Migration = focus + map

Joe Dellinger & Francis Muir

ABSTRACT

Depth migration is viewed as a two-step process: the data is *focused* and then the focused image is *mapped* to true depth. It is not necessary to know the complete velocity and anisotropy structure to focus the data, but to make a true depth image it is. As a simple example, we consider the case of depth migration in the presence of elliptical anisotropy with an arbitrary axis. In this case standard isotropic constant-velocity migration focuses the data exactly, but a linear transformation is still required to map the result to a true depth image.

INTRODUCTION

It is well known that depth information cannot be recovered from surface kinematic or single-component data if there is the possibility of unknown elliptical anisotropy with a vertical axis of symmetry. Less well known is that neither vertical *nor horizontal* position information can be recovered if the elliptical anisotropy may have an arbitrary axis of symmetry. The vertical scale is unknown, as is a horizontal shear.

Suppose we do have surface-to-surface seismic data recorded over such an elliptically anisotropic earth – how can we construct the true depth image? The trick is first to *focus* the data into a false space using the NMO velocity available from the data and an isotropic assumption, and then to *map* this solution onto the true depth space using a linear coordinate transformation derived from borehole or similar velocity data. The solution thus obtained is both dynamically and kinematically correct.

We first demonstrate the kinematics of this result using graphical methods, and then develop a mathematical argument. In both cases we will assume a two-dimensional world, understanding that there are simple extensions to three.

ELLIPTIC ANISOTROPY

A graphic illustration

A medium is *elliptically anisotropic* if wavefronts spreading away from a point source are ellipses rather than circles. To make an elliptically anisotropic wave equation, start with the regular isotropic wave equation and apply a linear transformation to the coordinate system. Graphically, a linear transformation is just a combined stretch and rotation.

A snapshot of a wavefield shows the wavefield at one instant of time. Since stretching a snapshot of a wavefield is the same thing as linearly transforming the underlying wave equation, we can get a snapshot of an elliptically anisotropic wavefield by simply stretching the corresponding isotropic snapshot.

As an example, we will examine the case of a virtual image created by a mirror.

In Figure 1 we show the standard isotropic case. The starburst at the top of the picture represents a point source, and the starburst at the bottom of the picture is its virtual image. If we stretch Figure 1 at a 45° angle to the reflector, and then rotate it so that the reflector is again horizontal, we obtain Figure 2. This figure must show how a mirror in an elliptically anisotropic medium works. Since the starbursts are supposed to represent *point* sources, they are not stretched.

Examining the two figures we can see that the "mirror image" of the point source is a point in both cases. The "mirror image" of the square near the left edge in Figure 2, however, is not a square but a strongly sheared parallelogram.

What implications does this have for migration? Since each point above the mirror maps into a perfect point below the mirror, isotropic migration must still be able to *focus* the image perfectly. However, we can also see by looking at the image of the square that even though the image is in focus it is "stretched". If we want a true depth picture we have to "unstretch" the image.

Mathematical analysis

Any elliptically anisotropic acoustic wave equation can be written in the form

$$(1+\kappa_1^2)\frac{\partial^2\phi}{\partial x^2} + 2\kappa_1\kappa_2\frac{\partial^2\phi}{\partial xz} + \kappa_2^2\frac{\partial^2\phi}{\partial z^2} = c^{-2}\frac{\partial^2\phi}{\partial t^2}, \tag{1}$$

where ϕ is the field variable, and κ_1, κ_2, and c determine the medium properties.

How do we migrate data recorded over an earth that satisfies this equation? If we assume the data were recorded on the surface ($z = 0$), then any transformation that leaves time and $z = 0$ unchanged will likewise leave the data unchanged. The linear transformation

$$\begin{aligned} x &= x' + \kappa_1 z' \\ z &= \kappa_2 z' \end{aligned} \tag{2}$$

leaves time and $z = 0$ unchanged and also simplifies equation 1 to

$$\frac{\partial^2 \phi}{\partial x'^2} + \frac{\partial^2 \phi}{\partial z'^2} = c^{-2} \frac{\partial^2 \phi}{\partial t^2}. \tag{3}$$

This is just the standard isotropic acoustic wave equation. Since it fits the surface data exactly, it can be used to focus our data as well as the correct equation 1. If we then apply equation 2 to map the image back to the correct coordinates, we will have correctly migrated the data.

VARIATIONS

Focusing in time

Despite uncertainty about what a time-migrated section really represents, time migration continues to be popular, at least in part because false structure is not created by erratic lateral changes in NMO velocity estimates, nor modified as NMO velocities are re-estimated. Perhaps the most compelling reason for focusing in time is that both NMO velocity estimation and horizontal interval velocity estimation via the Dix method are time dependent and in no way depend on the isotropic assumption. So we can replace the isotropic focusing stage by an elliptic scheme, where horizontal interval velocities are derived from NMO velocities as before, but the vertical velocity is held constant, and depth replaced by time. The second stage *false→true* depth-section mapping is now replaced by a *time-section → depth-section* mapping, reminiscent of the original depth migration techniques.

