Cable Tangent Stacking

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ABSTRACT

Velocity estimation proceeds by summation over a hyperbolic curve and then placing the sum at *zero-offset* time. Hyperbolas of differing velocities are tangent at *zero offset.* Alternately, the sum may be placed at the time of *another offset.* The offset can be chosen somewhere mid-cable where the hyperbola picks up most of its energy. This procedure, called *cable tangent summing* yields a velocity space in which interpolation and filtering are appropriately done at constant time. Equivalently, cable-tangent velocity space is a stretching transform of the usual velocity space. The stepout dt/dv of events in velocity space contains the information of reflection coefficient vrs. angle.

INTRODUCTION

Most people don't look at amplitude, phase, and stepout of events in velocity space, they just look at the coherence. Here we examine those concentrated blobs everyone likes to find in velocity space.

Velocity space is an invertible representation of the data, just like (τ, p)-space. In (τ, p)-space a kind of moveout correction can be done. Likewise in velocity space a kind of moveout correction can be done. A "moveout corrected" velocity space is convenient for interpolation and filtering over velocity. Theoretically, moveout correction in velocity space turns out to be equivalent to hyperbolic stacking to a nonzero offset travel time. This paper illustrates and examines the relationship.

A specific application of the relationship is not proposed, except I mention that I developed the subject while designing a temporal deconvolution filter to produce the best velocity space.

STEPOUT IN VELOCITY SPACE

Hyperbolic summation of a common midpoint gather transforms offset to velocity. The stepout dt/dv of events in velocity space is often ignored. The stepout will be seen to encode the average offset from which energy is gathered.

Each point in (t, x)-space yields a curve in (τ, v)-space. The sum of all the x-points on the cable corresponds to a sum of all the curves in (τ, v)-space. Let us examine these curves to understand what the average will be. The basic conic section is

$$t^2 \;=\; (z^2 + x^2)/v^2 \tag{1}$$

For constant z equation (1) is a hyperbola in (t, x). Customarily we convert depth z to travel-time depth $\tau = z/v$. Our analysis will use "sloth," $s = 1/v^2$ instead of velocity. With this notation, (1) becomes

$$t^2 \;=\; \tau^2 + x^2 s \tag{2}$$

This is a hyperbola in (t, x)-space, for any point in (τ, s)-space. Cable-tangent hyperbolas are defined by the equation

$$t^2 \;=\; (\tau^2 - x_r^2 s) + x^2 s \tag{3}$$

In (3), x_r is taken as a constant or as a function $x_r(\tau)$. Equation (3), like equation (2) is a hyperbola in (t, x)-space for each point in (τ, s)-space. Consider a fixed τ and the family of cable-tangent hyperbolas generated by varying s. Equation (3) says that where $x = x_r$ these hyperbolas all touch. When $x_r = 0$ the hyperbolas touch at their tops.

Let us define a particular x_r that increases linearly with time. This certainly isn't the only possibility. But it is reasonable and it illustrates the principal features of nonzero x_r.

$$x_r \;=\; \alpha\,\tau \tag{4}$$

Figure 1 shows some points in velocity (τ, s)-space and the implied curves in (t, x)-space. For display in figure 1, I chose

$$\alpha \;=\; \frac{2}{3}\,\frac{x_{\max}}{t_{\max}} \tag{5}$$

So, at the end of the time axis, the tangency is two thirds of the way out the cable. Inserting (4) into (3) yields the curves $t(x)$ plotted in figure 1.

