ELASTIC WAVEFIELD INVERSION

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By
Peter Mora
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I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a dissertation for the degree of Doctor of Philosophy.

Jon F. Claerbout
(Principal Advisor)

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a dissertation for the degree of Doctor of Philosophy.

Francis Muir

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a dissertation for the degree of Doctor of Philosophy.

Kiyoshi Yomogida

Approved for the University Committee on Graduate Studies:

Dean of Graduate Studies & Research

Elastic wavefield inversion

Peter Mora, Ph.D.
Stanford University, 1987

ABSTRACT

The goal of seismology is to obtain Earth properties from seismic data. I derive formulas based on nonlinear least squares iterations to find the elastic properties of the Earth corresponding to the synthetic wavefield that best matches the seismic observations. Primary P- and S-wave reflections, mode converted waves and Rayleigh waves are all theoretically useful in the inversions. Beginning with a starting guess, the Earth properties are iteratively updated using a preconditioned conjugate-gradient algorithm. The gradient direction is cast in terms of two wave propagations so Frechet matrices are not necessary and the calculations are easily implemented on fine grain parallel computers.

Although the derivation is in three dimensions and commences from the anisotropic elastic wave equation, I have chosen to concentrate on the special case of an isotropic Earth and solve for the compressional and shear wavespeeds. This case is important because the wavespeeds are the properties that have the largest influence on seismic waves. The sudden (high-wavenumber) variations in the wavespeeds at layer boundaries determine the amplitudes of reflected waves while the gross (low-wavenumber) variations in the layers determine the traveltimes of reflected and transmitted waves.

The method is tested by inverting 2D synthetic reflection seismograms (shot profiles), 2D synthetic transmission seismograms (VSP's) and real reflection seismograms (a shot profile). The wave propagations in the tests are done using elastic finite differences to allow for Earth models of realistic complexity. The synthetic data results indicate that low-wavenumber components in wavespeed converge slowly when only reflection data were inverted. This was not expected considering that traveltime curves of reflections should rapidly resolve the low wavenumbers. However, by analyzing an acoustic equivalent of the algorithm in detail, I have identified reflection-tomography-like terms and migration-like terms in the inversion formulas. I prove that the low wavenumbers are resolved by the tomography-like terms and suggest how to get an even rate of convergence of the high and low wavenumbers. I conclude that my iterative elastic inversion formulas obtain all

wavenumbers of the compressional and shear wavespeeds that are resolvable separately by migration and tomography.

I propose that the main topics for future research in seismic wavefield inversion are: how to eliminate the need for an initial model, how to model seismic waves more accurately, how to resolve additional Earth properties, and how to allow for non-Gaussian distributed noise and Earth properties.

Preface

Finally, both the theory and computer power make it feasible to throw away ad hoc methods and partial solutions. Seismic observations can be fed into computers and Earth models will pop out. This goal is on the verge of being realized! Read on.

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Contents

Abstract iv

Preface vi

Acknowledgements vii

List of figures xii

1 Dreaming of inversion 1

1.1 The seismologists' dream 1

1.2 Conventional wisdom fails 2

1.3 Inversion to the rescue 4

2 Observations of elastic waves 5

2.1 Is the earth elastic? 5

2.2 Primary reflections and mode conversions 5

2.3 Other elastic phenomena 12

2.4 Conclusions 12

3 Elastic inversion theory 14

3.1 Overview 14

3.2 Philosophy of inversion 15

3.2.1 Elastic waves or acoustic waves? 15

3.2.2 Inversion or conventional processing? 15

3.2.3 Linear or nonlinear? 16

3.3 Probability theory and least squares 18

3.3.1 Probabilistic inversion 18
3.3.2 Nonlinear least squares 19
3.3.3 Preconditioning 22
3.3.4 Conjugate directions 23
3.3.5 Inversion algorithm 24
3.3.6 Computational aspects for nonlinear functions 24
3.4 Inversion achieved by modeling 25
3.4.1 Overview 25
3.4.2 What is the adjoint operation? 26
3.4.3 What is the adjoint operation for the seismic problem? 28
3.4.4 What is the linearized elastic forward problem? 28
3.4.5 What is the elastic adjoint operation? 30
3.4.6 Interpreting the adjoint 32
3.4.7 The gradient direction 34
3.4.8 Model covariances 35
3.4.9 Data covariances 36
3.4.10 Different model parameters 37
3.5 Inversion described by pictures 39
3.5.1 Introduction 39
3.5.2 Transmission data 39
3.5.3 Reflection data 43

4 Results **50**
4.1 Overview 50
4.2 Noisy diffractions 51
4.3 Single component data 58
4.4 Reflection seismic data 61
4.5 Transmission seismic data 71
4.6 Field data 79
4.7 Conclusions 84

