Appendix A

Derivatives of event parameters

In the residual event migration of Chapter 4, derivatives of event time and stepout need to be computed with respect to the reflector position and the model parameters (equation (4.21)). Similar derivatives (of zero-offset time or pseudo depth) are encountered in the gradient calculation of Chapter 5 (matrices **A** and **B** in equation (5.15)). In both cases, derivatives need to be calculated for a fixed configuration of source, geophone, and reflector. The computations use the traveltime calculations of Chapter 3, and in the rest of the appendix I assume that source and geophone traveltime maps, $t_S(x,z)$ and $t_G(x,z)$, are readily available. Of course, source and geophone position are identical for the pseudo-depth derivatives of Chapter 5, and only one traveltime map is needed in that case.

A.1 Relating reflector movement to changes in events

For a given depth point $\mathbf{r}$ on a reflector in a constant-offset section, the 2×2 matrix in equation (4.21) tells how changing the depth-point position affects the event parameters t and p_y of the corresponding data point $\mathbf{d}$. The derivatives in the matrix are simply computed by finite-differences. For example, $\partial t/\partial x$ is given by

$$\begin{aligned} \frac{\partial t}{\partial x} &= \frac{\partial t_S}{\partial x} + \frac{\partial t_G}{\partial x} \\ &= \frac{t_S(x_{i+1},z_i) - t_S(x_{i-1},z_i)}{2\Delta x} + \frac{t_G(x_{i+1},z_i) - t_G(x_{i-1},z_i)}{2\Delta x}, \end{aligned} \tag{A.1}$$

with (x_i, z_i) the coordinates of the depth point on the grid of the traveltime maps, which has a lateral grid spacing of Δx. $\partial t/\partial z$ is calculated analogously, as are the derivatives of stepout with respect to x and z, where stepout is evaluated with equation (4.8).

A.2 Relating model perturbations to changes in events

Because $\partial t/\partial \mathbf{m}$ in equation (4.21) is calculated for fixed depth point, source and geophone position, it can be computed by Fermat's principle, as in traditional traveltime tomography. Given the rays ρ_S and ρ_G between depth point and source and geophone, respectively, the traveltime t of data point $\mathbf{d}$ is just the integral of slowness s along the rays

$$t = \int_{\rho_S} s\, dr + \int_{\rho_G} s\, dr, \tag{A.2}$$

with r the arclength along the ray. Invoking Fermat's principle, the change in traveltime for a perturbed slowness model $s+\delta s$ is now computed as the integral of slowness perturbations along the unperturbed rays,

$$\delta t = \int_{\rho_S} \delta s\, dr + \int_{\rho_G} \delta s\, dr. \tag{A.3}$$

For a discrete slowness parametrization, the above equation gives a linear relation between slowness and traveltime perturbations. Using the spline representation of section 5.2, the above relation gives the following expression for the derivative of traveltime with respect to model parameter $m_c = c_{ij}$:

$$\frac{\partial t}{\partial m_c} = \int_{\rho_S} f_i(x(r))\, g_j(z(r))\, dr + \int_{\rho_G} f_i(x(r))\, g_j(z(r))\, dr. \tag{A.4}$$

The rays ρ_S and ρ_G needed in the above equation are reconstructed from the traveltime maps: the traveltime gradients ∇t_S and ∇t_G are followed back from reflector point to source and geophone, respectively. The time gradients are readily available in the traveltime calculations of Chapter 3. Note that this ray reconstruction is much more efficient than conventional ray-tracing methods. A conventional method would have to be either a two-point method, which is generally complicated and computationally expensive, or, if a shooting method is used, it would require shooting many rays, which would then have to be sorted or interpolated to identify the exact source and geophone ray.

As for the derivatives of stepout with respect to the model parameters, they are computed with equations (4.7) and (4.8):

$$\frac{\partial p_y}{\partial m_c} = \frac{\partial t_{S+1}}{\partial m_c} - \frac{\partial t_{S-1}}{\partial m_c} + \frac{\partial t_{G+1}}{\partial m_c} - \frac{\partial t_{G-1}}{\partial m_c}, \tag{A.5}$$

where the traveltime derivatives are calculated with equation (A.4).