A hybrid scheme

Once the notion of anisotropy is accepted, it seems reasonable to estimate horizontal and vertical velocities separately — even if we have no hard information on vertical velocities. For example, it is customary to estimate velocities densely along a line. These estimates are then used to correct for NMO, or in what is much the same thing, as a basis for focusing. We understand, of course, that these lateral variations in NMO velocity may not really represent changes in vertical velocity, so it might be prudent to smooth these horizontal velocity estimates before using them as vertical velocities.

PHYSICAL REALITIES

So far we have managed to discuss elliptic anisotropy without considering the realities imposed by physics. Does stretching have meaning? It is possible to stretch one wavetype in isolation, such as S_H or P in transverse isotropy. The standard elastic wave equation cannot be stretched, however, because the orthogonality of particle motion between wave types would be lost in the process. This does not mean

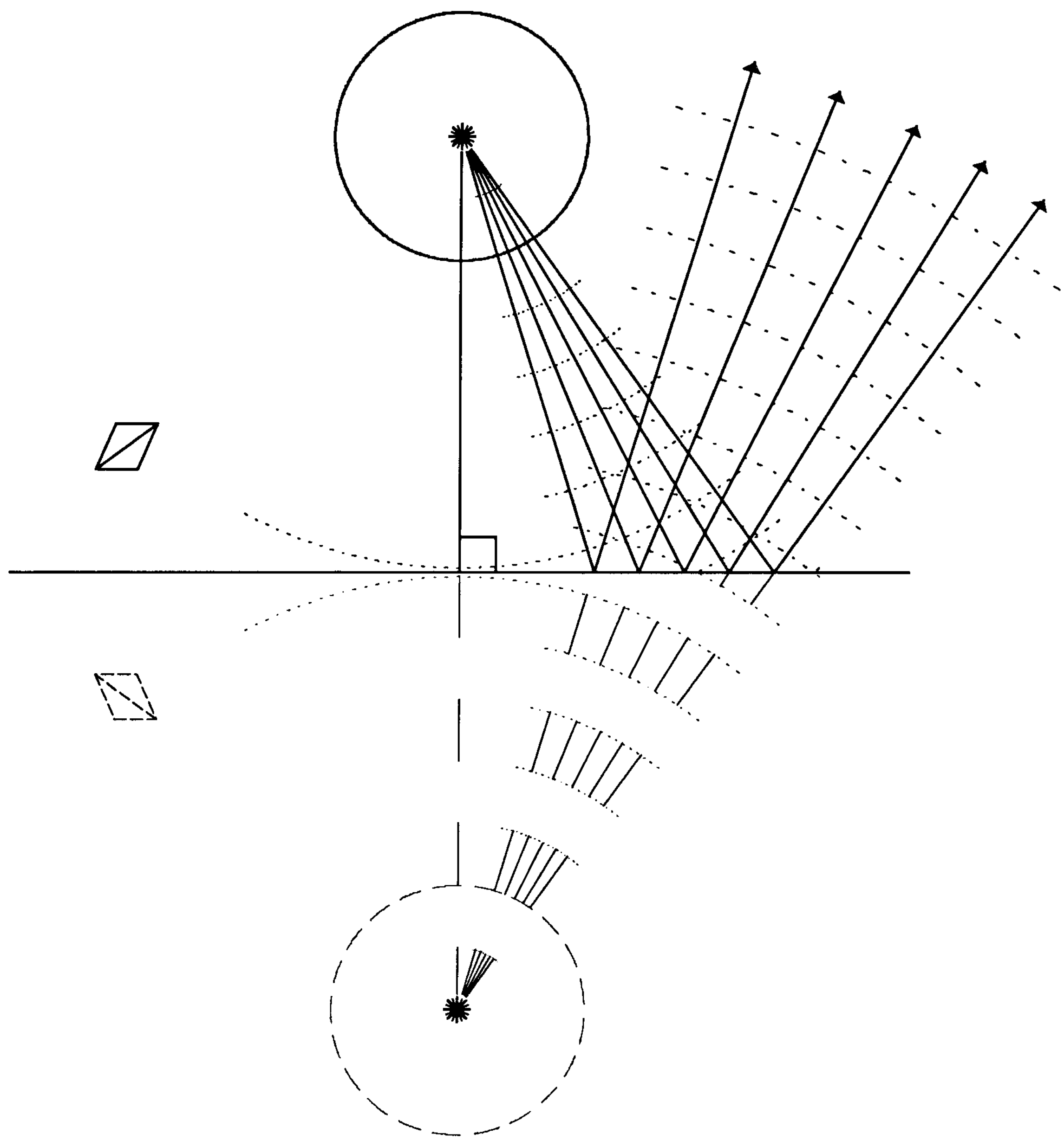

FIG. 1. A mirror in an isotropic medium. The starburst above the mirror represents a point source. One complete wavefront and several partial ones are shown propagating away from it, along with some associated rays. Below the mirror we see virtual images of objects above the mirror.

