TIME-SERIES AND ONE-DIMENSIONAL MULTIPLE REFLECTION

[condensed from SEP-12, SEP-20, p. 57-62, and Estevez-Claerbout (EC) Geophysics paper]

Any ***wave-field*** theory for multiple reflections must reduce, in layered media, to a time-series theory. Since a ***time-series*** theory is considerably easier to understand, it seems worthwhile to first describe the final time-series theory in order that certain simplifying assumptions and limitations will be clear from the outset.

In a strictly mathematical sense the inverse wave theory of Goupillaud and Kunetz completely solves the one-dimensional problem of suppressing multiple reflections and recovering reflection coefficients. However, their theory is an unsatisfactory starting point for a two- or three-dimensional inverse problem. To begin with, their method can hardly be said to "work" in any practical sense even in one dimension. I trace the main weakness to the assumption of the knowledge of a broad-band source. Also, their theory is somewhat complicated, stemming largely from incorporation of phenomena which are rarely observable, such as transmission coefficients and inner bed multiples. I advocate simplifying their theory by neglecting these phenomena, in order that more significant phenomena such as source waveform, receiver offset, and lateral variations may be incorporated. As the commercial success of predictive methods of multiple suppression become more frequent, we will be able to turn our attention back to those more rarely observable phenomena.

The basic idea of our approach will be similar to the one taken by Riley and Claerbout (1976). First, the seismic wave field is decomposed into downgoing and upcoming waves by obtaining separate equations for each type of wave. These equations are coupled at the reflectors through the reflection coefficients. With these coupled equations, accompanied by the appropriate boundary conditions, we can compute a reflection seismogram given a distribution of reflection coefficients (forward problem); or alternatively, given the reflection seismogram, we can determine the reflection-coefficient distribution (inverse problem).

Consider vertically incident plane waves and define

d_t = wave downgoing from the surface at time t (1a)

u_t = wave upcoming to the surface at time t (1b)

s_t = downgoing source excitation function (1c)

The free-surface condition is that the total downgoing wave is the initial excitation s_t plus the reflected, polarity-reversed wave that is upcoming at the surface

$$d_t = s_t - u_t \tag{2}$$

Some Things to Neglect

A considerable amount of mathematical simplification results when we make the practical assumption that the only multiples of interest are those with one or more reflections from the free surface. Throughout this whole study we will make this assumption with the full knowledge that is it not always valid. The reason why it is so often reasonable is that reflection coefficients in the earth tend to be much less than those of the free-surface, unit reflection coefficient. This assumption may be mathematically expressed in various, nearly equivalent ways. For example, it is practically the same thing to say that the downgoing wave is much larger in magnitude than the upcoming wave, not at each moment in time, but in the sense of power as a function of frequency. Specifically, we may define Z-transforms

$$D(Z) = d_0 + d_1 Z + d_2 Z^2 + d_3 Z^3 + \cdots \tag{3a}$$

$$U(Z) = u_1 Z + u_2 Z^2 + u_3 Z^3 + \cdots \tag{3b}$$

and state our assumption as

$$D\left[\frac{1}{Z}\right] D(Z) \gg U\left[\frac{1}{Z}\right] U(Z) \qquad (4)$$

for all real ω where $Z = e^{i\omega}$.

The reason why it is so advantageous mathematically to make this assumption is that once the downgoing wave (including all multiple reflections) is known at the surface $z=0$, the assumption that it grossly exceeds the upcoming wave means that reflectors at various depths do not significantly change the downgoing wave from its surface value. Any changes to it would be quadratically small, being the product of the reflection coefficients and the upcoming wave. So except for time shift, the downgoing waveform is presumed depth-invariant.

Another assumption which reduces much of the clutter for about the same loss in accuracy is to assume that any transmission coefficient ($=1+c$ where c is a reflection coefficient) may be replaced by unity. This is a quadratic error since the transmission downward ($=1+c$) times the transmission upward ($=1-c$) equals $1-c^2$. Instead of further attempts at justification of the approximations let us examine the consequences of the theory that they allow.

