# Analysis of issues in transmission tomography using ray trace tomography

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## ABSTRACT

Synthetic modeling using ray trace tomography indicates problems one should be aware of when applying other transmission techniques. The improvement of these problems may be a feature we should seek in other transmission tomographic techniques.


## INTRODUCTION

The use of ray paths and travel times seems to be a crude approach to seismic data inversion that is counter to the recent trend of increasingly honoring the physics of the wave equation by treating seismic data as a dynamic wave field. However, ray paths and travel times are a valid approximation to the wave equations under certain conditions. For these conditions, ray trace tomography is a valid transmission tomographic technique. It represents a simple approach at seismic data inversion that can efficiently address the characteristics of more general transmission techniques.

Reservations exist about the practicalities of applying ray trace tomography to data because it requires the picking of travel times and development of complicated computer programs. Whether the technique is viable or not, it is an efficient method for analyzing certain characteristics of transmission tomography in general.

Ray paths are adequate approximations for wave propagation through broad velocity variations, defined as those components of the velocity field with wavelength greater than that of the seismic signal. These velocity variations generally do not cause precritical reflections and can essentially be analyzed only with the transmitted energy. The dominant effect of broad velocity variations on a transmitted wavefield is in travel time. The amplitude effect is frequently subtle.

Ray trace tomography finds the the velocity field and reflector depths that best fit the travel times of the reflections on the prestack traces. The method works by discretizing the velocity field into a numerous number of cells and the reflector into a series of points. The velocity cells and reflector points are related to the travel times by ray paths traced through a reference model that represents a ''best guess" of the true model. The formulation does not take into account that changing the model affects the ray paths. The ray trace tomography formulation used here is described in Stork (1988).

For an inversion to improve on a model, it must generally be able to relate the arrivals in the synthetics with the corresponding ones in data. One method, used by Tarantola (1985) & Mora (1987), performs this relationship by subtracting the wavefield of the reference model from that of the data. This method produces a constructive contribution only when the arrivals in the synthetic wave forms and those in the data overlap. This criterion requires an accurate starting reference model, something not always available. In cases when the the arrivals in the synthetics do not overlap those of the data, a human must generally guide the computer to specify which waveforms to relate. This can be done in the frequency domain by ''unwrapping" the phase between the two arrivals or by specifying travel time deviations between the data and synthetics. To a first order approximation all of these methods, travel times, phase unwrapping, and data subtraction, are identical. Travel times are used here because of their simplicity and efficiency.

Ray trace tomography is used to analyze several characteristics of the application of generalized transmission tomography techniques to reflection seismology. The reflection seismology experiment, both surface and bore hole, is inherently a very imperfect data collection geometry. Some of the problems are: **1)** The sources and receivers are restricted to the earth's surface or a few number of hole in it. This restriction causes the ray coverage to only have a limited angular coverage. **2)** The ray paths used to sample the structure are affected by the images they are trying to resolve, which introduces a non-linear component into the inversion. **3)** Regions of incomplete ray coverage may contain structure that affects the signal. In general, an inhomogeneous ray coverage causes problems that are difficult to correct. **4)** The velocity analysis and reflector depth are strongly coupled and must be inverted for simultaneously. In some cases it is difficult to distinguish the two apart.

A complete analysis of all these potential complications is not possible here. They are merely introduced with the use of synthetic modeling.

## DISTORTION FROM LIMITED ANGULAR RAY COVERAGE

The limited angular ray coverage available from surface or down hole reflection seismology causes the tomographic inversion to smear the image along the ray paths. An example of this smearing for surface reflection data is shown in Figure 1b where the original model is in Figure 1a. For this example, there are no ray path errors since straight rays are used for both the forward modeling and the inversion. The vertical aspects of the letters have been well resolved while the horizontal aspects, particularly the bar of the "**A**" and the middle of the "**S**" have not been well resolved. This vertical smearing of the images can be explained as resulting from not having any horizontal rays to pin down the vertical location of an anomaly. Since geologic structures are generally horizontal in shape, the image is likely to be severly effected by the vertical smearing. In many cases, the image will probably not be as easily identifiable as this "**GAS**" example because the letters of **GAS** contain many vertical aspects, and because geologic structures generally do not spell convenient words.

In contrast to surface data, the ray coverage of cross hole data is horizontal, which is generally parallel to the geology. Figure 3b is an inversion of Figure 3a in a cross hole geometry. The model of Figure 3a is meant to be an example of horizontally dominant structure of most geology. Ray path errors are again small. Ray coverage of the model is shown in Figure 9a. Despite the limited angular ray coverage available, the inversion has produced a very accurate image.

This vertical smearing for surface data, however, has little effect on the migration of surface data. Zero-offset synthetic finite difference data produced from the original model of Figure 1a is migrated through the smeared inversion of Figure 1b and the result is shown in Figure 2a. Figure 2b is the migration through the constant velocity background for comparison. The original flat reflector at the bottom of the model is nearly perfectly reproduced.

## EFFECT OF REFERENCE MODEL ERRORS

The previous examples were performed using the correct ray paths. However, in realistic applications where the original image is not known, we, of course, can't use the correct ray paths. The only approach available is to trace rays through a "best guess" model. The errors of these reference ray paths from the correct rays will introduce errors into the the solution.

These errors of the ray paths are called non-linearities because they affect the linear system used to compute the solution. They are separate from the inversion artifacts caused by the incompleteness of the linear system, which are linear errors. The limited angular ray coverage, for example, causes the linear system to be incomplete.

The previous examples are rerun such that the velocity variations are allowed to effect the rays for forward modeling and the rays used for inversion are traced through constant velocity reference models. Rays of least travel time were used for the modeling. The inversion with non-linear errors of the **GAS** model is shown in Figure 4a. The letters of **GAS** have generally been fattened. This can be explained with the fast velocity of the letters attracting neighboring rays. The migration through this inversion, shown in Figure 5a, has been severly affected, although it is still better than the migration through the reference model shown in Figure 2b.

In an attempt to improve on the ray paths errors of the inversion, the rays are retraced through the inversion of Figure 4a and another inversion is performed. This procedure is repeated once again, and the result after a total of three ray tracings and inversions is shown in Figure 4b. The figure contain high frequency artifacts which should have been damped, but it can be seen that the letters **GAS** have been slightly reduced in width from the inversion in Figure 4a. The migration through this inversion, shown in Figure 5b, shows improvement over that in Figure 5a, but it still has many problems. Note that the migration under the letter "**S**" is most accurate. This is a result of the letter **S** containing mostly horizontal features which do not effect the ray paths error as much as vertical features. Thus, for surface reflection geometry, even though horizontal features are distorted the most by the vertical smearing of the limited angular ray coverage, their effect on the migration is less because they cause smaller non-linearities.

Since the rays of cross hole data are parallel to the direction of the trend of the velocity variations, they will be strongly affected. The inversion with non-linear effects, in Figure 6a, shows the result has been severely compromised, with very little of the original features distinguishable. Several large artifacts were created that dominate the solution. Inversions run with different damping showed little change. The amplitude fo the velocity variations are only ± 5%, hardly the worst case one can expect. Examples ray paths affected by the original model are shown in Figure 12 and 14.

The result of retracing the rays through this image and performing another inversion, shown in Figure 6b, shows only marginal improvement.

Usage of rays of fastest travel time is generally problematic because they do not sample the media very well. Figure 9b shows the ray coverage of the first arrival rays through the inversion of Figure 6a. These are the rays used for the inversion of Figure 6b. Even though the velocity variation of Figure 6a are generally of less amplitude than that of the original model, the rays are strongly concentrated in the high velocity channels. Several areas even in the center of the model are poorly sampled and will not be inverted well.

The original inversion is repeated with the addition of VSP data and surface data in Figures 7a and 7b. Each case shows improvement over the inversion with cross hole data only. The biggest improvement is that the large artifacts are removed. However, the results are still compromised by the non-linear effects of the incorrect ray paths and does not resemble the original model. Retracing the rays through the model of Figure 7b and performing another inversion in Figure 8a shows some improvement as parts of the original model are noticeable.

## EFFECTS FROM HETEROGENEOUS RAY COVERAGE

An inhomogeneous ray coverage has the potential for distorting an image in a very uneven and unpredictable fashion, making it difficult to interpret or use in any subsequent process. The method is a complicated one to analyze because of the lack of any uniformity. Some of the artifacts in the inversion in Figure 6a with only the cross hole data are probably a result of the heterogeneous ray coverage. The artifacts disappear with the addition of VSP and surface data. A plot of the ray density in Figures 9a, 9b, 10a, 10b, and 11a for the inversions shows how the coverage is very uneven when only the cross hole data is used and becomes more uniform when VSP and surface data added.

These effects are expected to be most serious for a VSP survey because the ray coverage is the most heterogeneous in this ray geometry. The **GAS** example is used again in the VSP geometry. Straight ray paths were used for the forward modeling and inversion so no ray path errors exist. Figure 16b shows the inversion using VSP data of the model in Figure 16a. The image is very severely distorted such that one really has no clue of the original image. Much of the distortion is due to the limited angular ray coverage, but comparison with the inversion of the image with only surface reflection data shows the VSP inversion to be much worse. The surface reflection data smearing is at least predictable and the image can be guessed.

An inversion using both the VSP and surface reflection data, presented in Figure 17a, does not show significant improvement over the inversion of only surface reflection data in Figure 17b. The ray distribution for each geometry is presented in Figures 18a, b, and c.

## POOR RESOLUTION OF AMBIGUOUS VELOCITY AND REFLECTOR DEPTH VARIATIONS FOR SURFACE REFLECTION DATA

A key goal of surface reflection seismology is the accurate mapping of reflector depth. In many different geologic regions, velocity variations are difficult to resolve. As a result, they are mapped into false reflector depths. Ray Trace Tomography indicates some of these velocity variations, but not all, can be resolved. In particular, smaller scale velocity variations lower in the section can not be resolved from reflector variations.

An example of this velocity-reflector depth ambiguity is presented in Figures 19 and 20. The original model, in Figure 19a, contains three velocity variations at different depths. The flat reflector at the bottom of the model is plotted on a separate scale so it can be vertically exaggerated. The reference model used for ray tracing had a flat reflector and a constant velocity. The inversion over the narrow eigenvalue range of (1.0-0.3) in Figure 19b has not inverted the velocity-reflector depth ambiguity and only placed half of the travel time variations into velocity and the other half into the reflector. Inversion to a much smaller eigenvalue in Figure 20b has resolved the reflector depth on the left side but not on the right side. However, the reflector on the right side has been generally raised so the average depth is correct. It appears that the broader scale reflector variations have been resolved but not the smaller scale ones.

## DISCUSSION

The use of ray trace tomography has presented several issues that one should be aware of when presently applying transmission tomography. Future research may want to address these issues. Perhaps different approaches can alleviate some of these problems.

The surface reflection geometry seems to suffer the least from these problems and may presently be the most promising application for transmission tomography. This geometry has the advantages that the image, which is invariably smeared, is not directly used for interpretation, but as input for migration which is mostly insensitive to the smearing. Since the mostly vertical rays of surface data are perpendicular to the mostly horizontal geology, the non-linear effects of ray path errors will generally be less severe than for other geometries. The ray coverage for a surface reflection survey is generally very uniform avoiding the complicated effects of heterogeneous ray coverage.

The most serious problem for using a cross hole geometry or even VSP geometry may be the non-linear effects of using the incorrect ray paths through a best guess reference model. It appears that the use of ray paths may aggravate these non-linear effects. The use of a waveform based transmission technique such as that of Woodward (1988) shows the potential for being more stable with non-linearities.

Since travel times alone are unable to resolve certain ambiguous velocity and reflector depth variations, additional information must be used, such as the amplitude information of the data. Again, a waveform based transmission tomography approach may improve on this problem.

## CONCLUSION

The more serious of the problems presented above are the poor inability to resolve the velocity-reflector depth ambiguity; the non-linear effect of using incorrect ray paths; and for VSP geometries, correction for the inhomogeneous ray coverage. Improved transmission methods may improve on these problems.

Data applications are needed to indicate the seriousness of these problems and the potential of a variety of techniques for correcting them.

## ACKNOWLEDGEMENTS

Much of this work is a reinterpretation of thesis research performed under Rob Clayton at the Caltech Seismology Lab. Much of this perspective is based on Rob Clayton's insight. Figures 1, 2, 4, 5, 19, & 20 are borrowed from the author's thesis.

Amoco was very supportive of this project and provided numerous SUNs to perform the modeling.

SEP's plotting package again significantly increased my productivity.

## REFERENCES

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Stork, C., 1988, Travel time tomographic velocity analysis of seismic reflection data: Ph.D. Thesis, Caltech.

Tarantola, A., 1984, Linearized inversion of seismic reflection data: geophys. prosp., **32**, 998-1015.

Woodward, M., 1988, Wave equation tomography: SEP-**57**.

**Figure 1:** **a)** Model used for inversion in Figures 1b, 4a, and 4b. Velocity variations are + 10%. A flat reflector exists at the bottom of the model. **b)** Inversion of GAS model using straight rays for forward modeling and inversion. As a result there are no non-linear errors. The smearing from the limited angular ray coverage has effected horizontal aspects of the letters the most.

