Midterm Exam: Sample Questions

Math 128A Spring 2002
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Your Name: _____

- Time: 75 minutes.
- Answer ALL questions.
- Please read carefully every question before answering it.
- If you need extra space, use the other side of the page.

1. (4 points) An important characteristic of computer precision is the *machine epsilon*. It is defined as the smallest number ϵ such that $1+\epsilon$ has a computer representation and $1+\epsilon > 1$. Find the machine epsilon for the IEEE double precision standard (11-bit exponent and 52-bit mantissa).

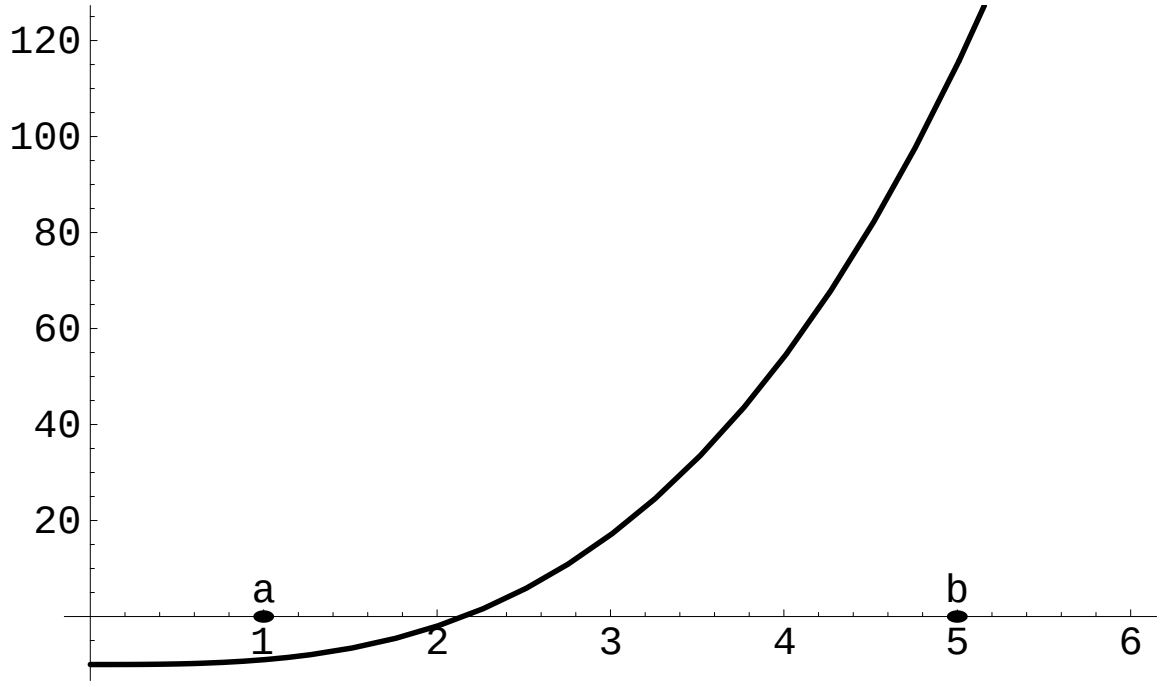
2. (10 points) Some computers do not have a hardware operation for division.

a. Show that one can approximate $c = \frac{1}{a}$ without doing any divisions by applying Newton's method for solving the equation $f(x) = 0$ with an appropriately selected $f(x)$.

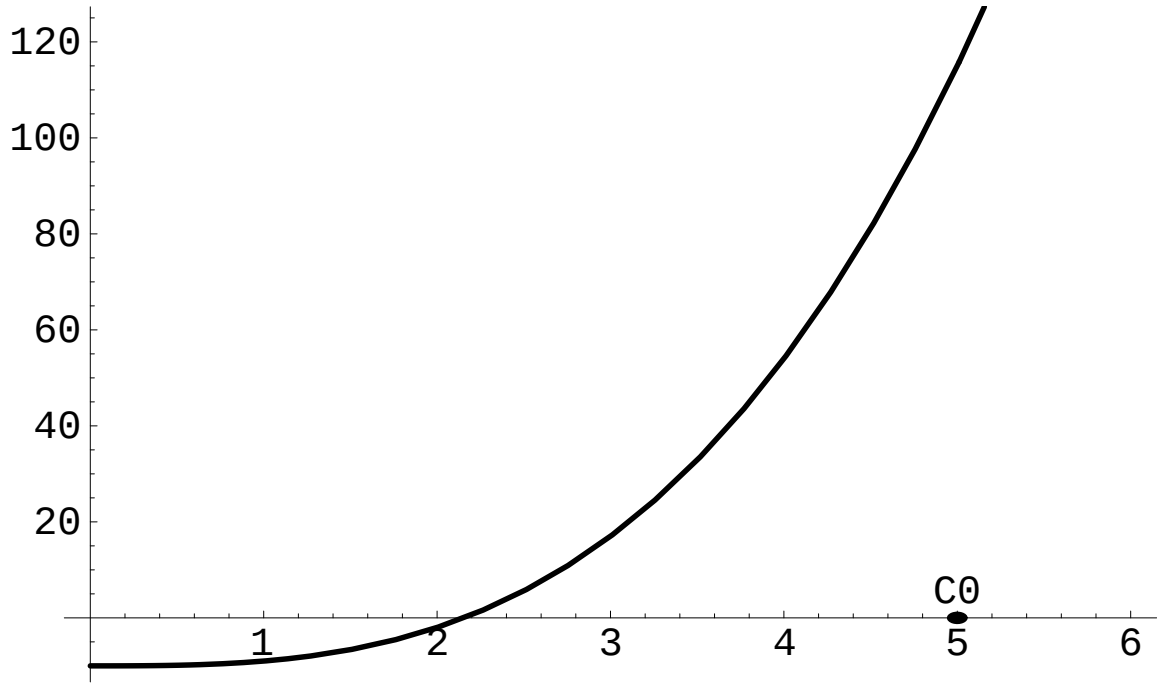
b. Starting with $c_0 = \frac{1}{2}$, find the next two iterations for approximating $c = \frac{1}{3}$.

3. (4 points)

a. The figure shows a function $f(x)$ and the initial interval $[a,b]$. Sketch the first three iterations of the bisection method applied to solving $f(x)=0$.



b. The figure shows a function $f(x)$ and the initial root estimate c_0. Sketch the next two iterations of Newton's method applied to $f(x)=0$.



4. (10 points) Prove that, if $f(x)$ is continuously differentiable, $f[x,x] = \lim_{y\to x} f[x,y] = f'(x)$ and, if $f(x)$ is twice continuously differentiable, $f[x,x,x] = \lim_{y\to x}\lim_{z\to x} f[x,y,z] = \frac{f''(x)}{2}$.

5. (8 points) Interpolate the function $f(x) = \sqrt{25 - x^2}$ at the nodes $x_1 = 0$, $x_2 = 4$ and $x_3 = 5$ with a quadratic polynomial $P(x)$. Find the relative error of $P(3)$.

6. (4 points) A function $S(x)$ defined on the interval $[a,b]$ is a quadratic spline if it is continuous together with the first derivative ($S(x) \in C^1[a,b]$) and the portion of $S(x)$ on each of the subintervals $[x_k, x_{k+1}]$ is a quadratic polynomial ($k = 1,2,\ldots,n-1$ and $a = x_1 < x_2 < \cdots < x_n = b$). How many boundary conditions are necessary for specifying the quadratic spline that interpolates $f(x)$ at the nodes $x_1, x_2, \ldots, x_n$?