Thin layer filtering and the O'Doherty-Anstey approximation

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ABSTRACT

Plane waves are normally incident on a sequence of horizontal layers. If the layers are lossless the shape of the frequency spectrum of the reflection response depends on the reflection coefficient series. The law of dependence can be found by solving the wave equation for the boundary and initial conditions of the seismic experiment. The O'Doherty–Anstey formula is an approximation to this law, and its validity would imply a lowpass spectrum of the reflection response if the reflectivity power spectrum has a highpass trend.

INTRODUCTION

A plane wave generated by a surface source is normally incident on a sequence of horizontal layers. Let the wave be a unit impulse in zero time. The pulse travels down through the layers and primary and multiple reflections generated within the horizontal interfaces are recorded as they arrive back at the surface. The recording starts at zero time (so the first event recorded is the incident pulse) and lasts for some finite time interval. Assume that there is no *intrinsic* attenuation (heat loss), so the amplitudes of the arriving reflections depend only on the reflection coefficients of the interfaces. If the whole earth below the recording location had constant impedance, the incident pulse would be the only recorded event. Then the frequency spectrum of the recorded time series would be white.

The existence of primary and multiple reflections when the earth is layered creates "notches" and generally "shapes" the frequency spectrum of the recorded time series (Schoenberger, 1974). That is, each of the frequency components of the incident pulse is subjected to a different gain and phase shift and the recorded reflection is the sum of these modified components. So each component is *attenuated* differently and this *dissipation* of energy is not caused by a physical process like

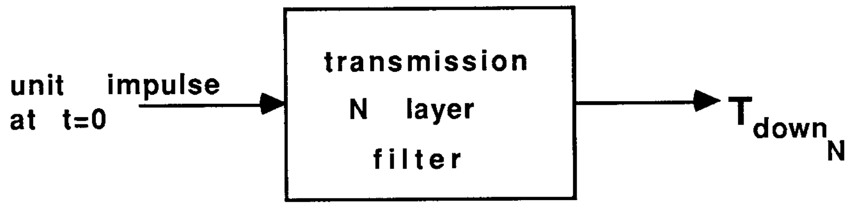


FIG. 1. Filtering of the incident pulse as it is transmitted below the layer sequence.

heat loss, but by constructive or destructive combinations of wave amplitudes in time. So it is reasonable to call this process "layer filtering" and we expect the filter to depend on the characteristics of the layers (impedances and thicknesses). These characteristics are described by the one-way travel-time series of the reflection coefficients, with the sampling interval a parameter.

LAYER FILTERING

N horizontal layers of equal one-way travel time Δx are "sandwiched" between two halfspaces with absorbing interfaces. A unit pulse is normally incident on the top of the first layer. The problem and the initial and boundary conditions are described in the paper preceding this in detail. (See also Claerbout, 1976, Chapter 8 and Resnick, (1986)).

Transmission filtering

The transmittance $\tau(t)$, which is the downgoing wave at the last interface (at one-way travel time $x = (N+1) * \Delta x$), is given in the Z-domain by

$$T_{down_N}(Z) = Z^{N/2} \frac{\prod_{k=0}^{N}(1+\rho_k)}{F_N(Z)} \tag{1}$$

where $F_k(Z)$ are given by equations (11),(12), and (13) of the previous paper. Figure 1 gives a "systemic" representation of the above equation that describes the sequence of thin layers as a transmission filter. $T_{down_N}(Z)$ is then the (transmission) *response* of the filter of N layers, that is the output of the filter when its input is a unit pulse at zero time.