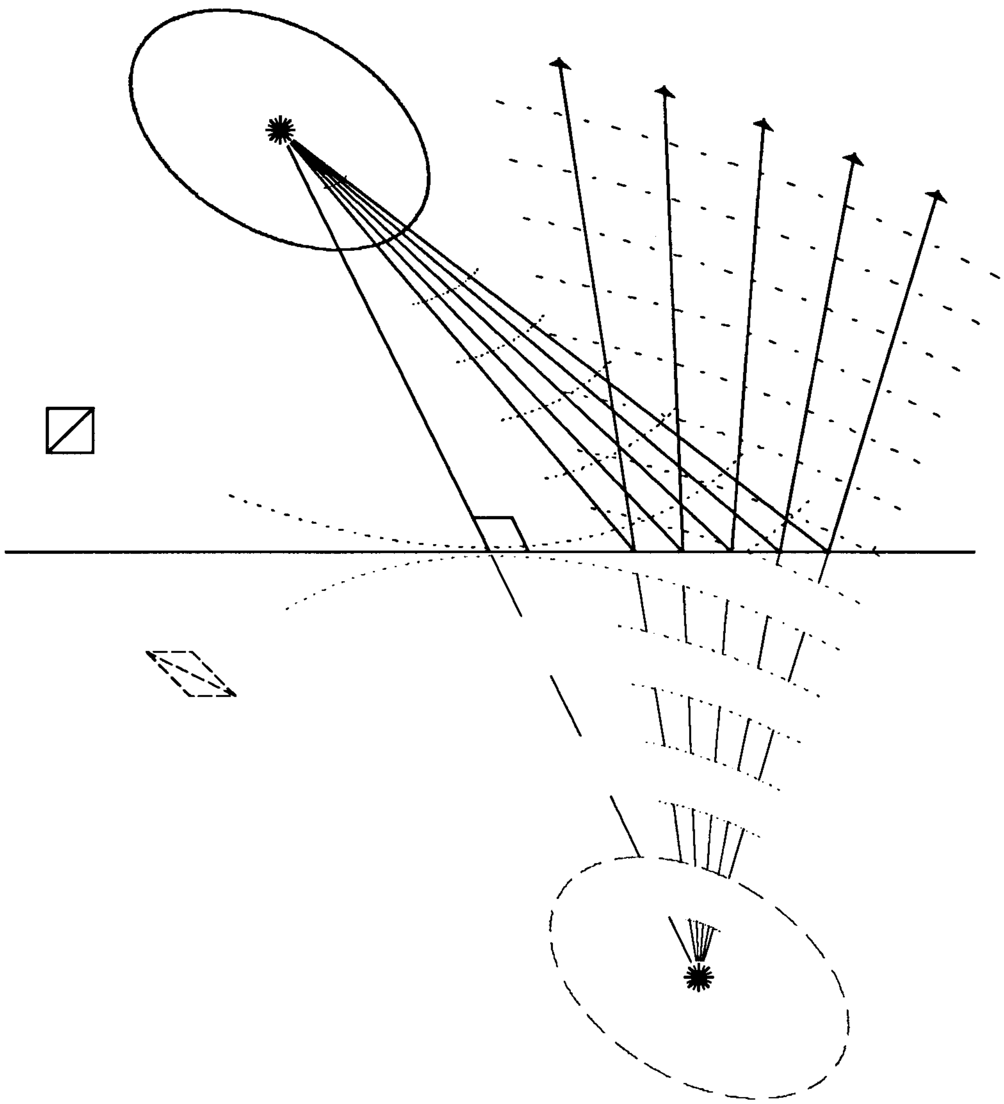

FIG. 2. A mirror in an elliptically anisotropic medium. It is just Figure 1 stretched and rotated. Note that while the image of the point source (the starburst) is also a point, the image of the square on the left is a strongly sheared parallelogram.

that the stretching operation has no usefulness. Geophysicists normally assume hyperbolic moveout on all events, even though it is not strictly true. In practice it is usually a good approximation for reasonable offsets. Assuming a hyperbolic moveout, however, is the same thing as assuming an elliptical wavefield. So this simple stretch model applies in any case where the standard hyperbolic approximation is used.

REFERENCES

Blair, J. M., and Korringa, J., Aberration-free image for *SH* reflection in transversely isotropic media: Geophysics, **52**, 1563–1565.

Helbig, K., Elliptical anisotropy – Its significance and meaning: Geophysics, **48**, 825–832.