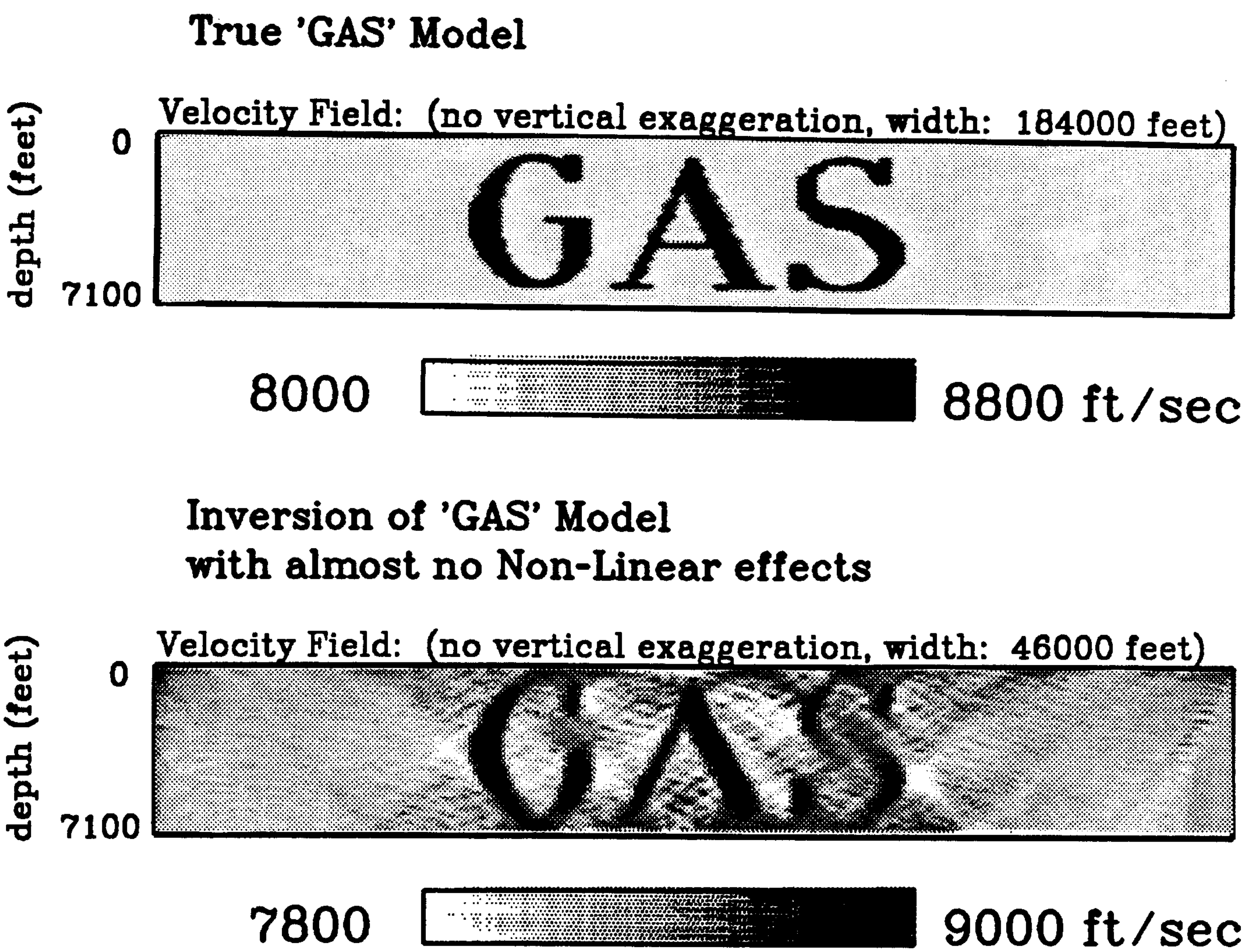
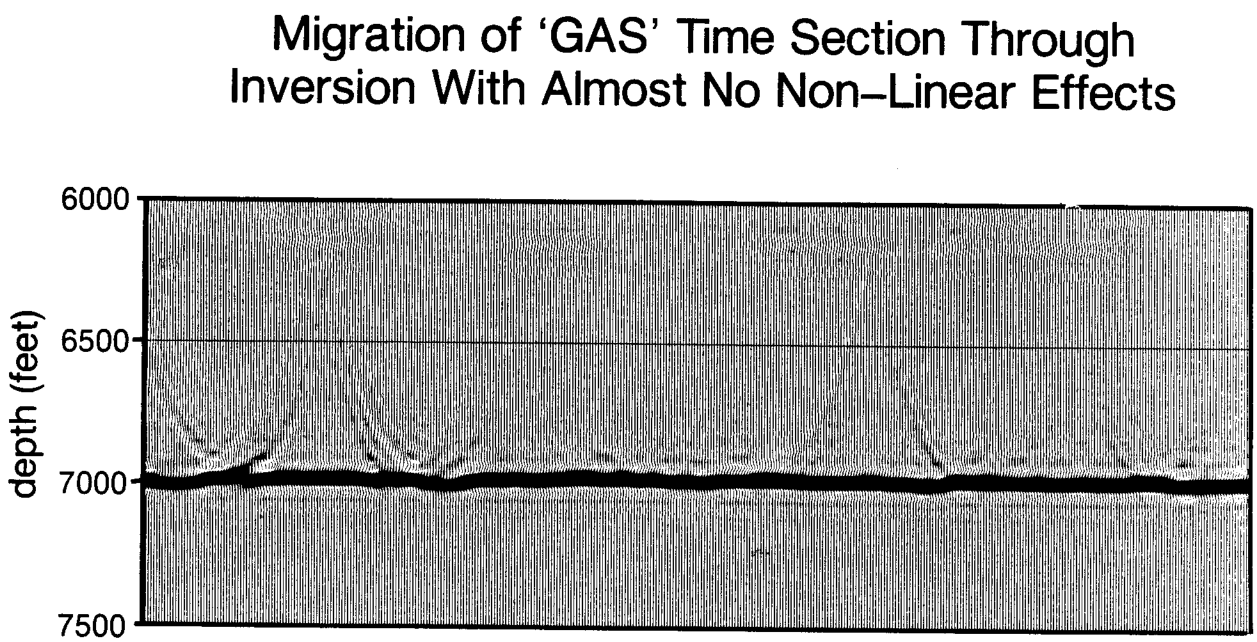


**Figure 2:** **a)** Migration of zero-offset finite difference data from the **GAS** model of Figure 1a through the inversion in Figure 1b without ray path errors. Result reproduces the flat reflector nearly perfectly. **b)** Migration of same data through the constant velocity reference model. This result serves as a comparison of the result without any tomographic inversion.



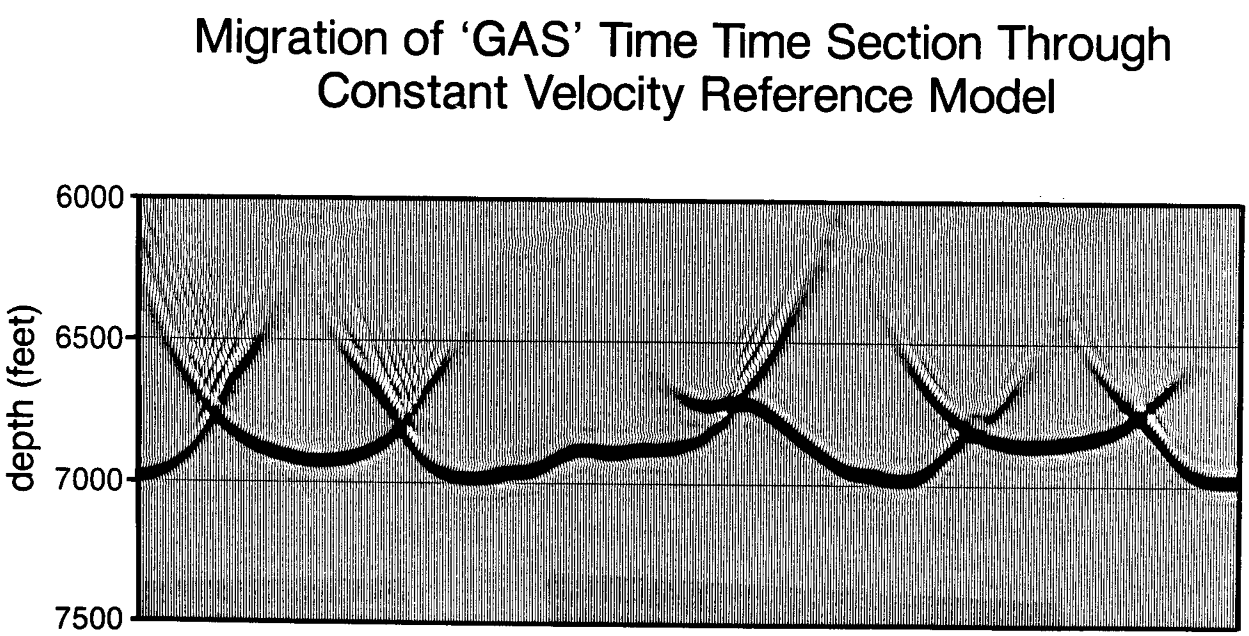



Figure 3: **a)** Original model used for cross hole geometry. Velocity variations are ± 5%. This Model is meant to represent mostly horizontal geometry with gradual lateral variations. **b)** Inversion of model using the correct rays. Since the structure is parallel to the rays, the limited angular ray coverage causes little smearing. Straight rays were used for the forward modeling as well as for the inversion.

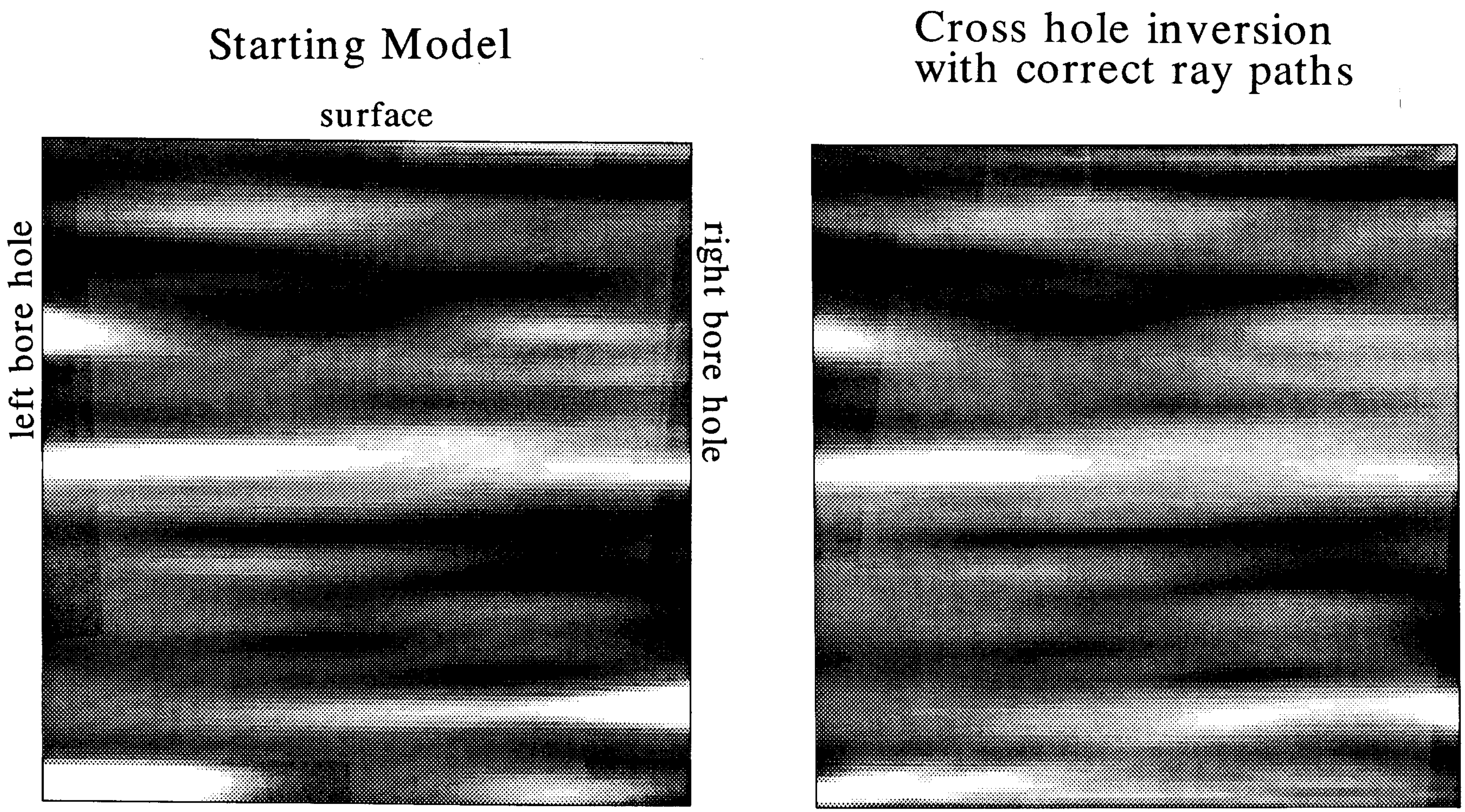


**Figure 4: a)** Inversion of GAS model using incorrect ray paths. Rays for forward modeling were allowed to be affected by the velocity variations. Ray used for inversion were traced through a constant velocity reference model, making them straight. The inversion is much worse than that without ray path errors in Figure 1b. The additional artifacts from the ray path errors are non-linear affects. **b)** In an attempt to improve on the artifacts from the ray path errors, rays are retraced through the inversion in a, used for another inversion, and the whole process is repeated one more time. Thus, this result represents 3 ray tracings and inversions. High frequency artifacts appear, but it appears the letters have been narrowed closer to their correct length.