Appendix B

MigVelAn: An interactive interface for migration-velocity analysis

The input to the velocity-estimation method is a set of reflectors in the migrated constant-offset sections, which are picked and modified interactively on a workstation. The picked reflectors form a surface in the prestack-migrated data space (representing reflector depth as a function of surface location and offset), and ideally one wants to interactively manipulate the surface in the three-dimensional data volume. However, although some work has been done in this area (Mallet et al., 1989), handling these surfaces in three dimensions on a workstation is quite complicated, especially if they are multi-valued (as in overhanging salt domes). Therefore, I limit myself to picking two-dimensional slices through the data cube, the constant-offset sections. Events can be displayed in the other dimension, though, so that picks can be verified. Some of the advantages of picking constant-offset sections are discussed in section 2.3.2.

The interface that I have written for picking the migrated data runs on top of the X-window system, and is coded in C++ with the help of a toolkit called InterViews (Linton et al., 1987; Dulac et al., 1988). The details of the interface are described in Van Trier, 1988; in this appendix I illustrate only how the data is manipulated with the program.

B.1 PICKING MIGRATED DATA

The standard picking procedure is to start with picking reflectors in the near-offset section. Reflectors are better defined in the near-offset section than in the far-offset sections, which have larger residual moveout. After the near-offset section is picked, the picks are projected onto the higher-offset sections, and they are adjusted after each projection. Thus, the main effort is in picking the near-offset section: residual moveout is generally small, meaning that only minor adjustments need to be made to the projected picks.

Figure B.1 shows a snapshot of the interface, displaying the migrated data described in Chapter 2. To view different constant-offset sections, the user can adjust the slider on the right side of the canvas. The section shown in the figure has an offset of 1213 m (see *x3*-field). The *Project*-buttons project picks either forward or backward (to far- or near-offset sections). Detailed adjustments to the picks can be made by zooming in on the section (Figure B.2). Reflectors are represented by linear segments that can be adjusted in the *Edit*-mode of the interface. (The linear segments are just used for manipulation; on final output, the interface models reflectors as spline curves fitted through the picks.)

If the picks in the constant-offset sections are satisfactory, the user can verify them in the offset-direction by pushing the *Cross Section*-button and using the mouse button to select a surface range. The interface then pops up a window that displays the CRP gathers (Figure B.3) in a so-called "deck" (an InterViews-object). As before, adjusting the slider on the right side of the window changes the gather that is displayed. Figure B.3 shows two gathers at two different surface locations. Residual-moveout curves are plotted on top of the gathers. At each surface location, these curves are found by a spline interpolation along offset of the picks in the constant-offset sections.

