ANISOTROPIC SEISMIC WAVE PROPAGATION

A DISSERTATION
SUBMITTED TO THE DEPARTMENT OF GEOPHYSICS
AND THE COMMITTEE ON GRADUATE STUDIES
OF STANFORD UNIVERSITY
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

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April 3, 1991

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Joe A. Dellinger
Stanford University, 1991

ABSTRACT

Traditionally, theoretical elastic-wave anisotropy has been studied analytically. While formal mathematical analysis can theoretically specify a wavefield exactly and completely, this very completeness often means that the results are expressed as pages of equations. These equations are often made more tractable by limiting the analysis to certain simple cases such as propagation along planes of symmetry or in highly symmetric media. Recent advances in computer power have made the study of theoretical anisotropy directly through *numerical* examples practical for the first time. To this end I present a gallery of examples of numerically calculated impulse-response surfaces and finite-difference wavefield snapshots. These examples are used to demonstrate and expand upon some of the theoretical properties of anisotropic elastic wave propagation predicted from geometrical or mathematical arguments. This philosophy of attack is applied to several varieties of anisotropy. Elliptical anisotropy can be completely modeled as linearly transformed isotropy. To the extent elliptical anisotropy is applicable, images of the subsurface generated by standard geophysical methods are sharp but distorted versions of the true depth picture. This is also true for the case of multiple dipping layers. For the case of two-dimensional transversely isotropic media I present examples spanning the wide range of wavefront behaviors possible in this symmetry system. I also present inequalities that can categorize the behavior from the elastic constants. Two-dimensional transversely isotropic equivalents of the isotropic wavetype-separation operators divergence and curl are derived and applied to finite-difference wavefields. The two-dimensional anisotropic operators work well although they are not as compact as the corresponding isotropic ones. The numerical examples show that mathematically tractable two-dimensional or symmetric cases are not representative of general three-dimensional anisotropy, however. In three dimensions wavetype-separation operators do not work for separating the two qS modes because of the obligatory presence of shear singularities tying the qS modes together. When a

transversely isotropic three-dimensional medium is perturbed to become orthorhombic a new event dubbed a "connection" can appear. This event acts to channel energy between the former qSV and SH modes outside of the symmetry planes, resulting in seismograms quite different in appearance from the unperturbed case.

Acknowledgments

A fool may ask more questions in an hour than a wise man can answer in seven years. – J. Ray (1670), English Proverbs

First of all, thanks to you for examining this rather long and theoretical thesis. (I'm hoping you're planning on reading more than just the acknowledgments; at *least* flip through and enjoy the nice pictures!) Thanks to everyone on my reading committee for wading through all this with such diligence. I hope you enjoyed it!

Among SEP members, Francis Muir and John Etgen deserve special thanks. Francis first got me interested in the subject of anisotropy in 1985, and it has kept me perplexed and entertained for several years now. John Etgen has been a fine friend as well as a brilliant collaborator. Many sections in this thesis answer questions that he asked. Most importantly, this thesis would have been impossible without the use of his finite-difference modeling programs. John spent many a latenight hour cursing in the terminal room while inserting features in his programs that I needed. Thanks, John. (Too bad I never was able to drag you and Jenni off to Yosemite.) Dave Nichols and Martin Karrenbach deserve special thanks for the many technical discussions we had; I can only hope I held up my end so well. Finally, thanks to Jon Claerbout for tolerating my esoteric investigations. Better catch them earlier next time.

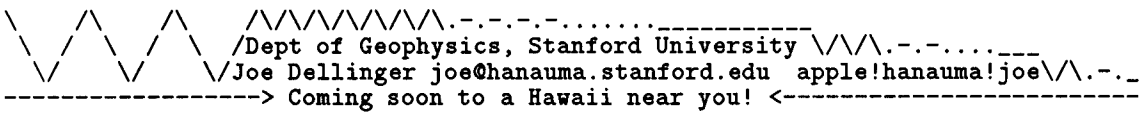
Having spent some time hacking at software myself, I know how much the people who really keep the SEP computer environment running tend to be underappreciated. Thanks to Steve Cole, Stew Levin, Dave, and Martin for all your efforts. This will be the last SEP thesis of the pre-workstation era. So long, Imagen.

Thanks to all my great friends at Breakers Eating Club, especially Roger, Scott, Tom,

Charles, and Craig. Hope to see you in Hawaii now and then! Thanks to all "the Dudes" for staying in touch despite being half a continent and a decade of my life away. Most importantly, thanks to my family for all your support. It's good to know someone is rooting for you no matter what!

Goodbye to the cave in the sky. I hope I'll never have to spend so much time in a windowless building again, especially in an office that would have had such a beautiful penthouse view of foothills and fog – *if only* there were a hole in the outside wall somewhere!

See you in Hawaii, or wherever I am by now. At the very least, **see you on the net!**



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