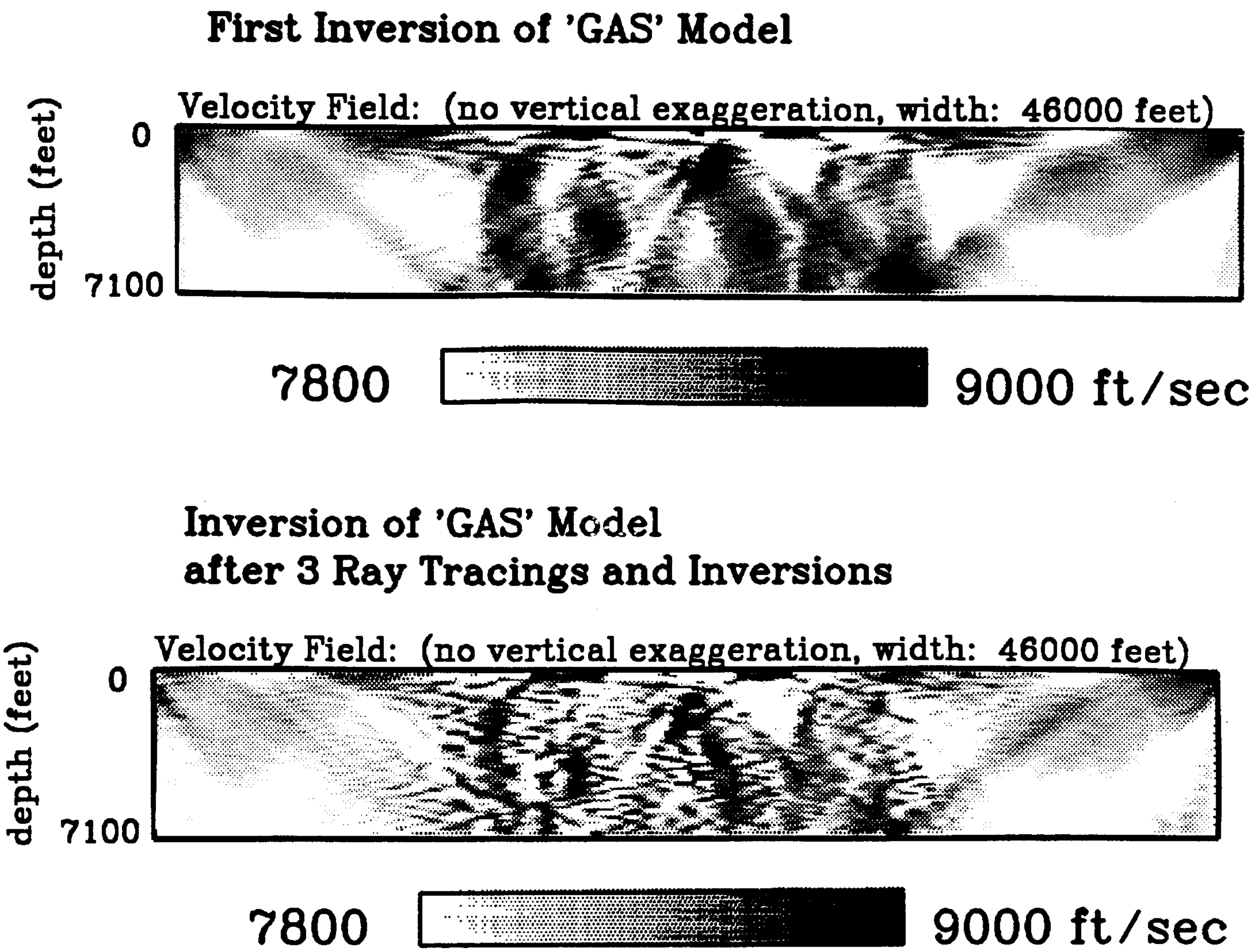


**Figure 5: a)** Migration of data through the inversion in Figure 4a with ray path errors. Result has many problems but is still better than the migration thought the reference model. **b)** Migration of same data through the the inversion in figure 4b. The inversion has been improved over the previous one especially on the right side under the letter S.

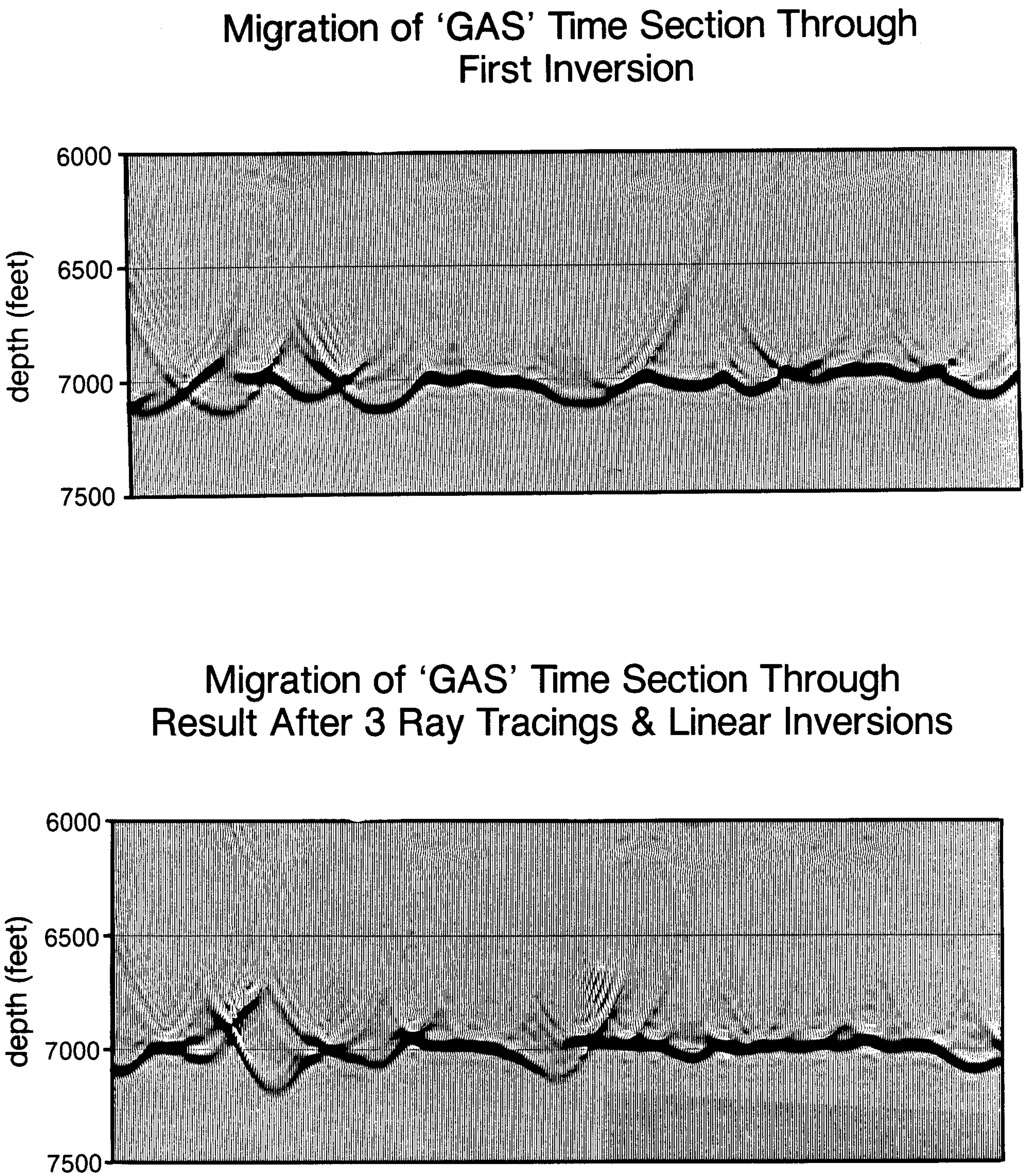


**Figure 6:** **a)** Inversion of model in Figure 3a using only cross hole data. The velocity variations of the original model were allowed to affect the rays used for modeling. Incorrect straight rays were used for the inversion, which have produced considerable artifacts to the inversion without ray path errors in Figure 3b. The result now bears little resemblance to the correct model. **b)** To improve on the ray path errors, rays are retraced through the previous model and another inversion is performed. Improvement is very minor.

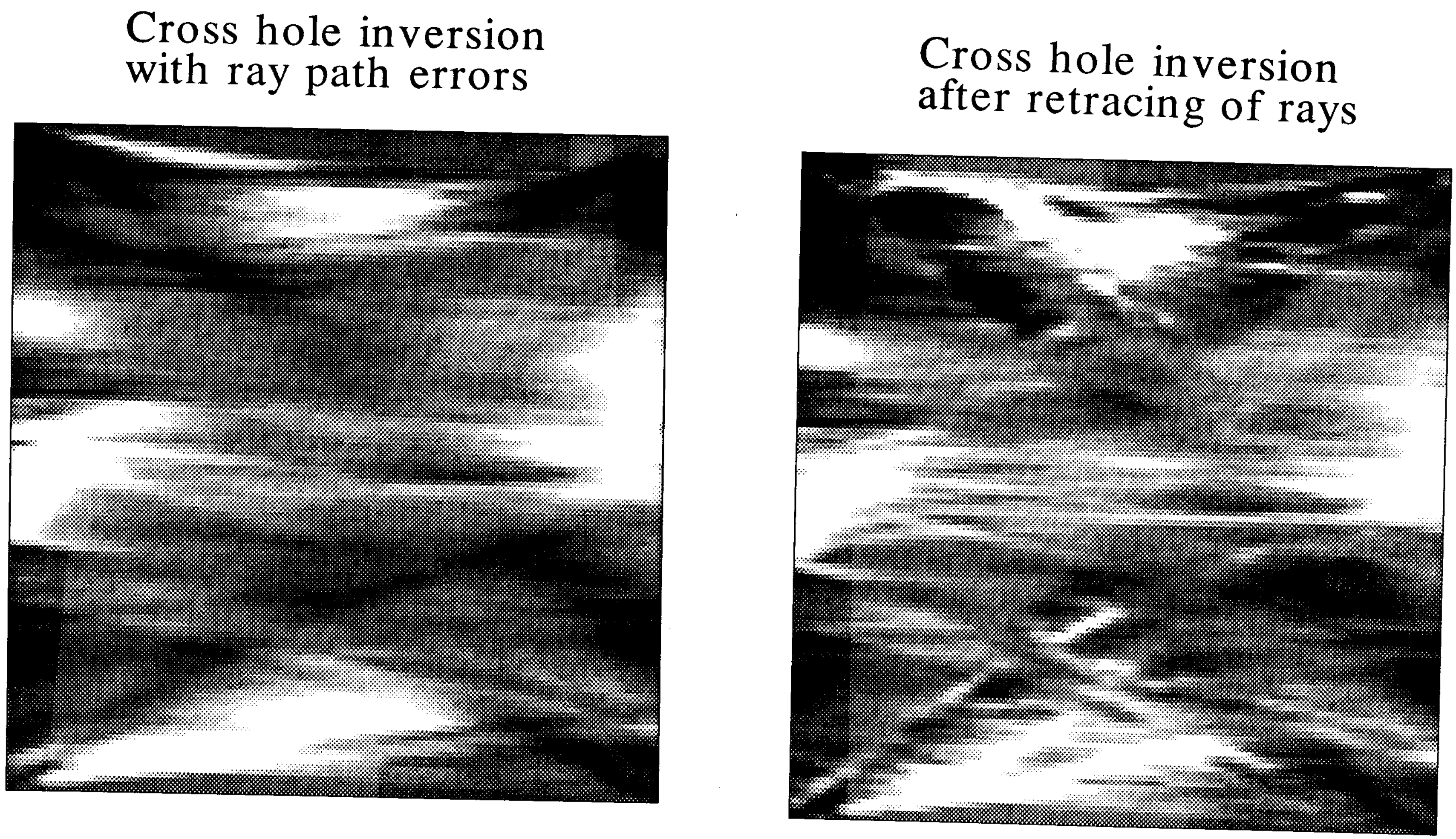


Figure 7: a) Inversion of model in Figure 3a using only cross hole and VSP data. Some of the artifacts from the inversion with only cross hole data have been reduced, but little of the structure of the original model is identifiable. b) Inversion of model using cross hole, VSP, and surface reflection data. Additional artifacts have been removed, but original image is not well inverted.