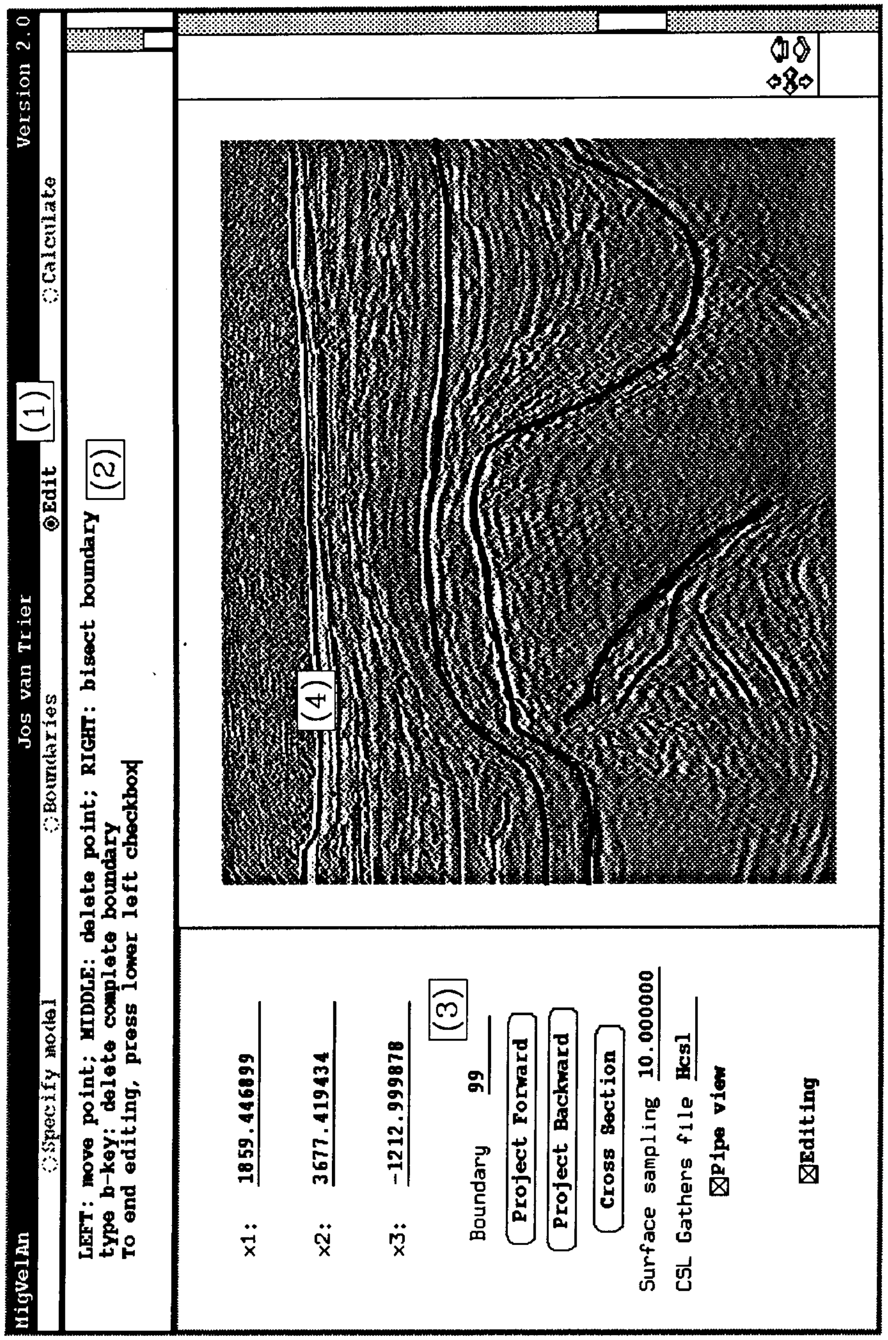

FIG. B.1. Interactive interface. The interface consists of four parts: (1) Mode-selection buttons; (2) Information window; (3) Mode-specific window for entering parameters or activating operations; (4) Canvas that displays the data. The different modes are *Specify model* for reading the data and specifying general parameters, ***Boundaries*** for picking reflectors, *Edit* for editing the picks and examining the data in the offset dimension, and *Calculate* for fitting spline curves to the picks and for writing the results.

