SNELL WAVES AND MULTIPLE REFLECTIONS

[condensed from SEP-15, p. 57-71]

A low-flying hypersonic aircraft sends a Snell wave into the earth. The Snell wave is slightly more complicated than a vertically incident wave, but it is much simpler than a spherical wave. Moveout correction does not do a sufficiently good job of correcting spherical waves to plane waves to enable satisfactory prediction and suppression of multiple reflections. But Snell-wave concepts provide a necessary link between theory and data.

Snell's Law and Observability of Snell's Parameter

Snell's parameter p is the sine of the angle θ between a ray and the normal to an interface divided by the medium velocity v. Snell's law says this parameter is conserved on transmission or reflection of rays. Snell's law is so basic that it applies in elasticity when a compressional wave is converted to a shear wave. As seismologists we have a special interest in stratified media, that is, media in which the velocity v(z) is a function of depth only. Consequently, Snell's parameter

$$p = \frac{\sin\theta(z)}{v(z)} \tag{1}$$

is a constant function of depth. For a ray traveling from a source to a receiver the Snell parameter p is a constant function of time, even if some legs of the journey are by shear waves.

Being constrained to make our measurements at the surface of the earth, we cannot make any direct observation of either the material velocity v(z) or the propagation angle θ, but the ratio (1) will be easily observed. Figure 1 shows that Snell's parameter p is the inverse of the speed at which the intercept of a wavefront with the earth's surface moves in the horizontal direction. That is,

$$p = \frac{\sin\theta}{v} = \frac{dt}{dx} = \frac{1}{\text{horiz. speed at } z=0} \tag{2}$$

5.2 Fig. 1

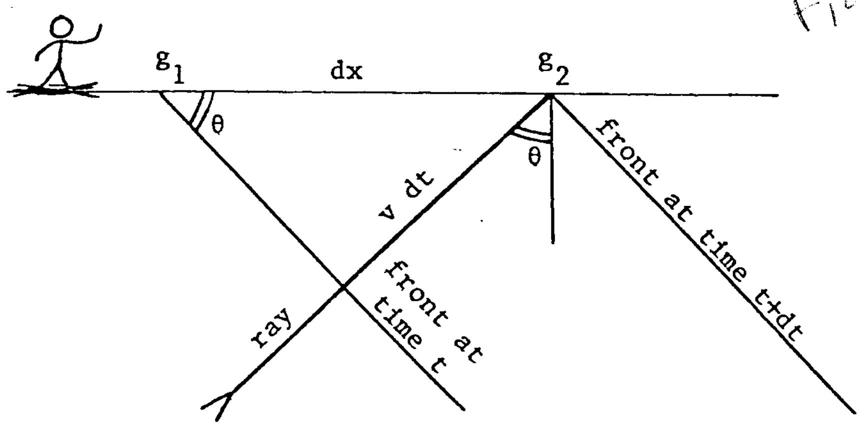

FIG. 1. Plane wave arrival at earth's surface showing that observation of dt/dx gives Snell's parameter $p = (\sin \theta)/v$.

The inverse to Snell's parameter p is known as the horizontal phase velocity. For a vertically incident plane wave this velocity is infinity. Less steep angles have slower velocities. When the velocity is zero, as under a parked airplane, the deformation of the earth dies out exponentially from the source. Intermediate velocities are such that, as long as the phase velocity exceeds the material velocity, we are discussing waves. When the phase velocity is less than the material velocity, the disturbances damp out exponentially away from the source, and the physical behavior becomes quasi-static deformation. A special case, surface waves or ground roll, is for phase velocities faster than that of the surface material and slower than that of the material at depth.

A Source for Snell Waves

Actually airplanes hardly move at speeds sufficiently high to be good sources of Snell waves. Mathematically, what we need is a continuously active point source which moves horizontally from $x = -\infty$ to $x = +\infty$ at a speed of $1/p$ [actually, for two-dimensional (x,z) analysis, we would need a line source along the third dimension y]. In a constant velocity medium the waves emitted from this source are plane waves with an angle from the vertical given by $\sin \theta = pv$. In a stratified medium $v(z)$ the wavefronts become curved and are no longer planar. Such wavefronts are so central to applied seismogram analysis in petroleum prospecting that they require a name. To prevent us from inaccurately referring to these wavefronts as non-vertically-incident plane waves, I propose to call them Snell waves.