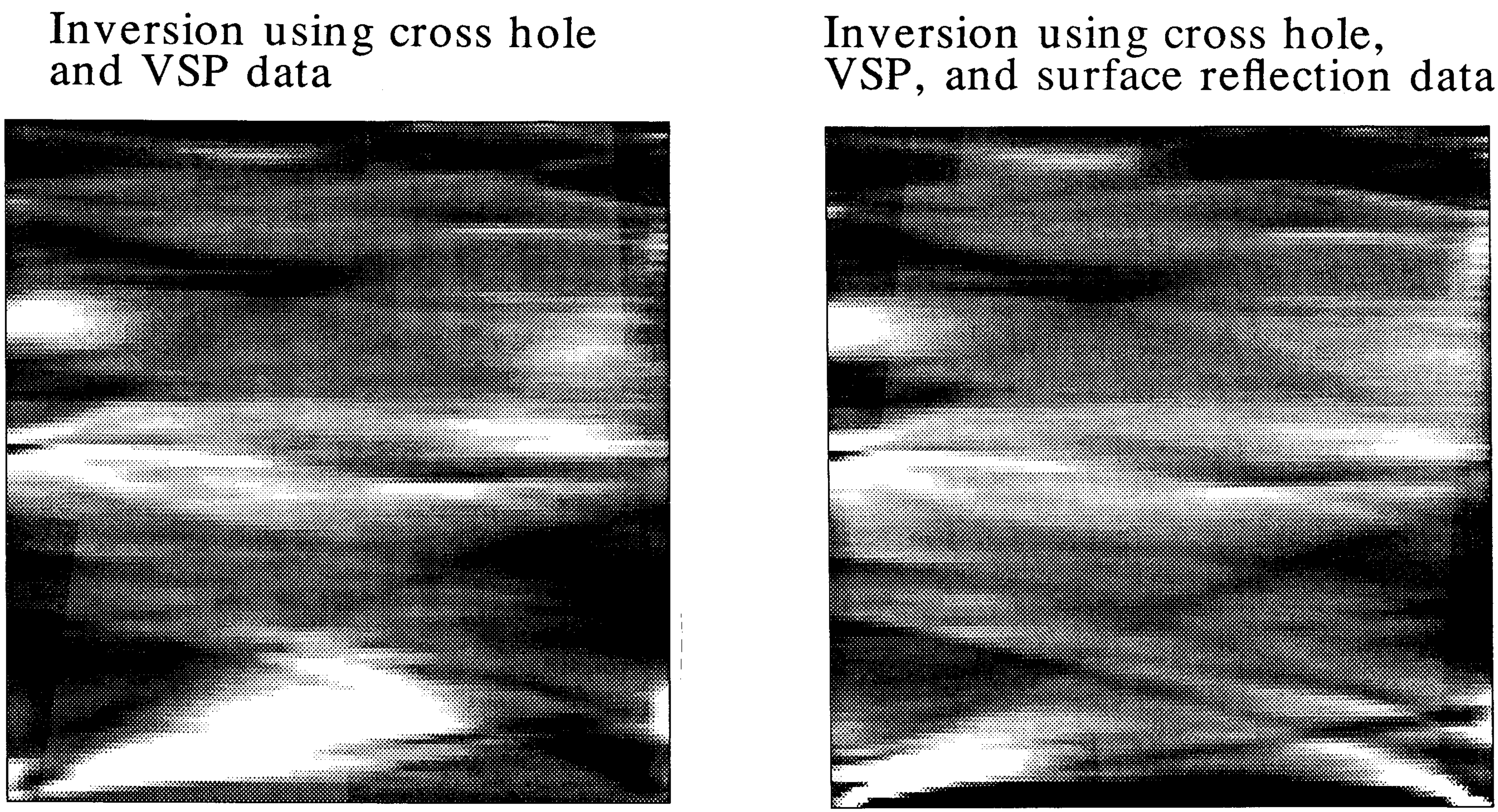
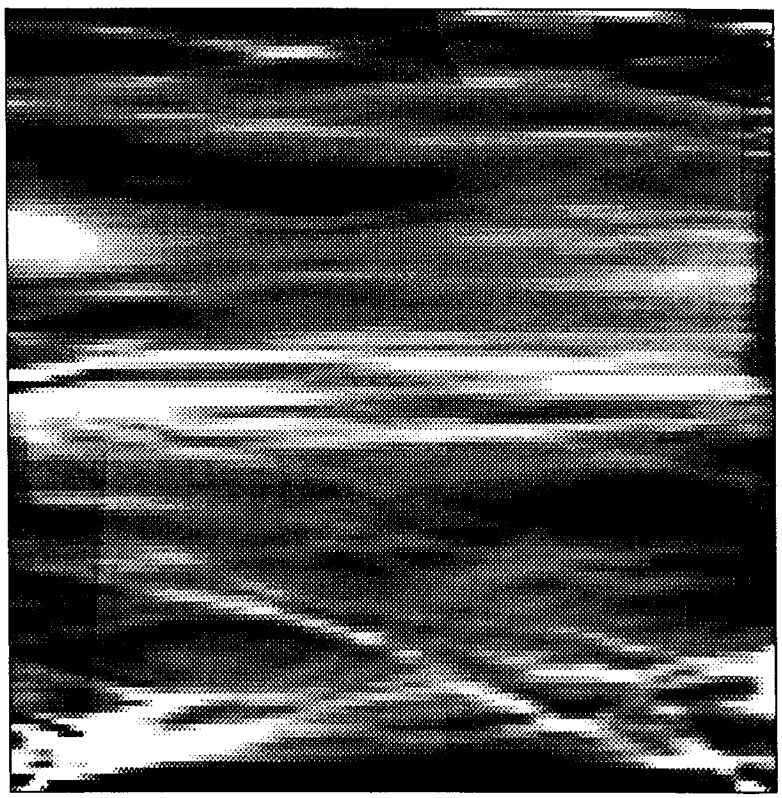


**Figure 8:** **a)** Result after additional ray tracing and inversion using cross hole, VSP, and surface reflection data. Some of the structure of the original model is noticeable, but the inversion is still poor.



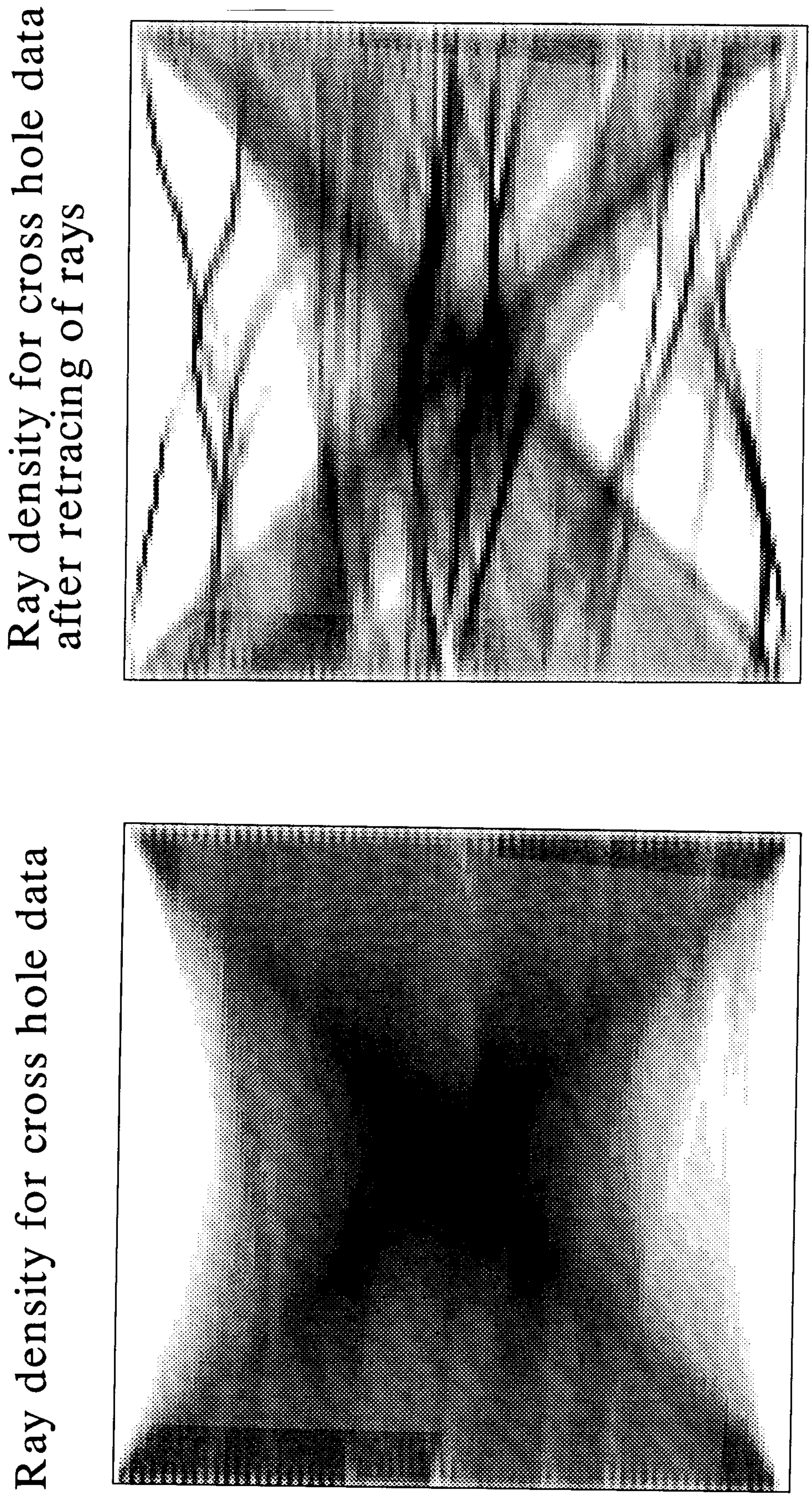



**Figure 9: a)** Density of ray paths in constant velocity reference model for cross hole data only. Density is greatest in the center. **b)** Density of ray paths through inversion of Figure 6a. These rays are used for inversion in Figure 6b. Many of the rays are narrowly confined to the high velocity channels producing a very uneven ray coverage. Only rays of fastest travel time were used.

Figure 10: a) Density of ray paths in constant velocity reference model for cross hole and VSP data. Ray coverage has improved at the top of the model over the cross hole data only. b) Density of ray paths in constant velocity reference model for cross hole, VSP and surface reflector data. Ray coverage has improved at the bottom of the model.

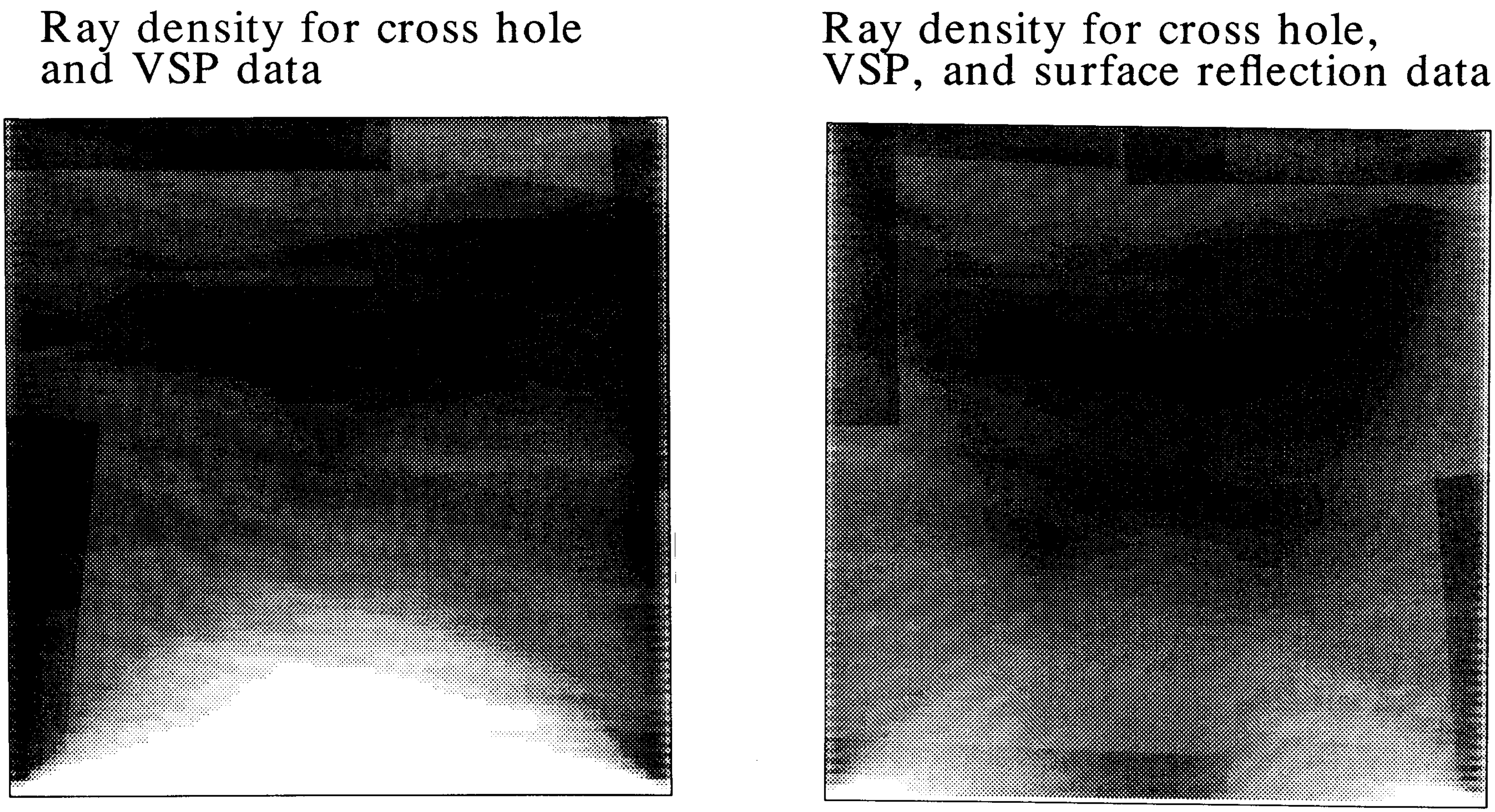


**Figure 11:** **a)** Density of ray paths for cross hole, VSP, and surface reflection data through model of Figure 7b. These rays are used for the inversion in Figure 8a. Ray coverage is more uniform than in Figure 9b.