$$t^2 \;=\; \tau^2(1 - \alpha^2 s) + x^2 s \tag{6}$$

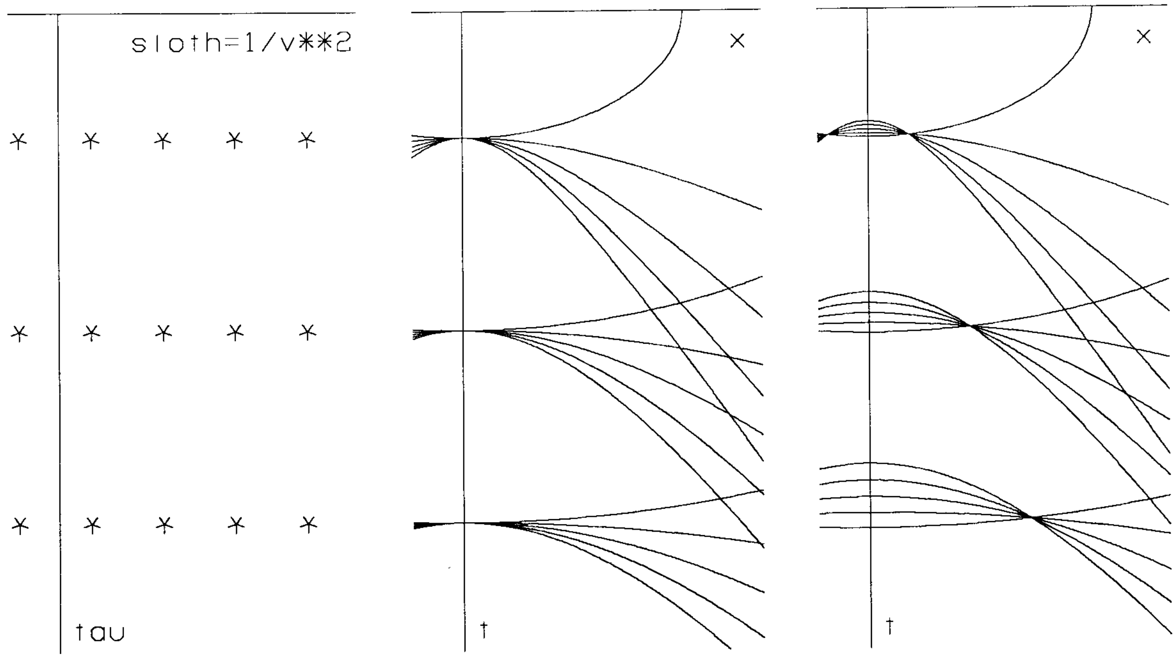

FIG. 1. Points in sloth space (left) corresponding to hyperbolas in (t, x)-space. Center shows the usual case $x_r = 0$. At the right is for $x_r = \alpha\tau$.

From figure 1 we notice that the families of hyperbolas tend to be uniformly spaced. They would be less uniformly spaced if we had chosen uniform intervals of velocity or inverse velocity. Thus uniform intervals of sloth are natural if economy of parameterization is important. On the other hand, the real earth tends not to fill a rectangular region in (τ, s)-space.

SLOTH SPACE

Equation (6) may be backsolved for τ.

$$\tau^2 \quad = \quad \frac{t^2 - x^2 s}{1 - \alpha^2 s} \tag{7}$$

For any point in (t, x)-space, equation (7) gives a curve $\tau(s)$ in velocity (sloth) (τ, s)-space. For α=0 these curves are parabolas. They are like a parabolic lamp reflector sending a beam to your left. Figure 2 shows examples for various points and curves. In the usual $x_r = 0$ velocity space, all curves have $d\tau/ds < 0$ so the average is certainly nonzero. In cable-tangent velocity space, curves roughly average to the horizontal.

Notice the curves corresponding to near zero offset data. They tend to concentrate in sloth space, implying that inner offsets carry little velocity information.

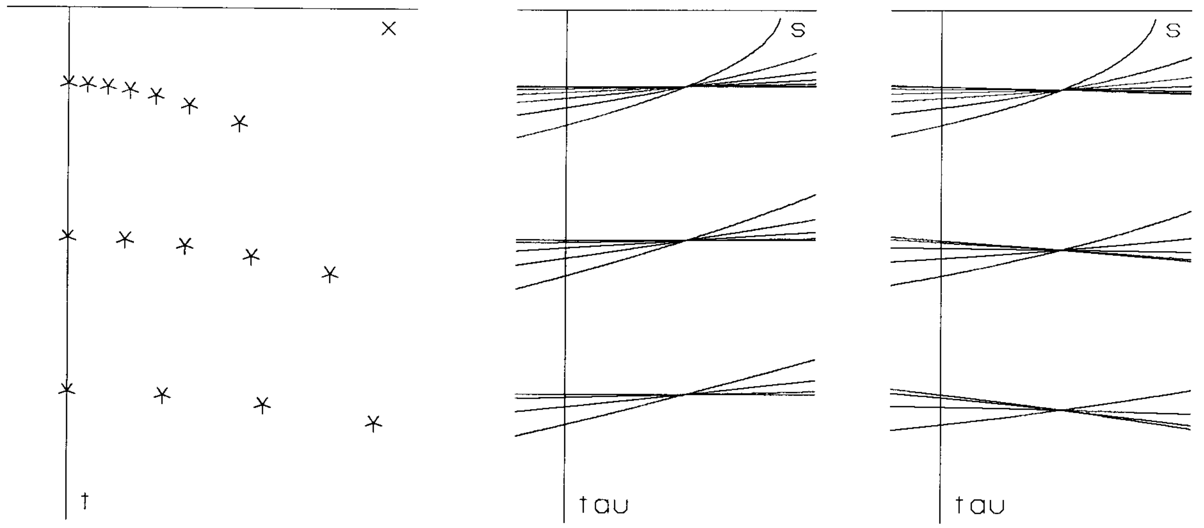

FIG. 2. Points in data space and their corresponding curves in sloth space for α=0 and for α=.8 *xmax* /*tmax* .

Comparing the location of intersecting curves in the α=0 plane to the other plane, you see how stretching the τ axis with increasing s can flatten events in sloth space.

While summing along a hyperbola you may as well compute another sum, the moment, x times the data you are summing. This will tell you how to do moveout in sloth space. After processing in sloth space, you will need to know how to do moveout or you won't be able to provide people with their usual final stack.

DETAILS

To select the points in (t , x)-space in figure 2, we select a midrange value of sloth s_{mid} . The hyperbola and line equations used were

$$t^2 = \tau^2 + x^2 s_{mid} \tag{8a}$$

$$r = \frac{x}{t} \tag{8b}$$

The coordinates of the points are found by first eliminating x from (8a,b).

$$t^2 = \frac{\tau^2}{(1 - r^2 s_{mid})} \tag{9}$$

Given a uniform mesh in (τ, r)-space, equations (8b,9) give parametric equations for t and $x = rt$ plotted on the left in figure 2.