Reflection filtering

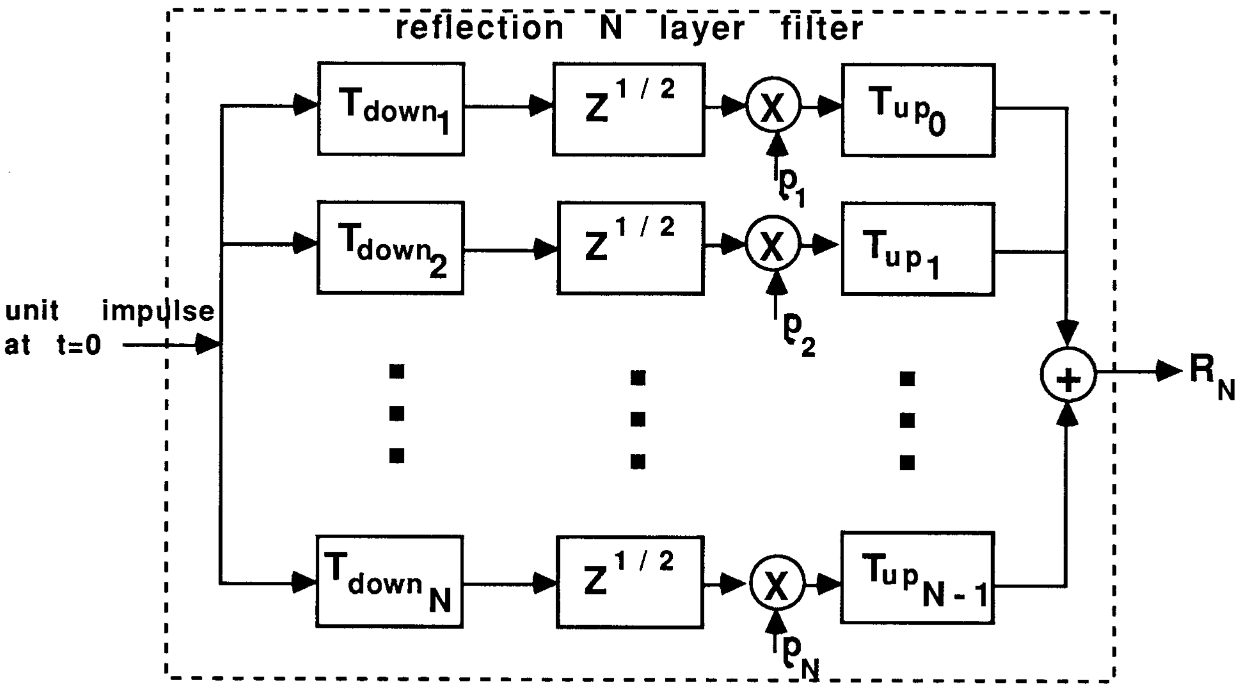


FIG. 2. Filtering of the incident pulse as it is reflected by the layer sequence.

The reflectance $r(t)$, which is the upcoming wave at the first interface (at one-way travel time $x = 0$) is given in the Z-domain by

$$R_N(Z) = \sum_{k=1}^{N} \frac{T_{down_k}(Z)}{(1+\rho_k)} \rho_k Z^{1/2} T_{up_k}(Z) \tag{2}$$

where $T_{up_k}(Z)$ is given by equations (9) to (13) of the previous paper. Figure 2 gives the "systemic" representation of the reflectance, as the (reflection) *response* of the filter of N layers.

THE O'DOHERTY-ANSTEY APPROXIMATION TO THE FILTER

Let ρ_i, $i = 0, \ldots, N$ be the reflection coefficient series of the N layer sequence. This is a discreet one-way travel-time series whose sampling interval is Δx. Its power spectrum then is

$$P_N(Z) = \sum_{n=0}^{N} Z^n \sum_{m=0}^{N-n} \rho_m \rho_{m+n} \tag{3}$$

Transmission filtering

Resnick (1986) derive the O'Doherty–Anstey formula in a deterministic way: *Under the assumption that* $|\rho_i| \ll 1$ they approximate equation (1) by

Then the transmission filter response is given by

$$T_{down_N}(Z) = Z^{N/2} \prod_{k=1}^{N} (1 + \rho_k) \exp(-P_N(Z)) \tag{4}$$

In the stochastic derivation of the formula, Banik, (1985), make the same assumption that the reflection coefficients are small and the extra assumption that the impedance is a stationary process. Then they solve the stochastic wave equation by mean field methods.

Reflection filtering

The same approximations as in the transmission filtering give the reflection filter response in the deterministic approach of Resnick et. al.:

$$R_N(Z) = \sum_{k=1}^{N} \rho_k Z^k \prod_{k=0}^{k-1} (1 - \rho_k^2) \exp(-P_k(Z) - P_{k-1}(Z)) \tag{5}$$

HOW GOOD AN APPROXIMATION?