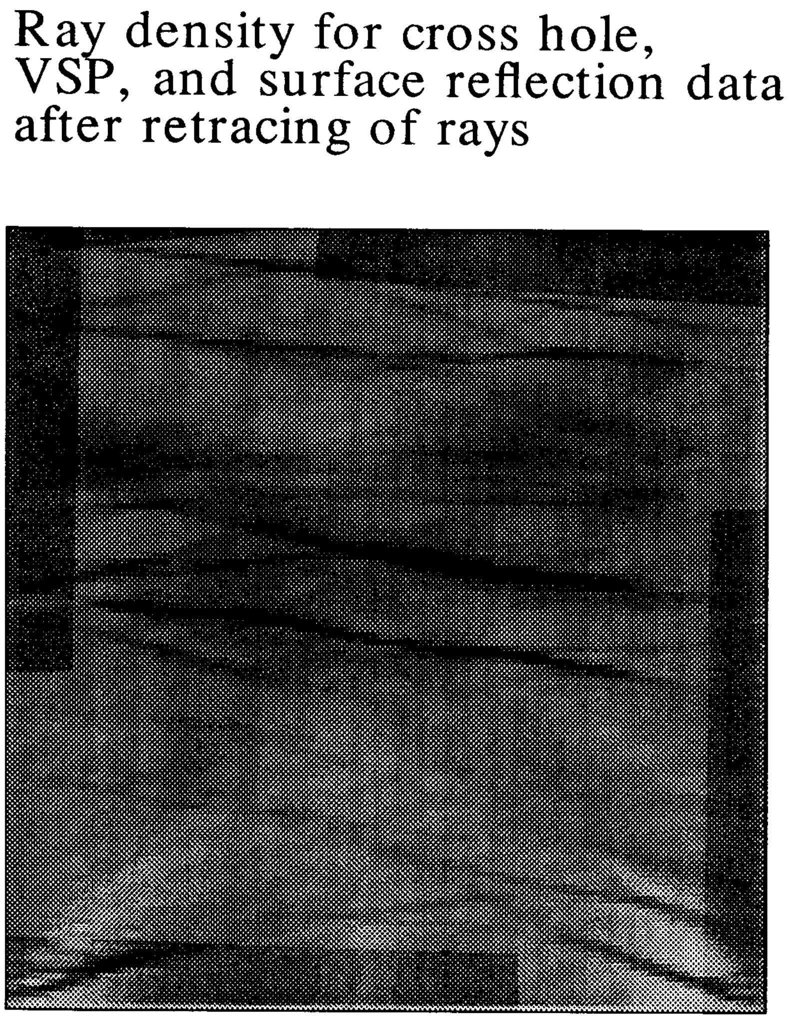
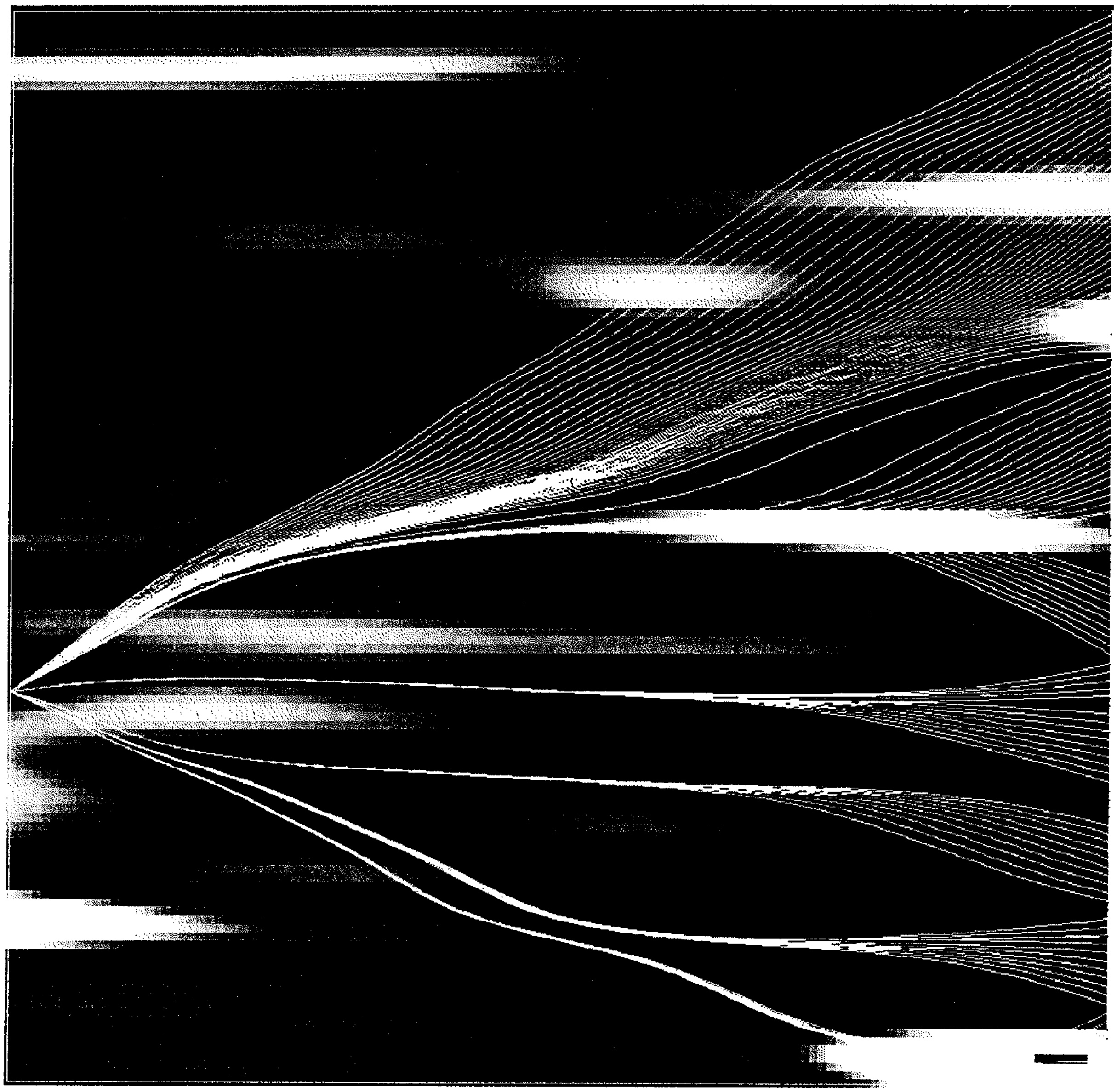


**Figure 12:** Color plot of cross hole ray paths for original model of Figure 3a. Rays of least travel time from a shot to receivers are shown. The rays are strongly attracted to high velocity channel despite the low amplitude of the velocity variations, ± 5%. Ray coverage is very uneven. Next figure is same plot in halftone.



