EXTRAPOLATION, INTERPOLATION AND SMOOTHING OF WAVE FIELDS

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The restoration of a missing seismogram from a reflection seismic dataset appears at first to be a somewhat pedestrian task. In fact it is an ideal warm-up for the major task of all seismology and exploration, that is, learning to process data which is *almost always* aliased, gapped, or truncated in one spatial dimension or another.

A General Point of View

Recorded data $\overline{p}$ is often an incomplete subset of the possible data p because of accidents, cable truncation, survey line termination, or spatial aliasing. Our purpose here is to attempt to *fill in* or *extend* the recorded data $\overline{p}$ to provide a full dataset p. We must first define a transformation f from data space to model space, say $P=f(p)$. We imagine the *model* P generated from the *extended data* p, to be a parsimonious representation of p. That is, there should be portions of P which contain everything of value and other portions which (practically) vanish. It is by zeroing these insignificant values that we provide the extra equations to combine with the observed data $\overline{p}$ to determine the full space of possible data p. We might, but we will not, decide beforehand what portions of P are to be zeroed. Instead we cautiously and gradually suppress weak values and enhance strong ones, all the while assuring that the extended dataset p agrees exactly with the observed dataset $\overline{p}$ where the domains overlap. Algebraically this amounts to iterative solution of a constrained optimization problem.

CDP Gather Example

Let $p(h,t)$ be a common-midpoint gather at the earth's surface $z=0$. Let the function f be downward continuation to $t' = 2 \int dz/v$, say to give $P(h,t')$. Model space is parsimonious because energy should concentrate near offset $h=0$. It may not be as parsimonious as it could be, however, because the velocity is wrong or because partial migration has not been done to remove the dip effect. But even then, model space should be quite parsimonious. We should be able to fill in and extend the dataset $\overline{p}$ by enhancing P where it is strong and suppressing it where it is weak.

Section Example

Consider a zero-offset data section. This data $p(y,t)$ is a function of midpoint y and traveltime t. Let $\overline{p}$ be limited to a small range of midpoints, say y_{80} to y_{90} or fewer, with a missing trace at y_{85}. After extending such data to $p(y,t)$ where the y-axis is limited only by computational feasibility, then a small number of dips may predominate. High-quality data is often defined as data with a dip spectrum that is fairly narrow-banded. Therefore, we define *model space* for this example by two-dimensional Fourier transformation from $p(y,t)$ to $P(k_y,\omega)$. Energy should concentrate along a few values of Snell parameter = k_y/ω, that is, along a few lines through the origin in (k_y,ω)-space. A few lines won't cover the space, so model space is parsimonious, even in the presence of spatial aliasing! In summary, if the dip spectrum were fairly narrow band ("good" data) then windowing (zero filling) would smear its dip spectrum. Squeezing down the small values should "unsmear" and ultimately lead to good interpolations *despite spatial aliasing.*

This seismic-section example has a curious similarity to the CDP-gather example in that lenses perform Fourier transforms.

The Minimum-Entropy Stretch

The object is to boost P where it is big and to shrink it where it is small. This will be done gently, on many iterations, after each of which the constraint $p = \overline{p}$ will be reimposed. Choice of the stretch function is somewhat subjective. As a consequence of the considerations in "Seven Essays on Minimum Entropy", I propose

$$P \leftarrow P\left[\frac{P^* P}{\ll P^* P \gg}\right]^{\varepsilon} \qquad 1 \gg \varepsilon > 0$$

If nothing is known *a priori* then $\ll P^* P \gg$ may be set to one and its value should subsequently drop out. In general, << >> denotes a very heavy smoothing, mainly to accommodate spherical divergence. If there are 1000 perfect CDP gathers (or, say, only 1000 extended CDP gathers) then their average (with h completely summed out) could be included in $\ll P^* P \gg$. In practice we might start with $\varepsilon = .1$, iterate to stability, then $\varepsilon = .05$, iterate to stability, etc.

Prior Information

There is a region of (h,t)-space to zap on grounds of possible velocities of rocks. Likewise, there is a region of (k_y,ω)-space which could be zeroed for the same reason. Also you always want to be trying to enhance at $h=0$. Gain control can be incorporated by introducing a factor t into both numerator and denominator $P^* P$. And so it goes. Each application has special prior information which vies for attention in comparison with the experience of many previous data samples. Previous experience provides a noise model with which to soften the strict ideology of "prior information." Regrettably, no complete recipe for relative use of experience versus theoretical information can be given yet.

Algorithm

$$p = \begin{cases} \bar{p} \\ 0 \end{cases}$$ Extend data with zeroes (unless you have a better idea)

Begin iteration reducing epsilon in stages

plot p

$P \leftarrow p$ Go into model space

$$P \leftarrow P \left[\frac{P^* P}{\ll P^* P \gg} \right]^{\varepsilon}$$ Reduce entropy

$p \leftarrow P$ Back into data space

$$\alpha = \frac{\sum \bar{p} p}{\sum p p}$$ Sums range only over regions where $\bar{p}$ is known

$$p = \begin{cases} \bar{p} \\ \alpha p \end{cases}$$ Restore recorded data where known; otherwise, scaled, extended data

End iteration

Comment: Note that the present choice of α minimizes $\sum(\overline{p} - \alpha p)^2$. The purpose of this is to account for bias of P^*P compared to its average, especially as it may be affected by the data domain.

Smoothing Data

You may not fully believe your data. This is up to you. If you wish to allow for certain errors in $\overline{p}$ it is easy to incorporate them at the last step of the algorithm. After the M.E. stretch, p is inconsistent with $\overline{p}$. The last step of the algorithm restores consistency. You could do the restoration in some incomplete way to allow for errors in $\overline{p}$. How you do incomplete restoration will define your noise model.

Attractive Features of Trace Restoration Research

This work is a prototype for many applications in which effects of spatial aliasing tend to dominate. Major success could ultimately revolutionize 3-D survey processing or processing of regional refraction lines.

Minimum-entropy debubble work has the serious drawback that the function f is not a unitary operator, as it is in trace restoration.

Temporary removal of selected traces from any given dataset provides an excellent benchmark for testing algorithms and definitions of entropy in a realistic field environment. Thus, unlike in debubble work, the "correct answer" can be known in field data trials as well as in synthetic data trials.

Synthetic Data for Trace Recovery by Minimum Entropy