It has been demonstrated that the power spectrum of the reflection coefficient series is not white but has a droop in low frequencies (< 100Hz) (Velzeboer, (1981), Walden,(1985)). If this is so in most cases and if the Anstey approximation is also valid in those cases, we come to the conclusion that the transmission and reflection (layer) filters both attenuate high frequencies (Ziolkowski, (1985)).

In Figures 3 and 4 we show the reflection response given by the exact solution and the difference between this and the one given by the Anstey approximation for the reflection coefficients of Log A (see previous paper). Note that the approximation error is about 10%.

In Figure 5 we give the amplitude of the frequency spectrum of the reflection response for a number of reflectivities. Each reflectivity contains all the coefficients of the previous one and some more until we finally construct the reflectivity of Log A.

MULTIPLE SUPPRESSION

In the paper preceding this, (Serakiotou, 1988), we present the idea for a technique that would partially suppress multiples caused by a thin layer sequence. One of the experiments we did was to compute the (one-dimensional) reflection response of velocity Log A (Figures 3 and 5 of previous paper) and of Log B (Figure 4 and 6 of previous paper) with the assumption of constant density. Then we subtracted the response of Log B from the response of Log A to get a better image of the isolated reflectors (Figure 7 of previous paper).

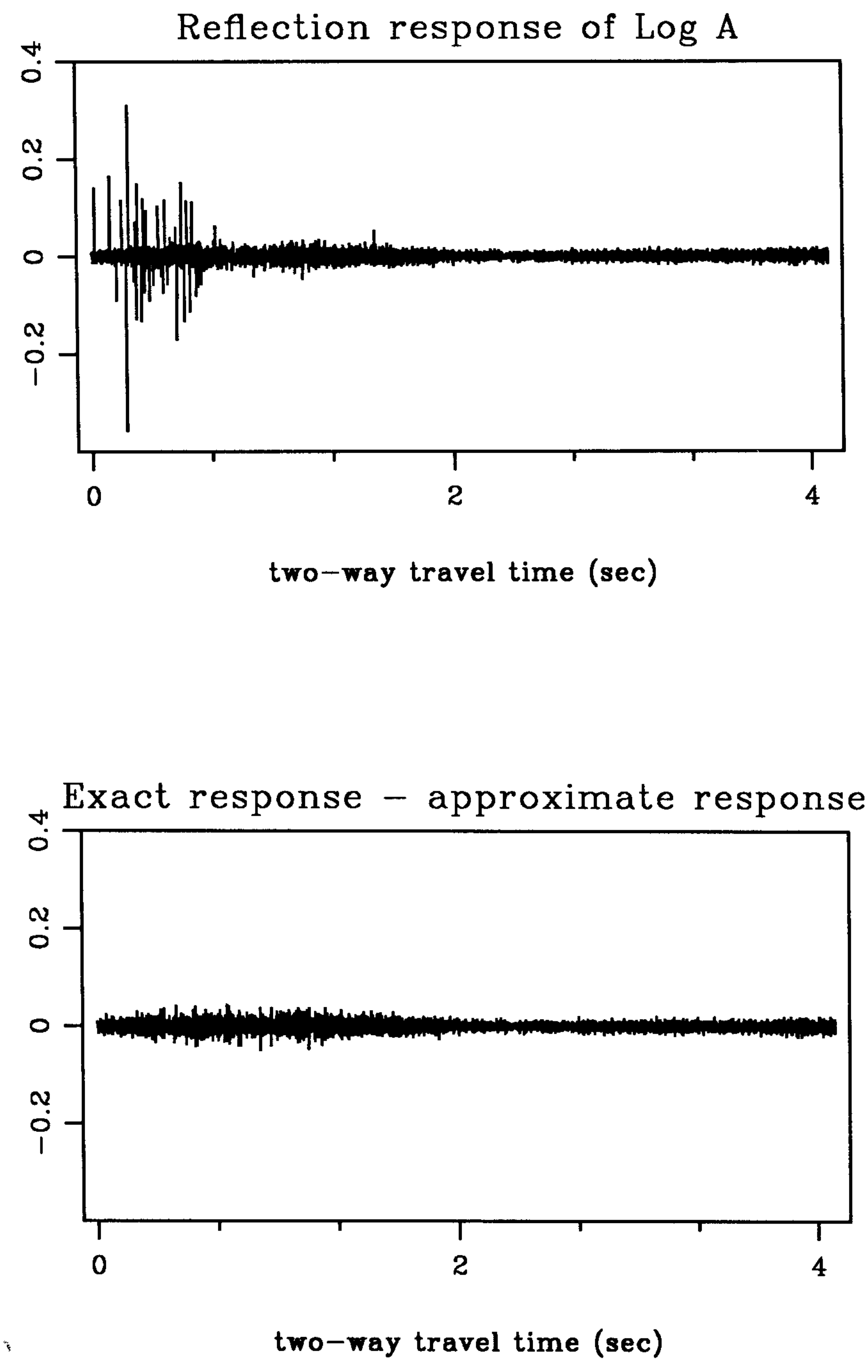


FIG. 3. The reflection response of Log A given by the exact solution and the difference of this and the Anstey approximation.

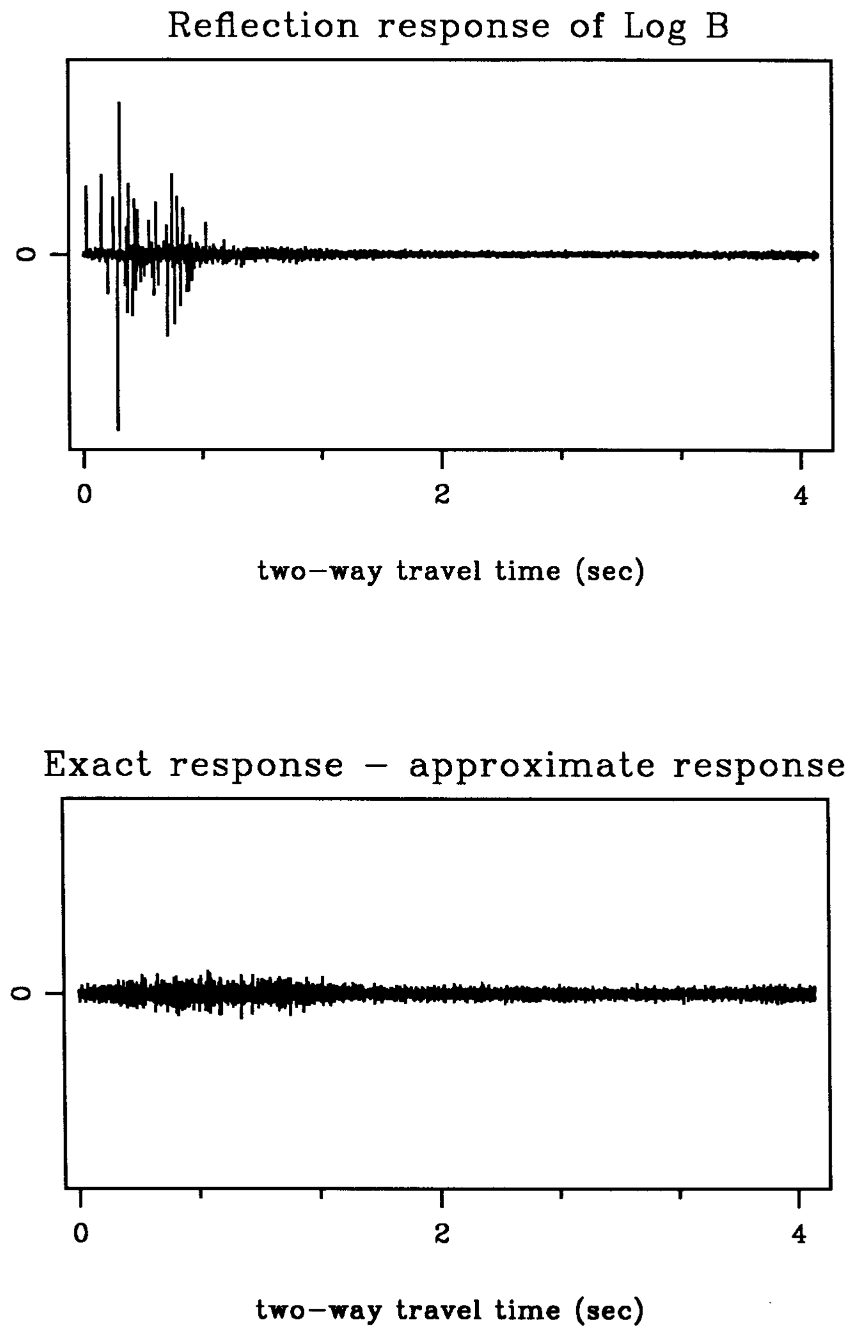


FIG. 4. The reflection response of Log B given by the exact solution and the difference of this and the Anstey approximation.