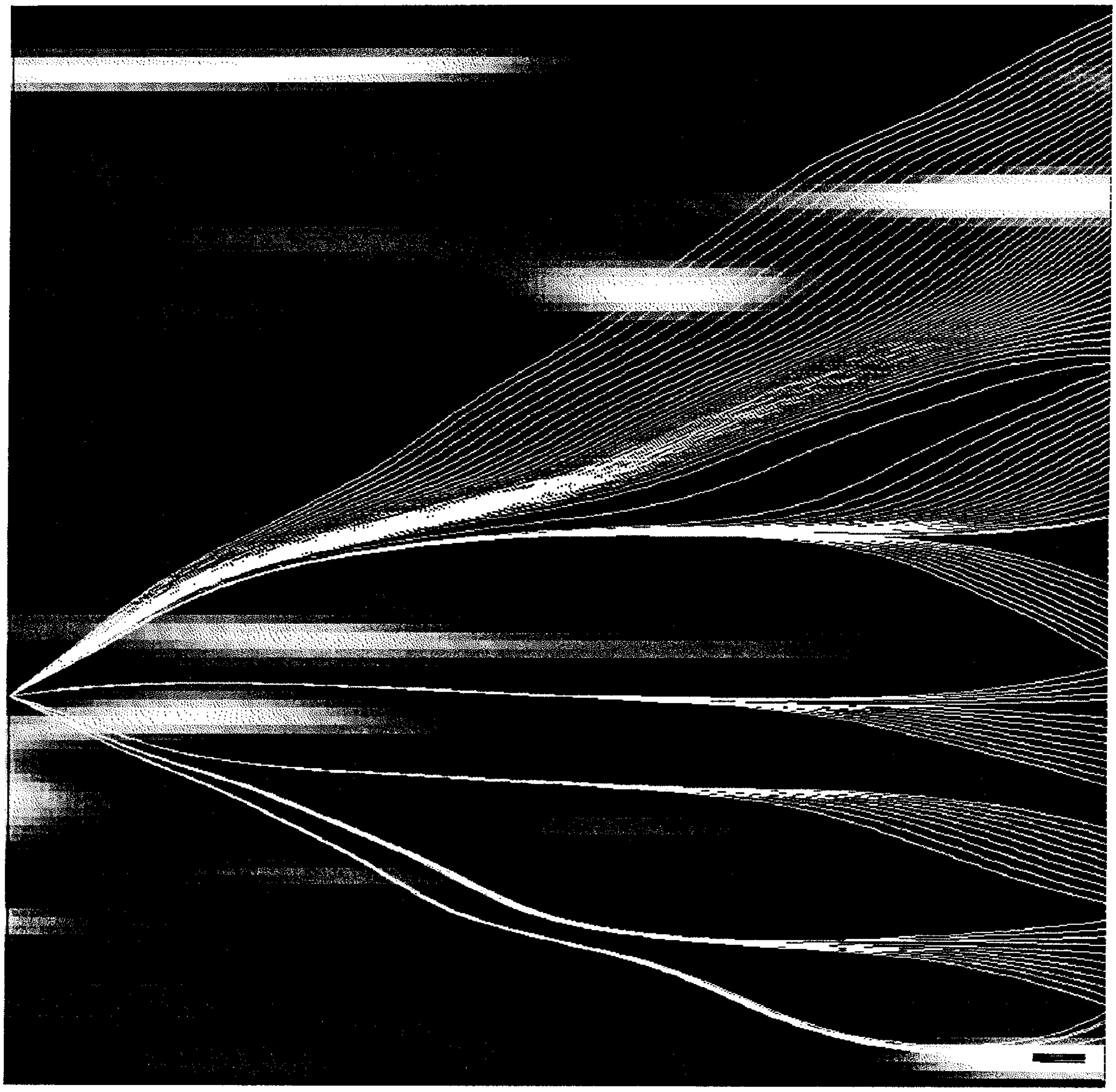


**Figure 13:** Halftone plot of same rays as in previous figure.

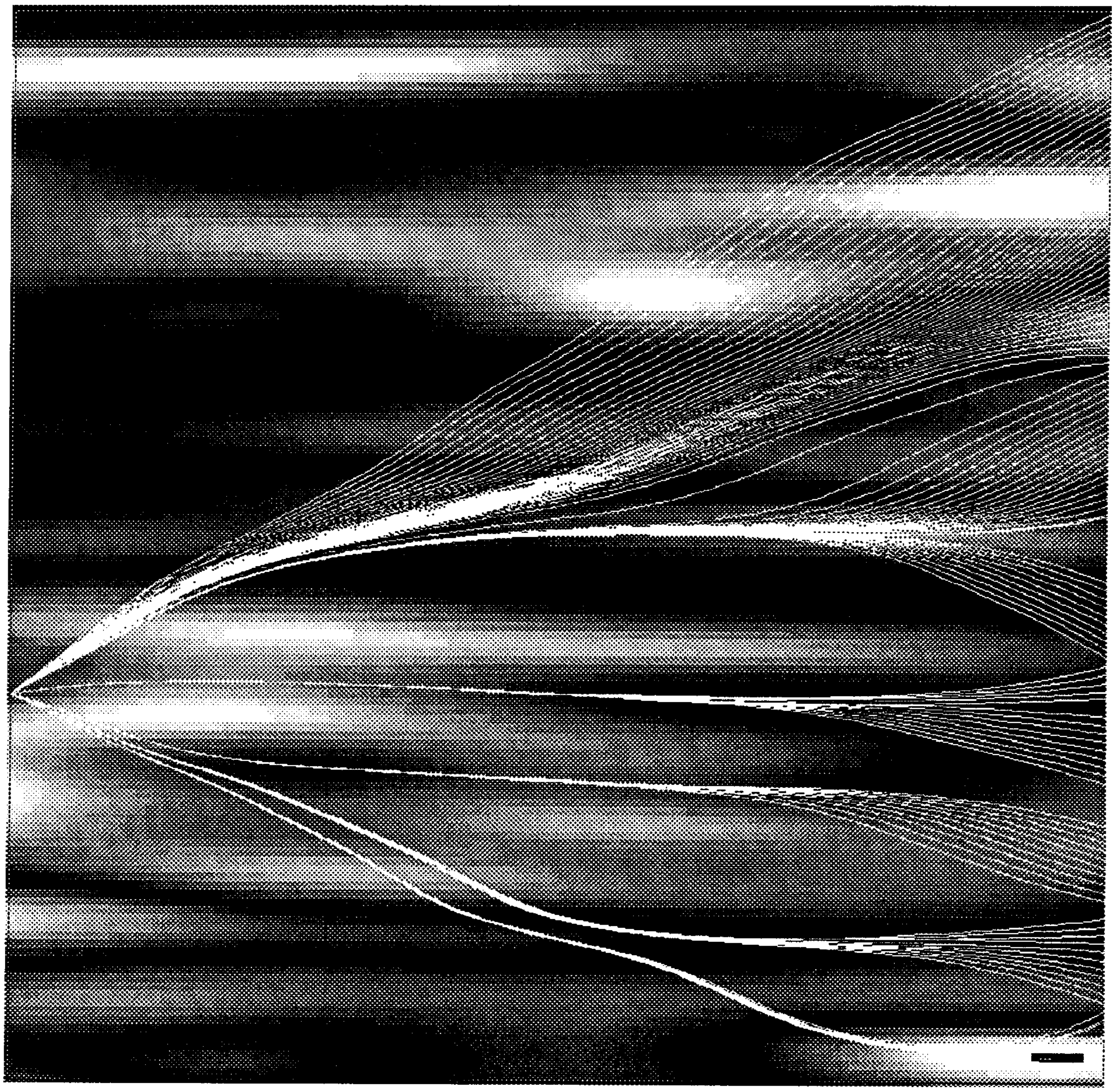
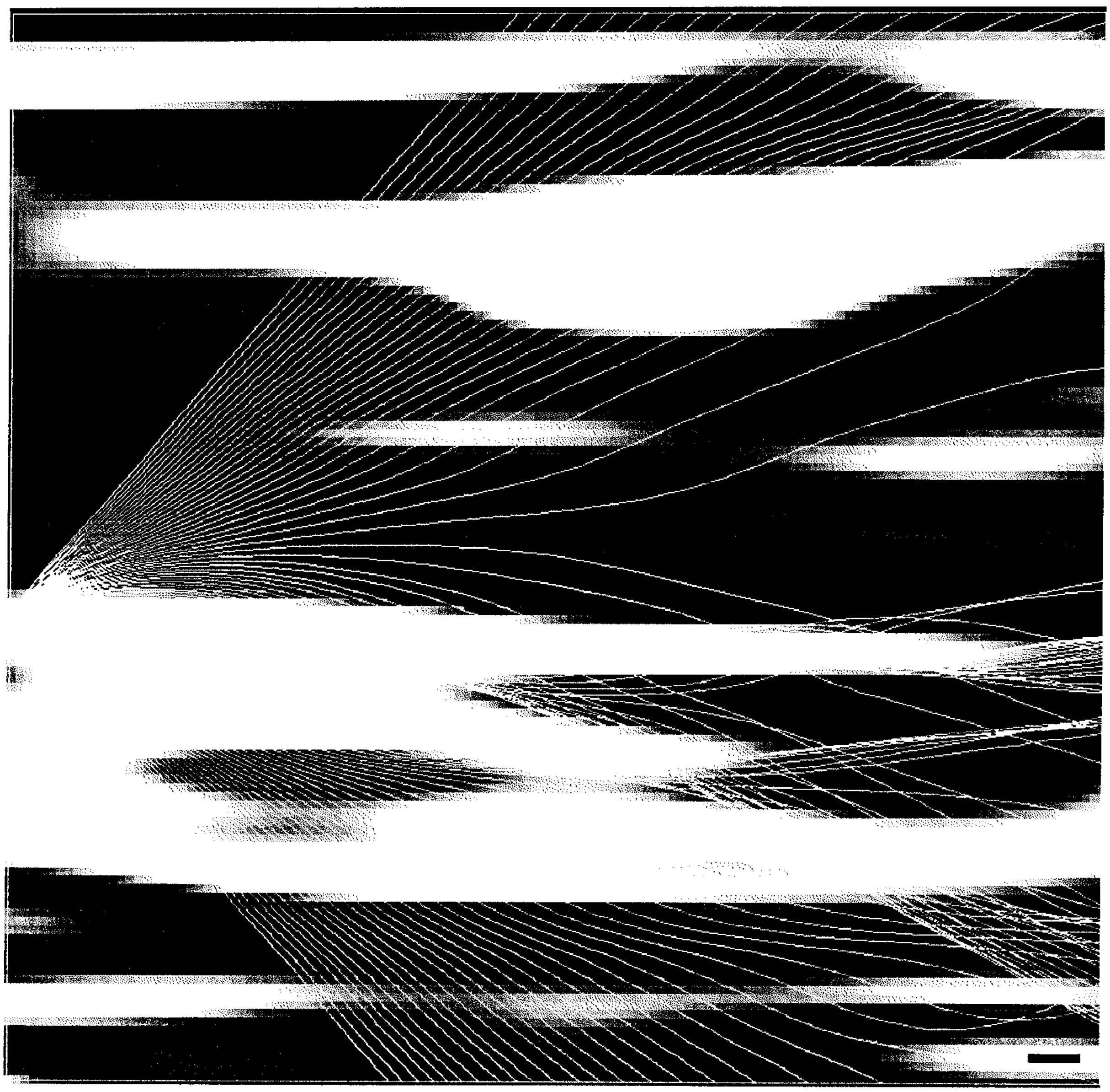
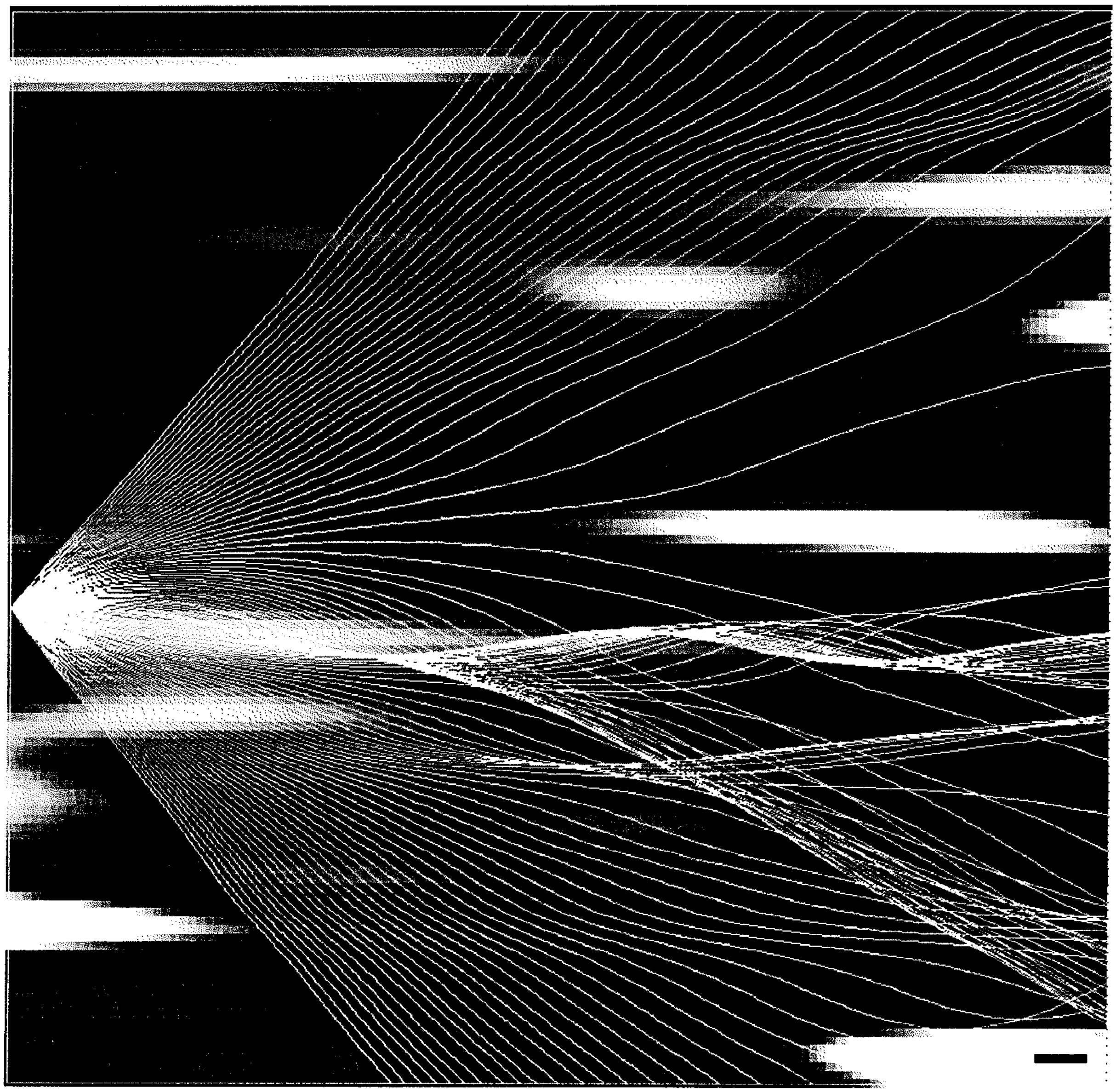


**Figure 14:** Color plot of ray fan for original model of Figure 3a. Rays are fired out evenly spaced in angle from shot point. Rays density increases in the low velocity channels indicating greater energy concentration.