5 Solution to the low wavenumber problem **85**
5.1 Overview 85
5.2 What is the low wavenumber problem? 86

5.2.1 The problem 86
5.2.2 The problem with the problem 86
5.2.3 Resolution of the problem through assumptions 86
5.2.4 Resolution of the problem through inversion in principle 87
5.2.5 Resolution and understanding through analysis 87
5.3 Are the low wavenumbers resolvable? 87
5.3.1 Insights from pictures 87
5.3.2 Non-constant background scattering 88
5.3.3 Non-constant background inversion 94
5.3.4 The resolved wavenumber spectrum 94
5.3.5 The tomographic terms revealed 98
5.3.6 Relationship to iterative inversion 100
5.3.7 Illustration of inversion 103
5.3.8 Inversion = migration + tomography 107
5.4 Conclusions 109

6 Directions of the future **110**
6.1 Is the seismic inverse problem solved? 110
6.1.1 What is the seismic inverse problem? 110
6.1.2 Solving for the initial model 111
6.1.3 Accounting for everything 111
6.1.4 Statistics 111
6.1.5 Parallelism, physics and inverse physics 112

7 Conclusions **113**

A Adequacy of finite differences **114**
A.1 Introduction and overview 114
A.1.1 Finite difference scheme 114
A.1.2 Questions of adequacy for inversion 114
A.1.3 Answers 115
A.2 Finite differences compared to Haskell-Thompson 115
A.2.1 Differences in the assumptions 115
A.2.2 Comparison of results 116

A.2.3 Comparison of P-P and P-S reflections 118
A.2.4 Comparison of results using a free surface B.C. 118
A.3 Are free surfaces needed for P-P and P-S amplitudes? 122
A.4 Conclusions 124

B Verification of the elastic adjoint operation **125**
B.1 Overview 125
B.1.1 Testing adjoint implementations 125
B.1.2 1D acoustic tutorial 125
B.1.3 1D acoustic results 126
B.1.4 2D elastic results 126
B.2 Tutorial: the 1D, constant velocity acoustic case 126
B.2.1 Mathematical proof of the adjoint 127
B.2.2 Numerical calculation 128
B.2.3 Algorithm 128
B.2.4 Pseudo code 130
B.2.5 Numerical test 131
B.3 2D elastic dot product test 137
B.3.1 Summary 137
B.3.2 Linearized forward problem 137
B.3.3 Numerical result 139
B.4 Conclusions 139

Bibliography **141**

List of Figures

2.1 Two-component field shot gather supplied by C.G.G. 7
2.2 Two-component synthetic shot gather computed by elastic theory. 8
2.3 Velocity model used to generate the synthetic data. 9
2.4 Difference between gathers with and without the deep reflector. 10
2.5 Two-component synthetic shot gather computed by acoustic theory. 11
2.6 Field shot gather with Rayleigh waves. 12
3.1 The "camembert" model used to generate transmission data. 40
3.2 Vertical component residual transmission data. 40
3.3 The transmission-data gradient calculations pictorially. 41
3.4 P and S-wave velocity perturbation for the transmission data inversion. . . 42
3.5 Layer/halfspace model used to generate reflection data. 44
3.6 Vertical component residual shot gather for layer/halfspace model. 44
3.7 The reflection-data gradient calculations pictorially. 45
3.8 P-wave and S-wave velocity perturbation for the reflection data inversion. . 46
4.1 Diffractor model relative to a homogeneous background. 52
4.2 Two-component shot gather for the diffractor model. 53
4.3 One and seven iteration P-wave velocity inversion results of diffraction data. 54
4.4 One and seven iteration S-wave velocity inversion results of diffraction data. 55
4.5 One and seven iteration density inversion results of diffraction data. 56
4.6 Two-component mismatch after inverting the diffraction data. 57
4.7 One and seven iteration P-wave velocity inversion result using the vertical component diffraction data. 58
4.8 One and seven iteration S-wave velocity inversion result using the vertical component diffraction data. 59