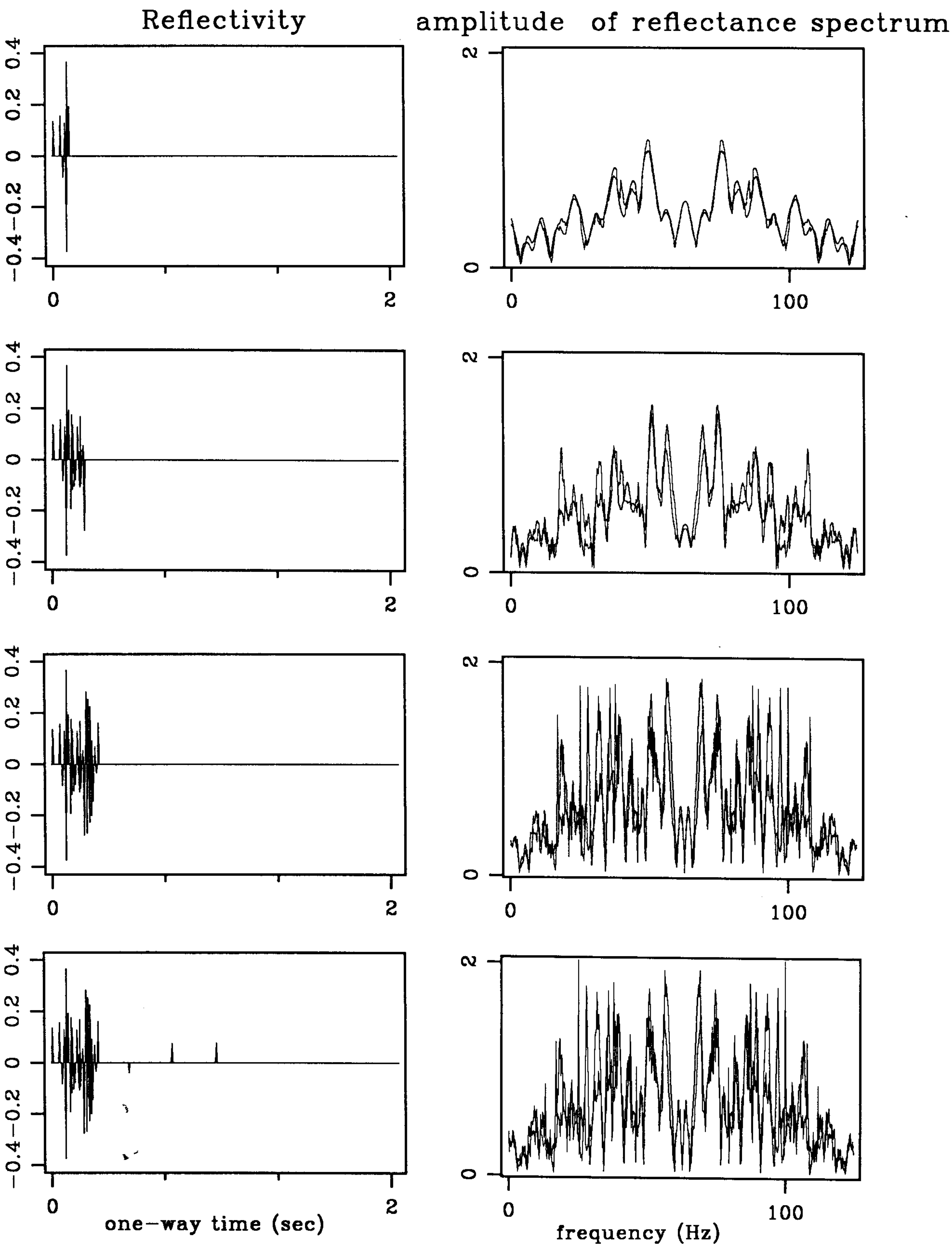
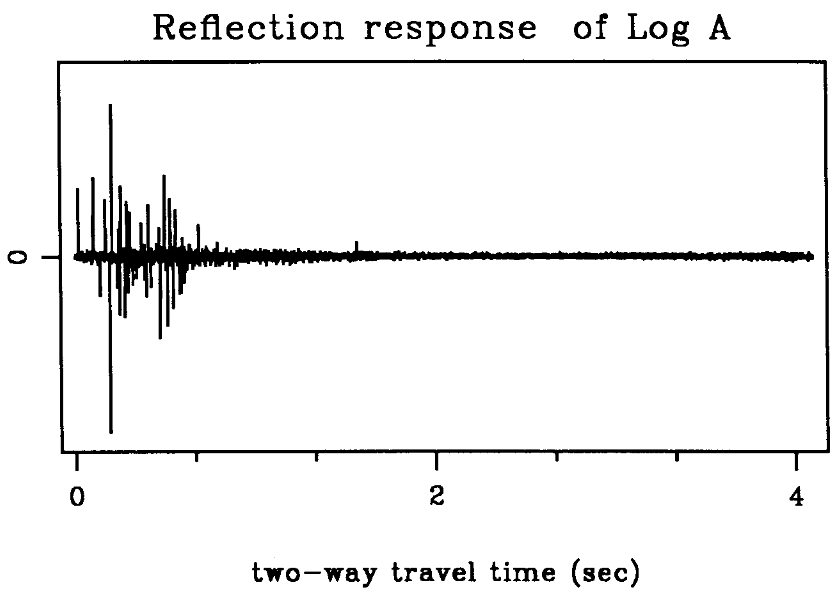