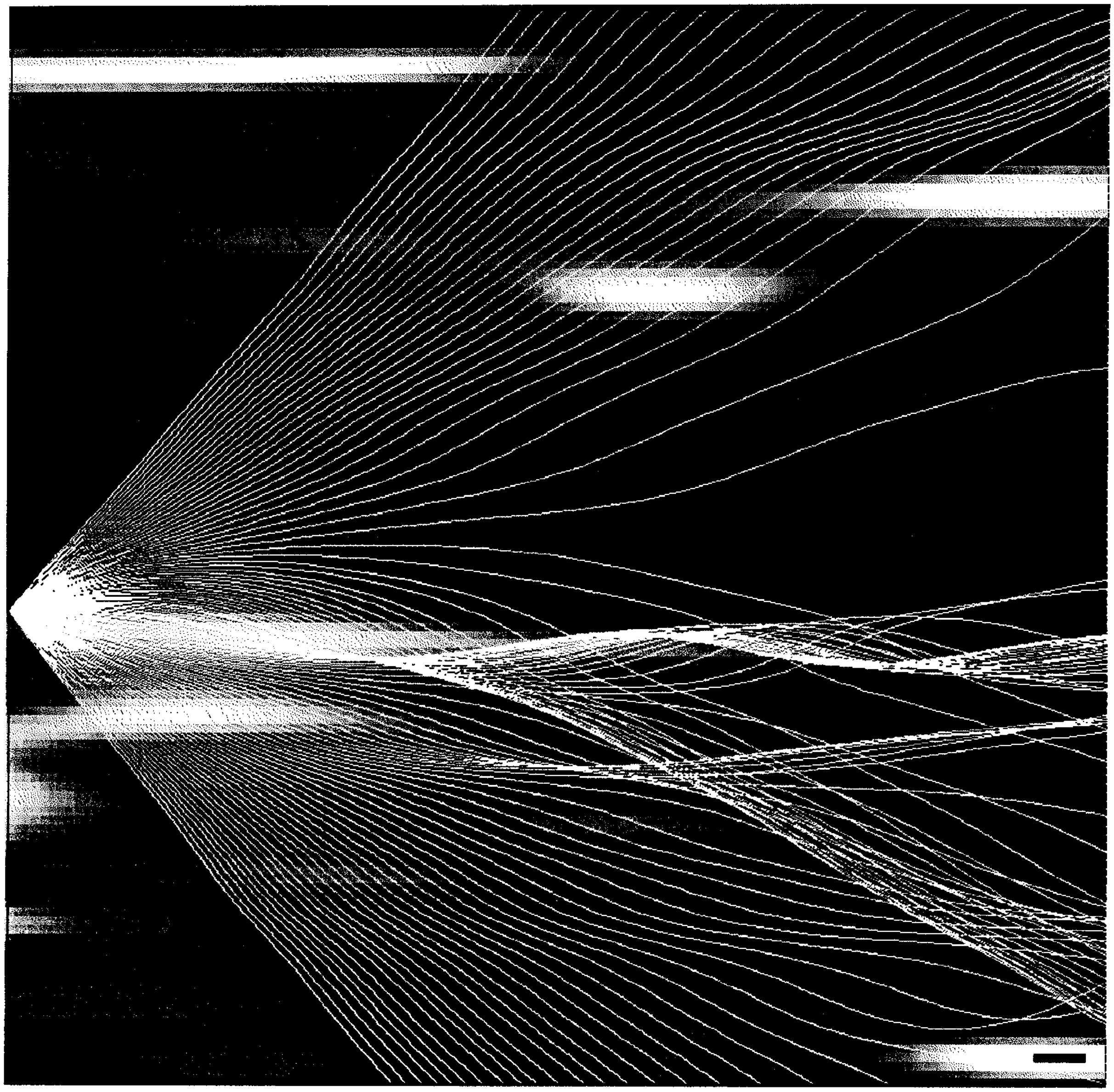


**Figure 15:** Halftone plot of same ray fan as in previous figure.

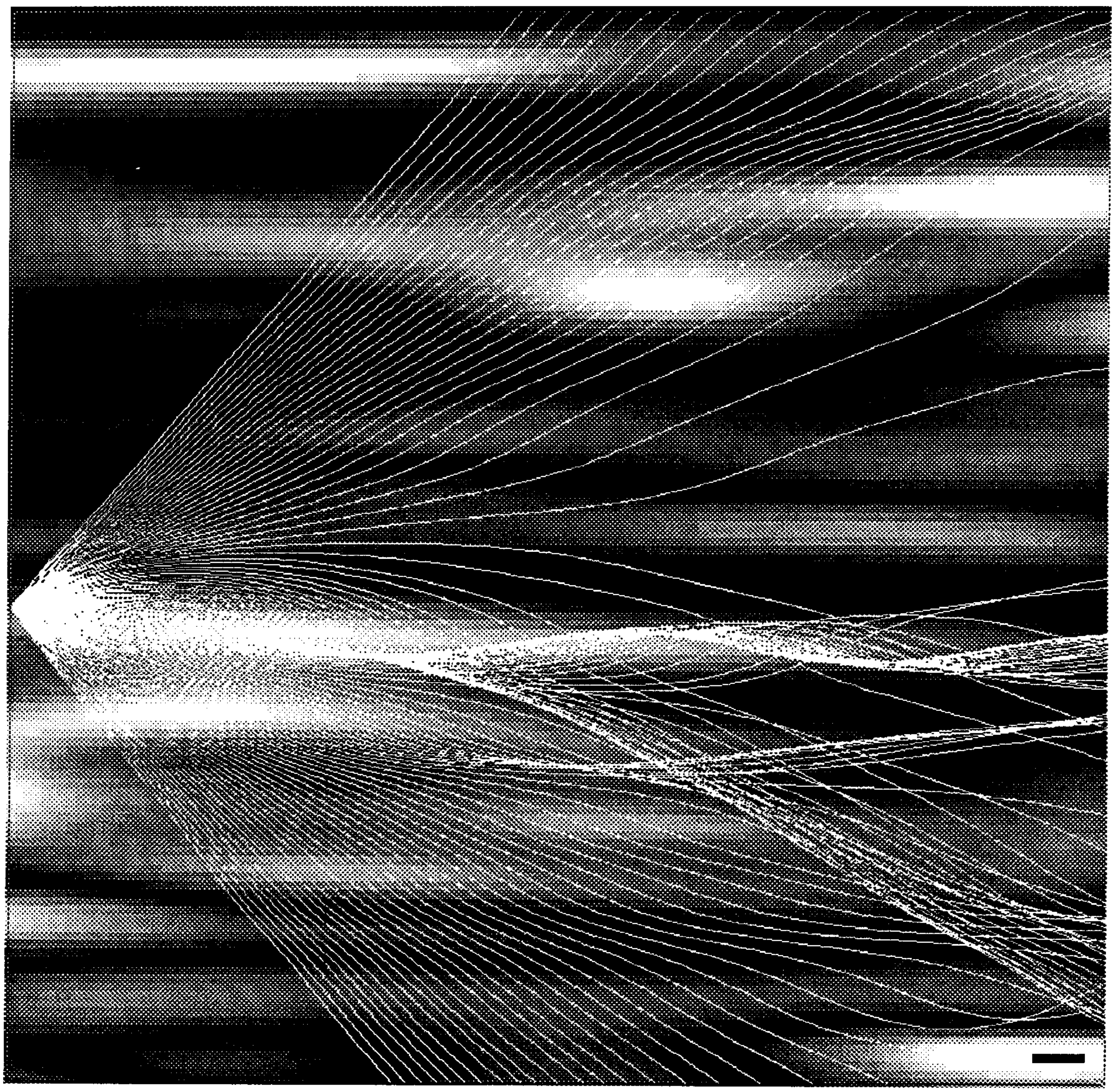


**Figure 16:** **a)** Original **GAS** model used for synthetic VSP inversions. Straight rays will be used for the forward modeling as well as the inversion so there will be no ray path errors. **b)** Inversion of **GAS** model using only VSP data. Image has been smeared so no part of the original, even near the bore hole, can be identified.

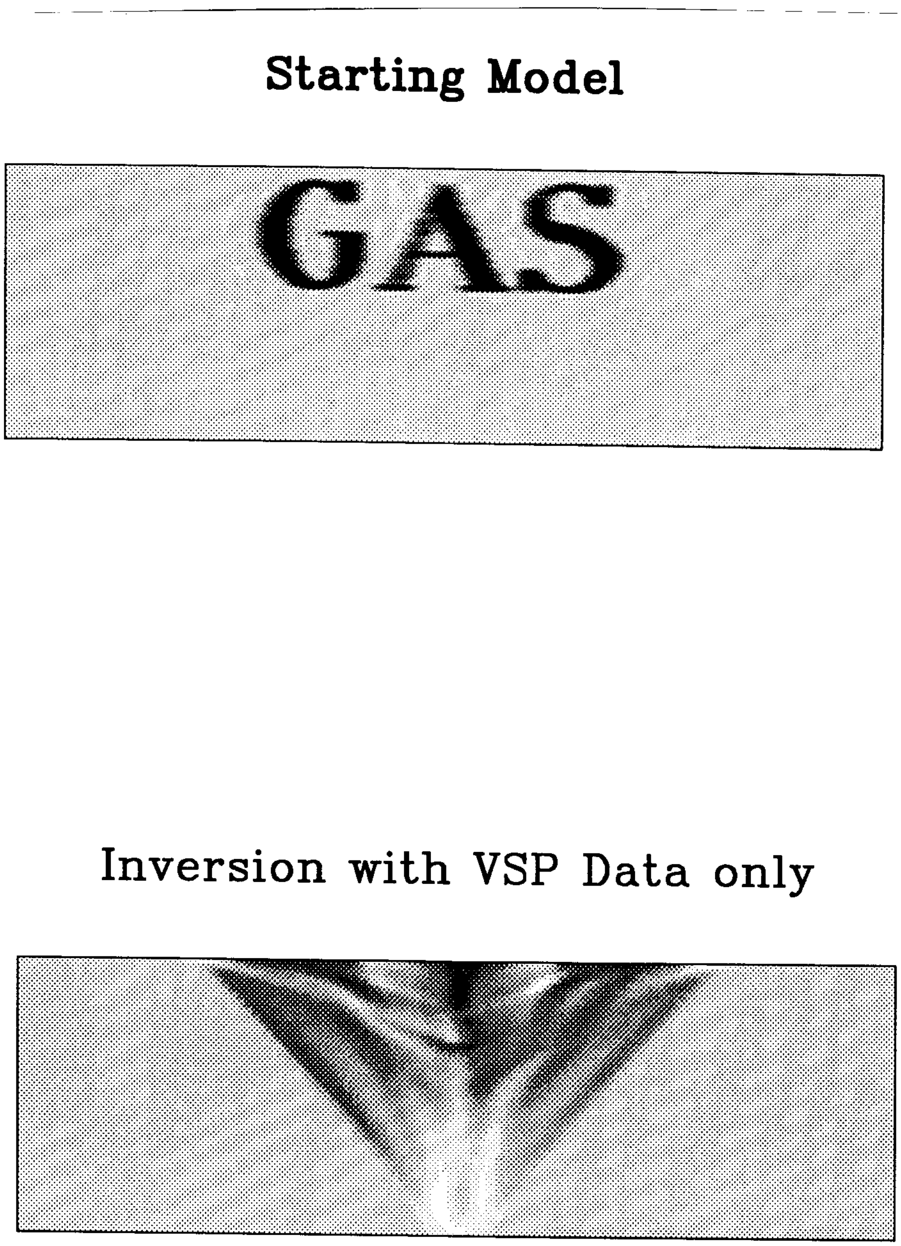
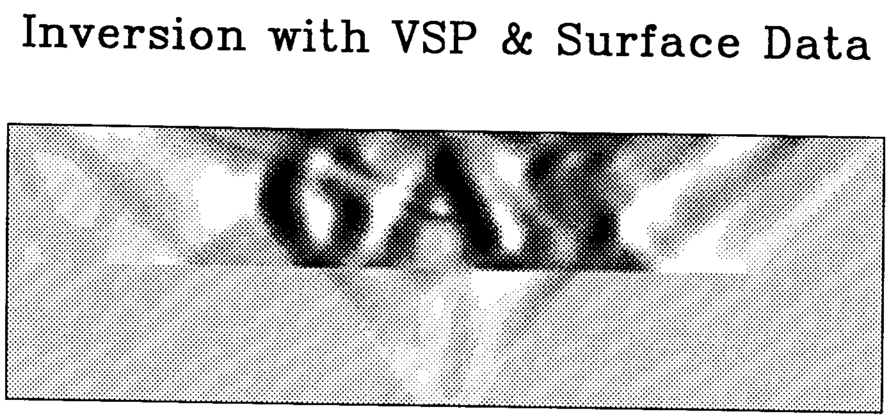


**Figure 17:** **a)** Inversion of **GAS** model using VSP and surface reflection data. Image is not much better than inversion with only surface reflection data in following figure. Main improvement is the horizontal bar of the **A**. **b)** Inversion of using only surface reflection data.



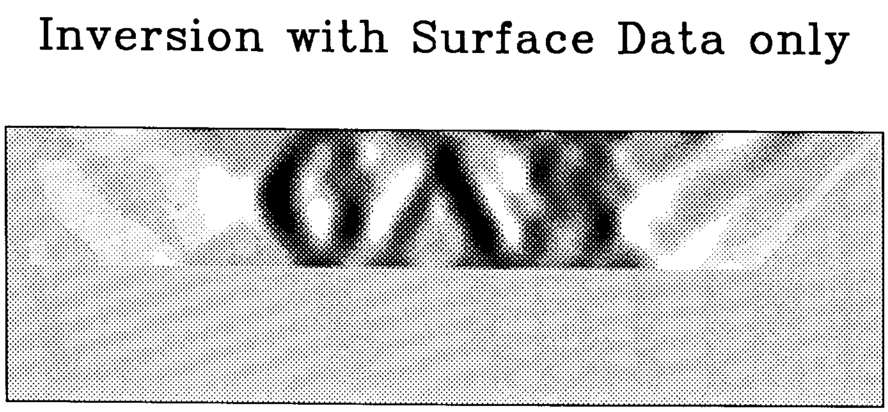



**Figure 18:** **a)** Ray density for VSP data only. Ray coverage is quite heterogeneous. **b)** Ray density for VSP & surface reflection data. **c)** Ray density for surface reflection data only.

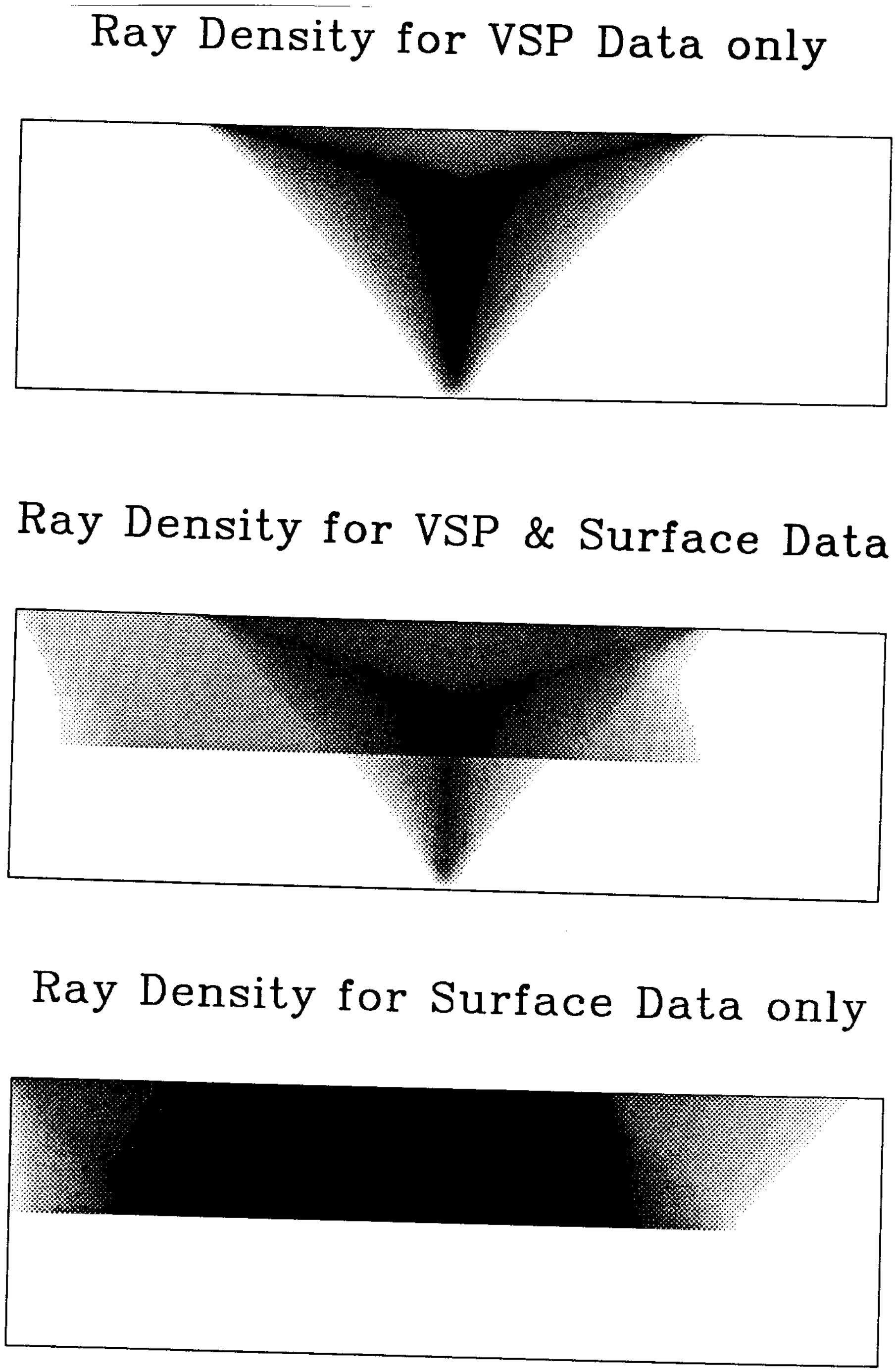
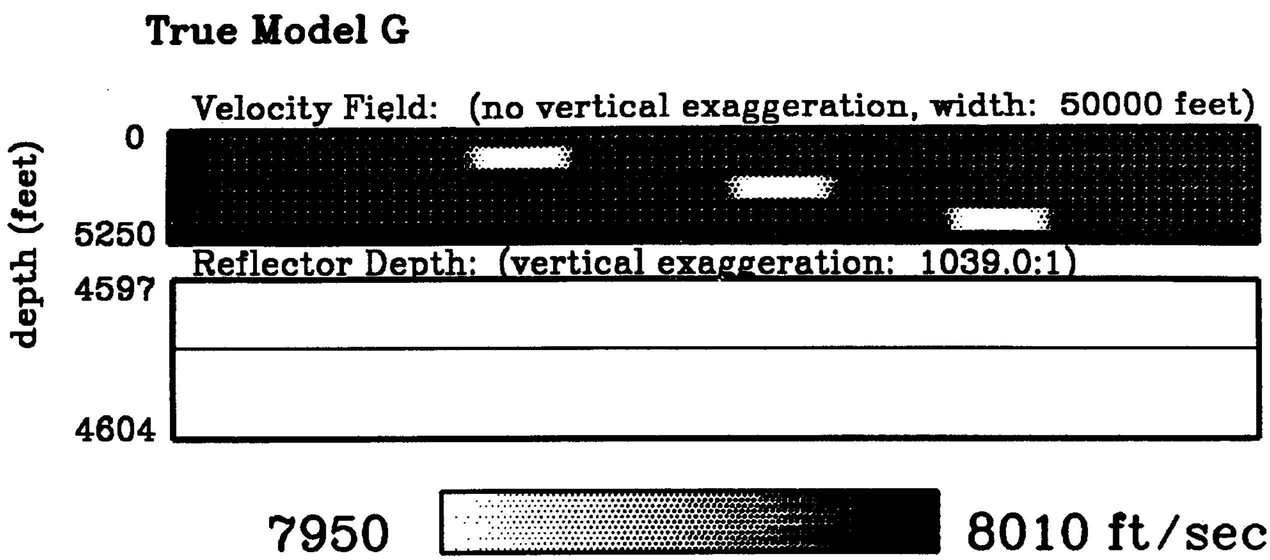