The "Noah" Relations

Examination of figure 1 suggests the equation

$$u_3 = c_1 d_2 + c_2 d_1 + c_3 d_0 \qquad (5)$$

For an arbitrary time t this becomes

$$u_t = \sum_{z=1}^{t} c_z \, d_{t-z} \qquad (6)$$

Eliminate the upcoming wave u_t with the free-surface condition (2)

$$s_t - d_t = \sum_{z=1}^{t} c_z \, d_{t-z} \qquad (7)$$

and introduce the simplifying definition $c_0 = 1$, obtaining the equation of a convolution

$$s_t = \sum_{z=0}^{t} c_z \, d_{t-z} \tag{8}$$

The appearance of a convolution suggests the definition of more Z-transforms in the fashion of (3a,b), say

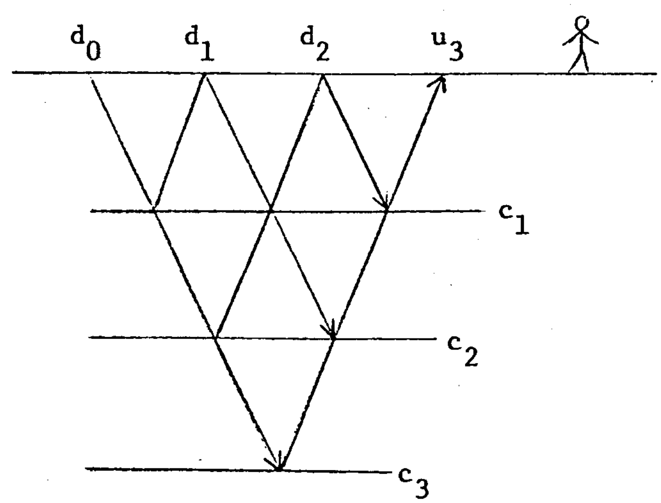

FIG. 1. Ray-path geometry for upcoming wave u_t at the surface at time t=3 in terms of waves d_t downgoing from the surface at earlier times and reflection coefficients c_t at uniformly spaced traveltime depths.

$$S(Z) = s_0 + s_1 Z + s_2 Z^2 + \cdots \tag{3c}$$

$$1 + C(Z) = 1 + c_1 Z + c_2 Z^2 + \cdots \tag{3d}$$

Defining (9), equation (8) may be obtained from identification of the coefficient of Z^t :

$$S(Z) = [1 + C(Z)] \, D(Z) \tag{9}$$

The computation of synthetic seismograms (forward modeling) may be done by solving (9) for D and invoking polynomial division

$$D(Z) = \frac{S(Z)}{1 + C(Z)} \tag{10}$$

Inversion

Inversion, i.e. computing the reflection coefficients from the waves U and D at the surface, involves substituting the surface condition S(Z) = D(Z) + U(Z) into (9) to get the source-independent equation

$$C(Z) = \frac{U(Z)}{D(Z)} \tag{11a}$$

Since U and D are not separately observed it is also convenient to solve for C in terms of S and U, namely

$$1 + C(Z) = \frac{S(Z)}{D(Z)} = \frac{S(Z)}{S(Z) - U(Z)} = \frac{1}{1 - U(Z)/S(Z)} \tag{11b}$$

The form of (11b) correctly suggests that stable recursive behavior may be expected when S is minimum phase.

Source Waveform Estimation

The practical problem is to estimate S(Z) in such a way that (11) does not diverge. This is a well-known problem which has a well-known, not completely satisfactory answer. In summary, (10) may be written

$$D = S(1 - C + C^2 - \cdots \tag{12a}$$

$$= S - SC + SC^2 - \cdots \tag{12b}$$

Often the seafloor reflection is sufficiently strong and clear for the term SC to be identifiable on the data as a primary reflection, say P, and SC^2 to be identifiable as a multiple reflection, say M. Then we have

$$S = \frac{P^2}{M} = \frac{S^2C^2}{SC^2} \qquad (13)$$

where (13) could be solved by least squares

$$MS \approx P^2 \quad \text{or} \quad P^2S^{-1} \approx M. \qquad (14)$$

Exponential Tilt

A nice feature of all the Z-transform equations in this approximate theory is that they are invariant under exponential scaling of time functions - that is, all functions may be multiplied by $\exp(\alpha t)$. The result is equivalent to replacing $Z = \exp(i\omega\Delta t)$ by $Z = \exp[(\alpha + i\omega)\Delta t]$.