FIG. 4. The amplitudes of the frequency spectra of the reflectivities in the left are shown in the right column, computed by both the exact solution and the approximation.



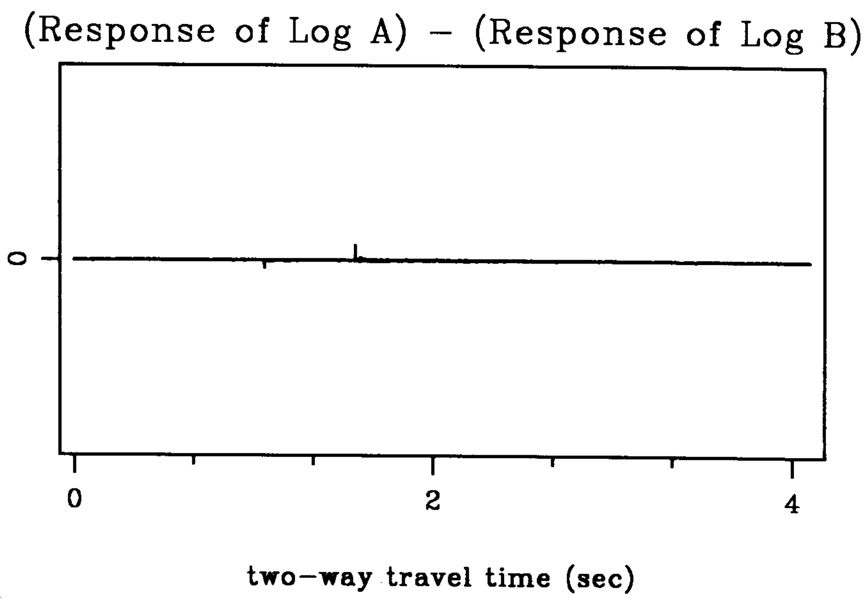


FIG. 5. The reflection response of Log A and the remainder of the subtraction of the response of Log B from Log A. Compare with the results of the exact solution in Figures 5 and 7 of the previous paper (Serakiotou, 1988).

Here we repeat the same experiment using the O'Doherty–Anstey approximation to do our computations. The results are very good and are shown in Figure 6. The computations are done faster than when using the exact formulas. Even the aliasing problem in the remainder of the subtraction seems less severe but this is expected, since the amplitudes of the wave train are underestimated by the approximation.