**Figure 19:** **a)** Starting model with only velocity variation. The velocity variations are gradual so their effect on the ray paths in minimal. The flat reflector at 5000 feet is plotted on a separate scale so that it can be vertically exaggerated. The reference model had a constant velocity and the correct flat reflector. **b)** Inversion of starting model with a very narrow eigenvalue range. Half of the original travel time deviations were put into velocity and the other half into reflector. The left side contains broad scale reflector artifacts while the right side contains smallĕr scale artifacts.



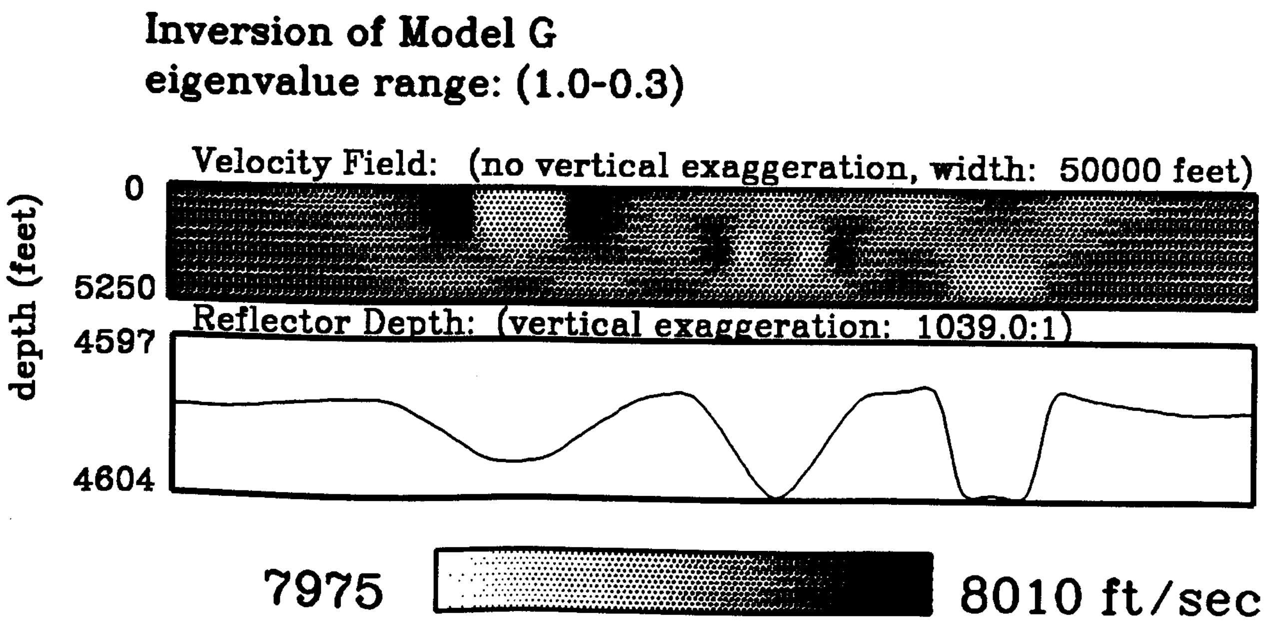
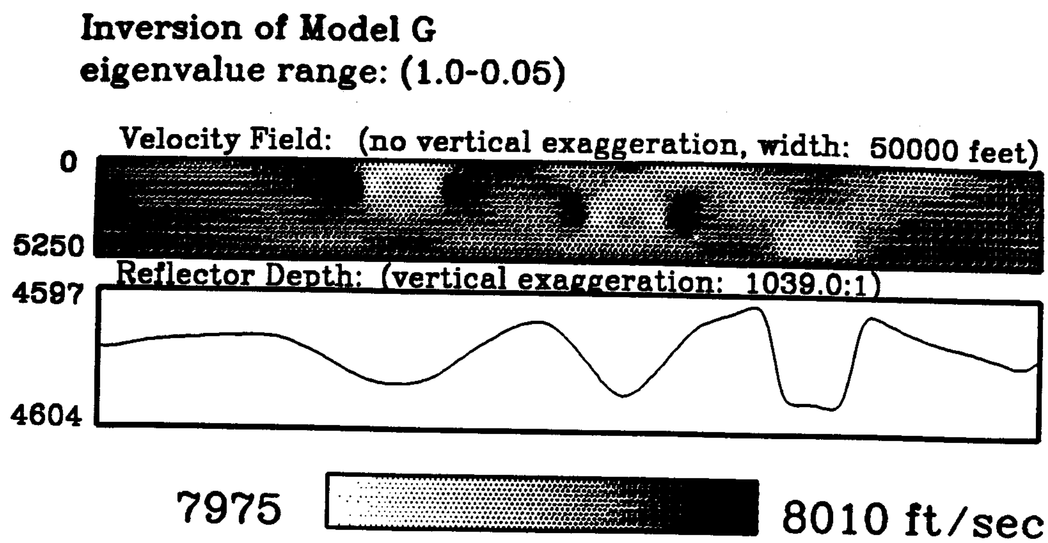
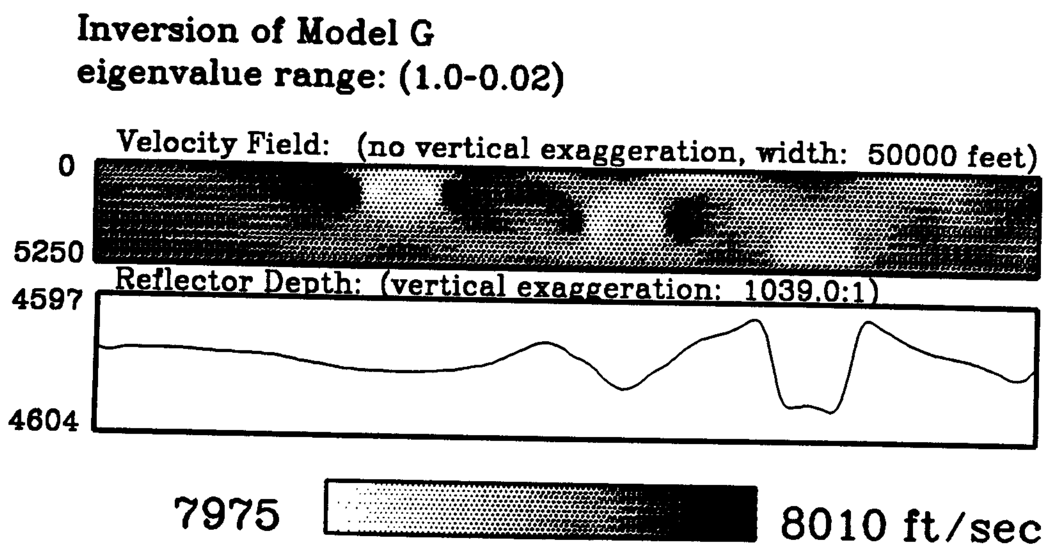



**Figure 20:** **a)** Inversion to smaller eigenvalue. The left side of the reflector is partially inverted and the right side is starting to be lifted up. **b)** Inversion to yet smaller eigenvalue. The reflector's left side has now been accurately inverted. The right side has been lifted up so the average reflector depth is correct. The broad aspects have been inverted, but not the narrow aspects. Most of the inversion has taken place between the eigenvalue of 0.05 of the previous figure and 0.02 of this figure.



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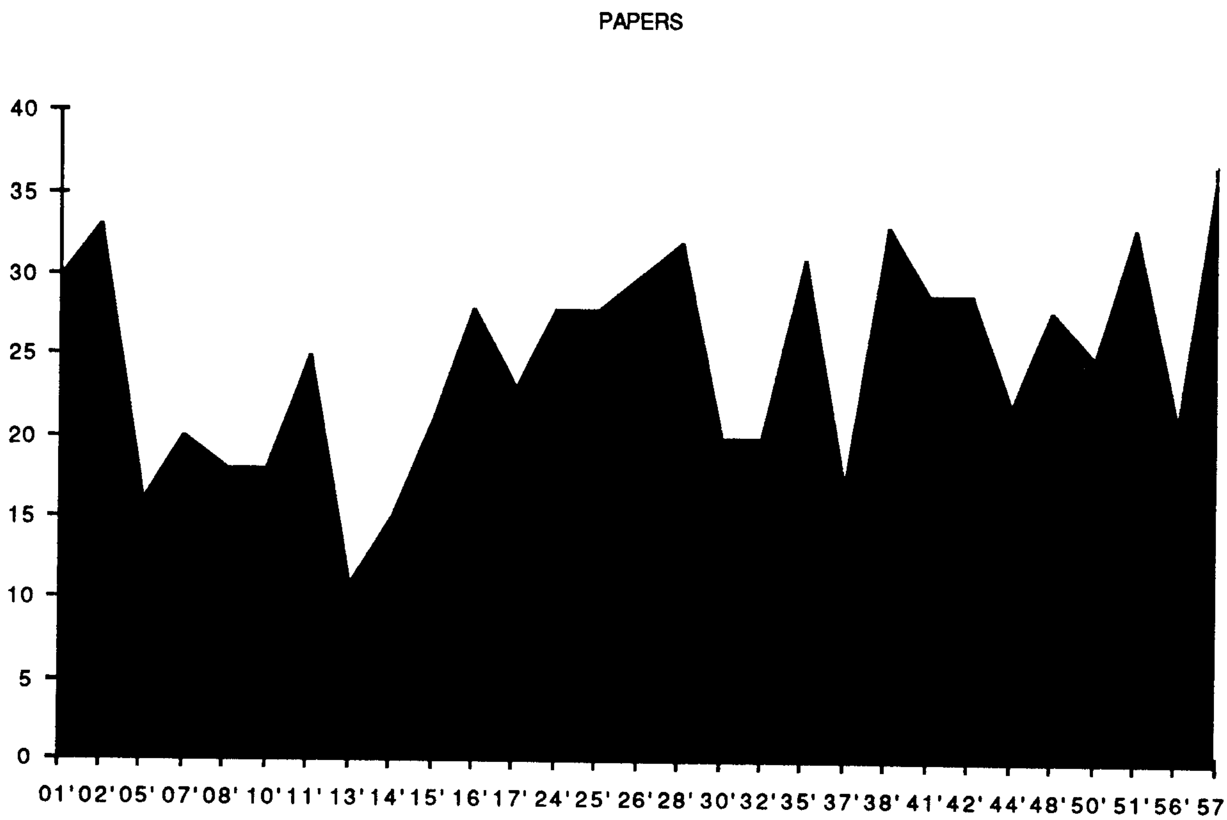

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