac{\partial \tau'}{\partial \zeta}\right)^2 = \frac{2}{\tilde{V}^2(x', \zeta)}\frac{\partial \tau'}{\partial \zeta} \tag{5}$$

The first step of this project is to retrieve and to understand how V. Khare obtains the equations mentioned in his paper and particularly the equation (5) from the wave equation.

Then I will try to apply to the 3-D case all the transformations described in the 2-D case, that is to say, to express the paraxial approximation of Eikonal equation for 3-D time migration and to find the inverse process, the 3-D time *de-migration*.

The next step will be to use a finite difference method with a synthetic model in order to have a rough but clearest idea of the possibilities I can take out of this equation. At last, it could be interesting to contemplate testing other methods for this process and/or to use real data.

CONCLUSIONS

The ray-tracing algorithm for 3-D map *de-migration*/migration produces good results provided that the maximum dip of the layers under investigation is under the limit corresponding to the chosen velocity model.

The limitation occurs because of the geometrical definition of the method. This problem drives me to investigate Eikonal equation for time migration in order to go further in this project. At present time, it is hard to say whether this new study will provide an algorithm strong enough to perform *de-migration* and map migration of any kind of time model.

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REFERENCES

Hatton, L., Larner, K. L., and Gibson, B. S., 1981, Migration of seismic data from inhomogeneous media: Geophysics, **46**, no. 5, 751–767.

Khare, V., 1991, A new ray-theoretic interpretation of time migration: 61st Annual Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts, 1492–1496.

Yilmaz, O., 1987, Seismic data processing: Soc. Expl. Geophys., Tulsa, OK.