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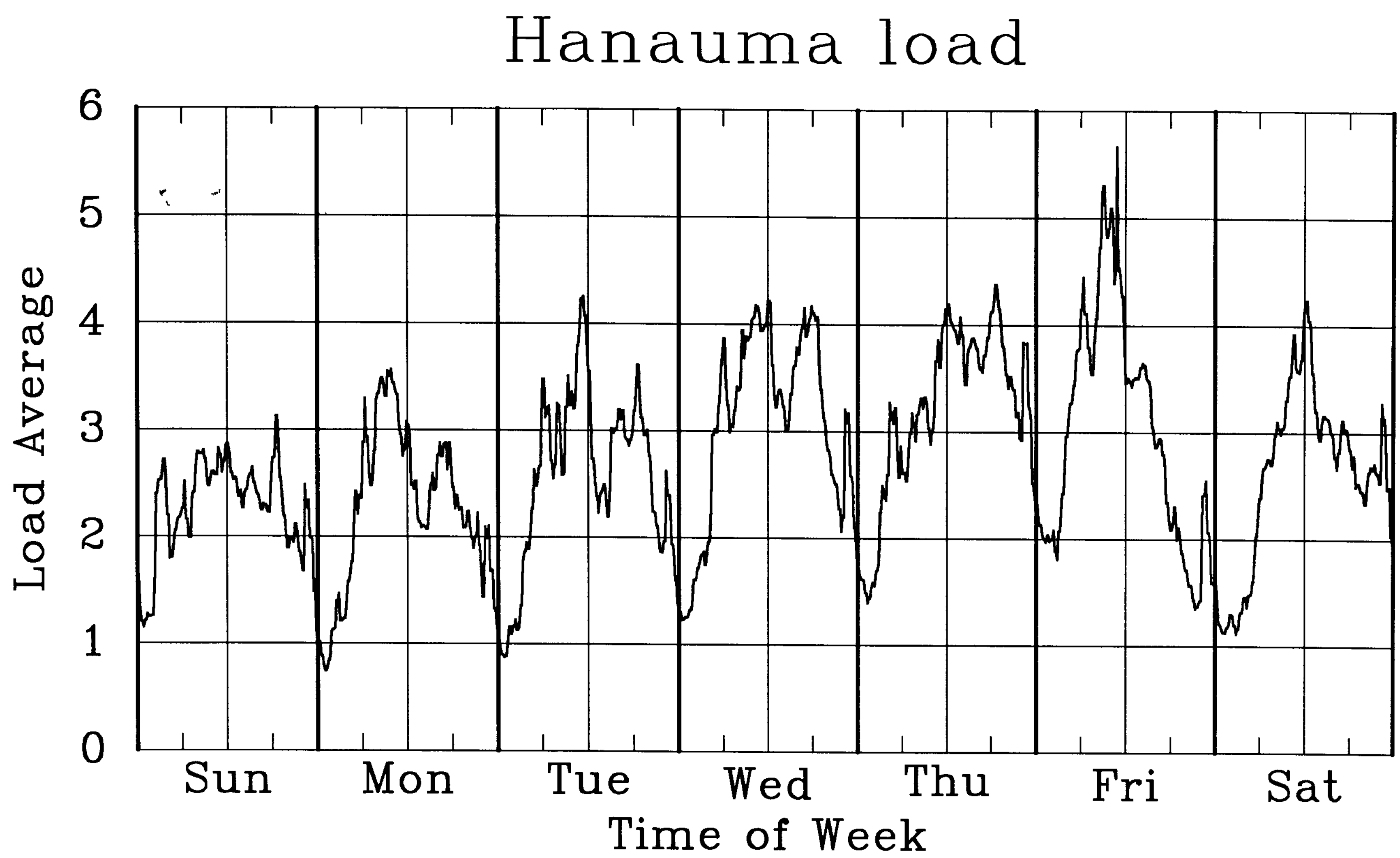

Load Average on Hanauma as a function of Time of Week for the period Feb 22, 1988 through April 18, 1988. Days are considered to begin and end at 6AM; this time is marked by the **thick** vertical bars. For some reason people don't rush off to dinner at 6PM on Thursdays and Sundays.