APPENDIX B

EXTRACTING RELIABLE SIGNAL

The essential features of signal/noise separation are explained by its application to velocity stacks and VSP's. I shall now stress the assumptions that should be observed when other applications are made. In minimizing objective function (A.3), an iterative optimization should converge on the most "important" details of the model parameters first. The most important details are those that can be most reliably determined from the data. To recognize the reliability, we should ask whether perturbations of parameters account for features of signal or noise.

B.1. The essential ingredients

To make the calculations of the Bayesian estimate and reliability possible, we must assume that p_{n_i} are Gaussian. Otherwise perturbations of the model parameters will not be a linear function of the data. We can iteratively extract the non-Gaussian details of the noise, such we did for the ground-roll in Chapter 1 and the tube wave in Chapter 3. The assumption of Gaussianity will become increasingly accurate for the residual noise. Most importantly, the assumption makes the gradient in equation (A.5) become a linear function of the residual data events.

$$\min_{\mathbf{s}} J(\mathbf{s}) = \min_{\mathbf{s}} \left\{ \sum_j \ln\, p_{s_j}(s_j) + \sum_j \frac{1}{2C_{n_j}^2} [d_j - f_j(\mathbf{s})]^2 \right\}$$

$$\frac{\partial J(\mathbf{s}^0)}{\partial s_i} = \frac{p_{s_i}{}'(s_i{}^0)}{p_{s_i}(s_i{}^0)} - \sum_j F_{ij}^0 \frac{1}{2C_{n_j}^2} [d_j - f_j(\mathbf{s}^0)] \tag{B.1}$$

Define the following random variables as the residual signal, noise, and data.

$$\mathbf{s}^{res} \equiv \mathbf{f}(\mathbf{s}) - \mathbf{f}(\mathbf{s}^0) \;\; ; \;\; \mathbf{n}^{res} \equiv \mathbf{n} - \mathbf{n}^0 \;\; ; \;\; \mathbf{d}^{res} \equiv \mathbf{s}^{res} + \mathbf{n}^{res} \tag{B.2}$$

$\mathbf{n}^0$ is previously extracted noise. Write the gradient $\mathbf{d}'$ as a single linear transform F of the residual data $\mathbf{d}^{res}$.

$$d_i{}' \equiv \sum_j F_{ij}\; d_j{}^{res} + c_i \;\; ; \;\; s_i{}' \equiv \sum_j F_{ij}\; s_j{}^{res} + c_i \;\; ; \;\; n_i{}' \equiv \sum_j F_{ij}\; n_j{}^{res} \;\; ;$$

$$\text{so that } \mathbf{d}' = \mathbf{s}' + \mathbf{n}' \quad . \tag{B.3}$$

Primes indicate a linear transformation of residual variables.

Because of the assumption that noise is Gaussian, signal and noise have remained additive in the gradient. Now let us estimate how much of a given sample of the gradient is only transformed signal. I postpone the estimation of pdf's for the transformed signal and noise.

When the necessary pdf's are known, the Bayesian and reliability estimates are simple. Our Bayesian estimate is defined as the expected value of signal when the sum of signal and noise is known. The Bayesian estimate of the signal s' in a sample of the linearly transformed data d' is given by equation (3.14) in the text.

Though we may say that we now have the expected amounts of signal in the perturbations, we have not yet determined how probable these amounts are. Define the reliability of a Bayesian estimate as the probability that the actual value is within a fraction c of the estimated value. Equation (3.15) gives the definition. Accept only those sample perturbations with a sufficiently high reliability.

B.2. Iterative linearization for stability

Some components of the MAP perturbation may be unreliable because the forward transform $\mathbf{f}$ largely destroys them. Another statistical simplification will allow us to suppress these sources of instability.

As we saw, the estimate of reliability requires only that the residual noise distribution be Gaussian. The signal distribution remained arbitrary. Let us instead assume that residual signal is also Gaussian, and let us also temporarily replace the forward transform $\mathbf{f}$ by a linearization about $\mathbf{s}^0$, as shown in equation (3.9). The temporary objective function changes into equation (3.10).

Now repeated applications of the gradient will remain a linear function of the noise. Objective function (3.10) may be minimized by use of a conjugate gradient algorithm; the cumulative perturbation $\mathbf{d}' \equiv \Delta \mathbf{s}$ solves the linearized least-squares inversion for the signal. Because of the linearity the estimations of reliability can still be applied to this perturbation.

The iterative linearization becomes necessary when unstable components completely obscure reliable details in the gradient perturbations. This linearization suppresses poorly determined high frequencies that often obscure the inversion of impedance functions.