4.9 One and seven iteration density inversion result using the vertical component diffraction data. 60
4.10 Horst/reef model relative to a linear background function. 62
4.11 A two-component shot gather for the horst/reef model. 63
4.12 One and ten iteration P-wave velocity inversion result using the horst/reef reflection data compared to the filtered P-wave velocity model. 65
4.13 One and ten iteration S-wave velocity inversion result using the horst/reef reflection data compared to the filtered S-wave velocity model. 66
4.14 One and ten iteration density inversion result using the horst/reef reflection data compared to the filtered density model. 67
4.15 Log through the reef (i.e. property versus depth): true model (bold line), initial model (fine line), one iteration result (dotted line) and ten iteration result (broken line). 68
4.16 Vertical wavenumber spectrum of the P-wave velocity log through the reef after one and ten iterations. 69
4.17 Vertical wavenumber spectrum of the true P-wave velocity log through the reef. 69
4.18 Two-component mismatch after ten iterations for the horst/reef example. . 70
4.19 A two-component offset VSP for the horst/reef model. 72
4.20 One and ten iteration P-wave velocity inversion result using the horst/reef transmission data compared to the true P-wave velocity model. 73
4.21 One and ten iteration S-wave velocity inversion result using the horst/reef transmission data compared to the true S-wave velocity model. 74
4.22 One and ten iteration density inversion result using the horst/reef transmission data compared to the true density model. 75
4.23 Log through the reef (i.e. property versus depth): true model (bold line), initial model (fine line), one iteration result (dotted line) and ten iteration result (broken line). 76
4.24 Vertical wavenumber spectrum of the P-wave velocity log through the reef after one and ten iterations. 77
4.25 The normalized sum of the square error as a function of iteration number. . 77
4.26 Two-component VSP mismatch after ten iterations for the horst/reef example. 78

4.27 Mismatch for the two-component field shot gather at iteration zero. 80
4.28 Mismatch for the two-component field shot gather at iteration three. 81
4.29 The normalized sum of the square error as a function of iteration number. . 82
4.30 Three iteration field data inversion result. 83
5.1 Basic plane wave experiment showing plane waves incident on a velocity anomaly embedded in a layer over a halfspace. 88
5.2 Raypaths corresponding to the four different scattering terms. 93
5.3 The part of the wavenumber spectrum of the squared slowness model that can be resolved using a single plane wave source. 95
5.4 The part of the wavenumber spectrum of the squared slowness that can be resolved using a point source (i.e. a sum of plane waves at all angles). . . . 95
5.5 The part of the wavenumber spectrum of squared slowness that can be resolved from the migration-like and tomographic-like terms in the inversion formulas. 97
5.6 Shot gather from a circular-anomaly/layer-halfspace-background model. . . 98
5.7 The true model compared to the ten iteration inversion result for the circular-anomaly/layer-halfspace-background data using a single shot gather. 99
5.8 One of five shot gathers generated from a circular-anomaly/homogeneous-background model. 103
5.9 True model compared to the one and fifteen iteration inversion results, from the circular-anomaly/homogeneous-background data, using five shot gathers. 104
5.10 One of five shot gathers generated from a circular-anomaly/layer-halfspace-background model. 105
5.11 True model compared to the one and fifteen iteration inversion results, from the circular-anomaly/layer-halfspace-background data, using five shot gathers. 106
A.1 The velocity model. 116
A.2 The pressure source - vertical receiver shot gather via FD (c.f. Figure A.3). 117
A.3 The pressure source - vertical receiver shot gather via Sherwood (c.f. Figure A.2). 117
A.4 The velocity model. 118
A.5 The pressure source - vertical receiver shot gather via FD (c.f. Figure A.6). 119

A.6 The pressure source - vertical receiver shot gather via Sherwood (c.f. Figure A.5). 119
A.7 The difference between Figures A.5 and Figure A.6 (Figure A.5 - FigureA.6). 120
A.8 The vertical source - vertical receiver shot gather via FD (c.f. Figure A.9) computed using a free surface boundary condition. 121
A.9 The vertical source - vertical receiver shot gather via Sherwood (c.f. Figure A.8) computed using an air-solid surface boundary condition. 121
A.10 The difference between Figures A.8 and Figure A.9 (Figure A.8 - Figure A.9). 122
A.11 The vertical source - vertical receiver shot gather via FD (c.f. Figure A.8) computed using an absorbing surface boundary condition. 123
A.12 The difference between Figures A.11 and Figure A.8 (Figure A.11 - Figure A.8) to see the effect of the free surface on the reflections. 123
B.1 The density perturbation vector $\delta\rho = \mathbf{x}$. 132
B.2 The density perturbation vector $\delta\rho_1 = \mathbf{x}_1$. 132
B.3 The displacement perturbation vector $\delta u = \mathbf{y} = \mathbf{A}\mathbf{x}_1$. 132
B.4 The displacement perturbation vector $\mathbf{A}\mathbf{x}$. 133
B.5 The adjoint vector $\delta\hat{\rho} = \mathbf{A}^T\mathbf{x}$. 133
B.6 The background wavefield $u(x,t)$. 133
B.7 The perturbation wavefield $\delta v(x,t)$. 134
B.8 The back-propagated residual wavefield $\psi(x,t)$. 134
B.9 The linearized forward problem $\mathbf{A}\mathbf{x}$. 138
B.10 The linearized forward problem $\mathbf{A}\mathbf{x}$ shown in Figure B.9 boosted by a factor of 10^3. 139