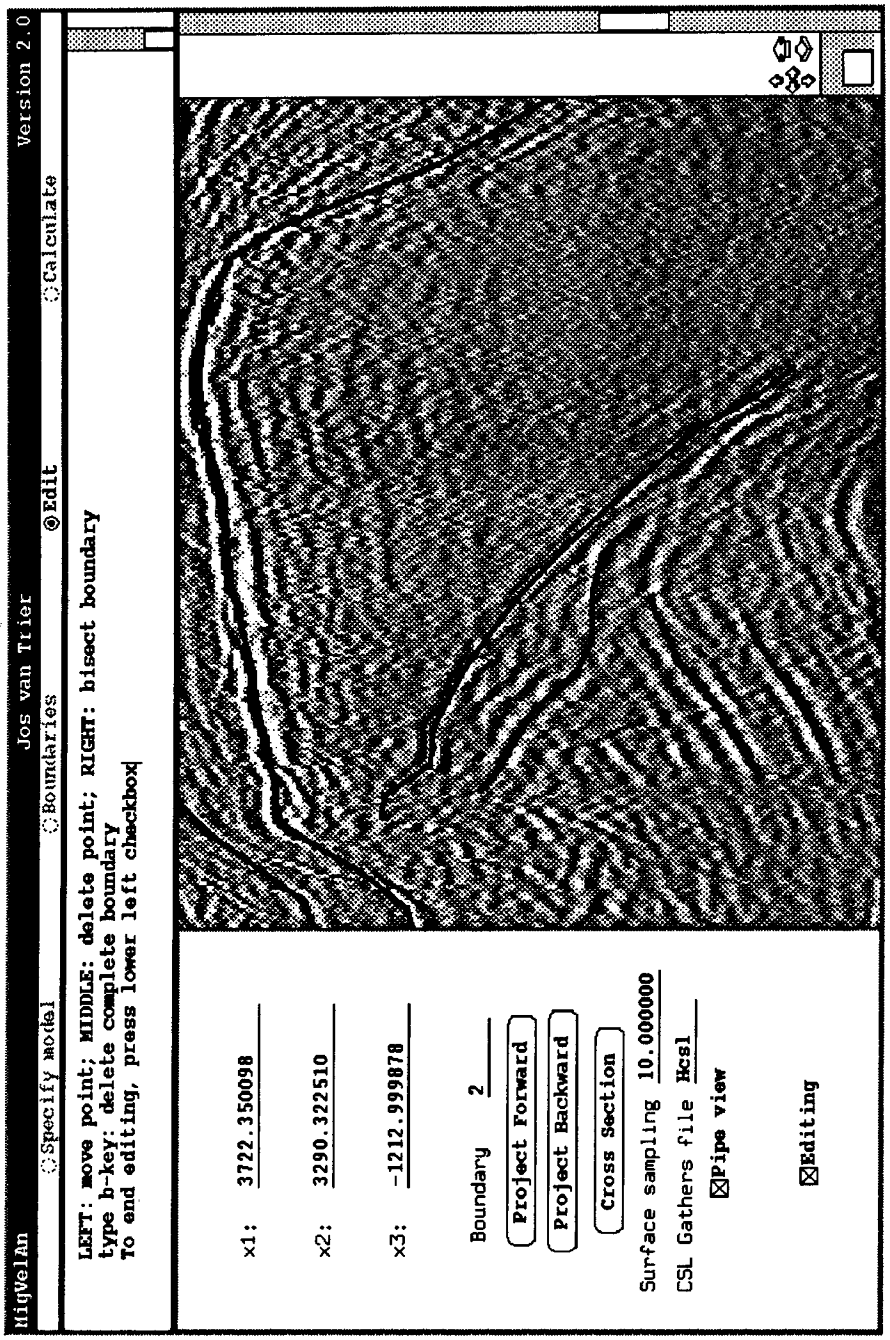

FIG. B.2. Same as Figure B.1, but with zoomed-in section.

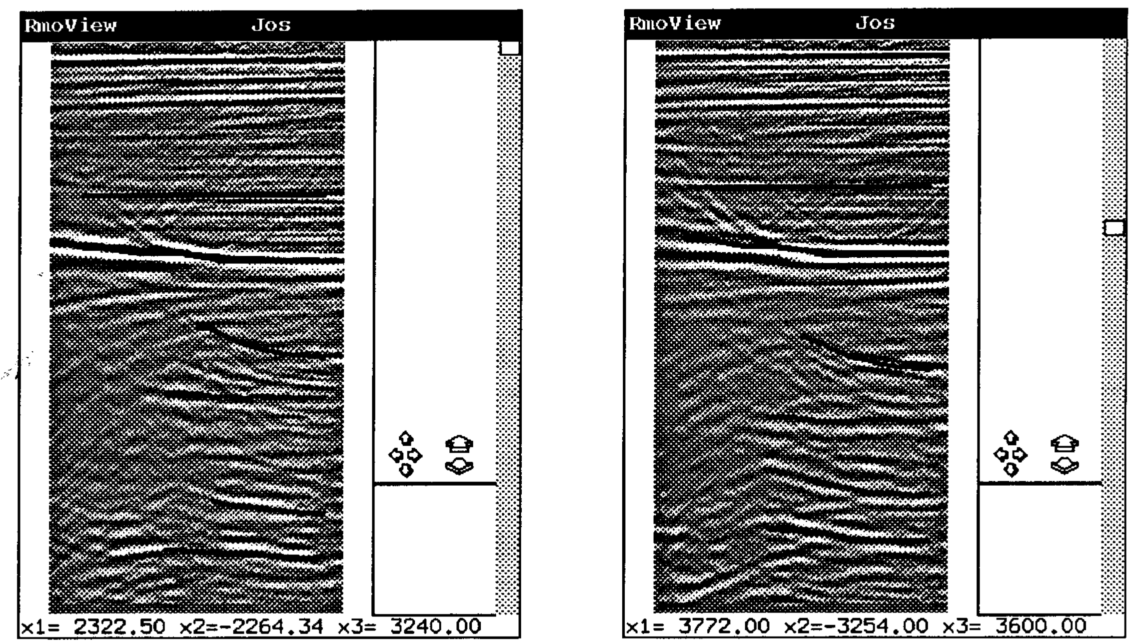

FIG. B.3. Residual-moveout curves in CRP gathers. The plot shows two instances of the same window: one displaying a CRP gather at a surface location of 3240 m (left) and one displaying the CRP gather at 3600 m (right). To adjust the surface location, the user moves the slider on the right side of the window.

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