Take a surface Snell wave source to have a horizontal phase velocity p_0^{-1}. That the wave disturbance seen at any depth z_0 also moves horizontally at the same speed follows from the time-shifted identity of any x with any other x. Thus, Snell's law (2) is merely a geometrical consequence of the fact that the horizontal phase velocity at any one depth must, for stratified media v(z), equal that at all other depths.

Spatial One-Dimensionality of Snell Waves

The nice thing about a source of vertically incident plane waves (p=0) in a horizontally stratified medium is that the ensuing wave field will be spatially one-dimensional. In other words, an observation or a theory for a wave field would be of the form $P(z,t)\cdot \mathrm{const}(x)$. What is true, but not quite so obvious, is that Snell waves for any particular non-zero p value are also spatially one-dimensional. That is, with

$$t' = t - px \tag{3a}$$

$$x' = x \tag{3b}$$

$$z' = z \tag{3c}$$

spatial one-dimensionality is given by the statement

$$P(x,z,t) = P'(z',t')\cdot \mathrm{const}(x') \tag{4}$$

Obviously when an apparently two-dimensional problem can be reduced to one dimension great conceptual advantages result, to say nothing of computational economic advantage. Before proceeding, study equation (4) until you realize why the wave field can vary with x but be a constant function of x' when (3b) says $x = x'$.

Equations (3a,b,c) are a coordinate transformation from (x,z,t) space to (x',z',t')-space. Equation (3a) is simply a definition of linear moveout. In later papers we will consider more complicated coordinate transformations. (The spatial coordinates could follow the path of a ray and move at the speed of a front.) In these more advanced papers the readers are asked to delve into such arcane matters as how to manipulate Fourier transforms in the (x',z',t') Snell coordinates and how to express the wave equation and solve it by finite differences in (x',z',t') coordinates. In order to motivate study of these complicated matters, this paper will study the useful ***geometrical*** aspects of linear moveout.

Decomposition of Spherical Waves into Snell Waves

We are all familiar with the idea of creating an impulse function by a superposition of sinusoids of all frequencies. The three-dimensional generalization of this is the creation of a point source by means of superposition of plane waves going in all directions. As seismologists we have special affection for mathematical models in which the velocity is solely a function of depth $v(z)$. So instead of thinking of plane waves of some angle parameter θ we think of these waves as characterized by their Snell parameter p. Instead of an angular bandwidth $\Delta\theta$, we have a slowness bandwidth Δp. The advantage of p over θ is that it doesn't change with depth z.

Not only can a point disturbance be thought of as a superposition of plane-wave disturbances but a plane wave can be thought of as a superposition of many Huygens secondary point sources. In fact, as will be described later, a Snell wave can be simulated by an appropriate superposition, called slant stack, of conventional exploration data. Actually, just the simple process of propagation spreads out a point disturbance to where, from a distance, the waves appear to be nearly plane waves or Snell waves.

Linear Moveout

Another name for the Snell parameter is the ***stepout*** of an event. It has units of inverse velocity and is often given in units of milliseconds per meter (seconds per kilometer). Simple measurements made on field data contain information about Snell waves. Looking on field data for events of some particular stepout p amounts to scanning hyperbolic events trying to pick the places where they are tangent to a straight line of slope p. The search and the analysis will be facilitated if the data is replotted with ***linear moveout***. That is, energy located at offset f and time t in the (f,t)-plane is moved to offset f and time $t' = t - pf$ in the (f,t')-plane. This is depicted in figure 2. The linear moveout converts all events stepping out at a rate p in (f,t)-space to "horizontal" events in (f,t')-space. The presence of horizontal timing lines facilitates search, identification and measurement of the locations of the events.

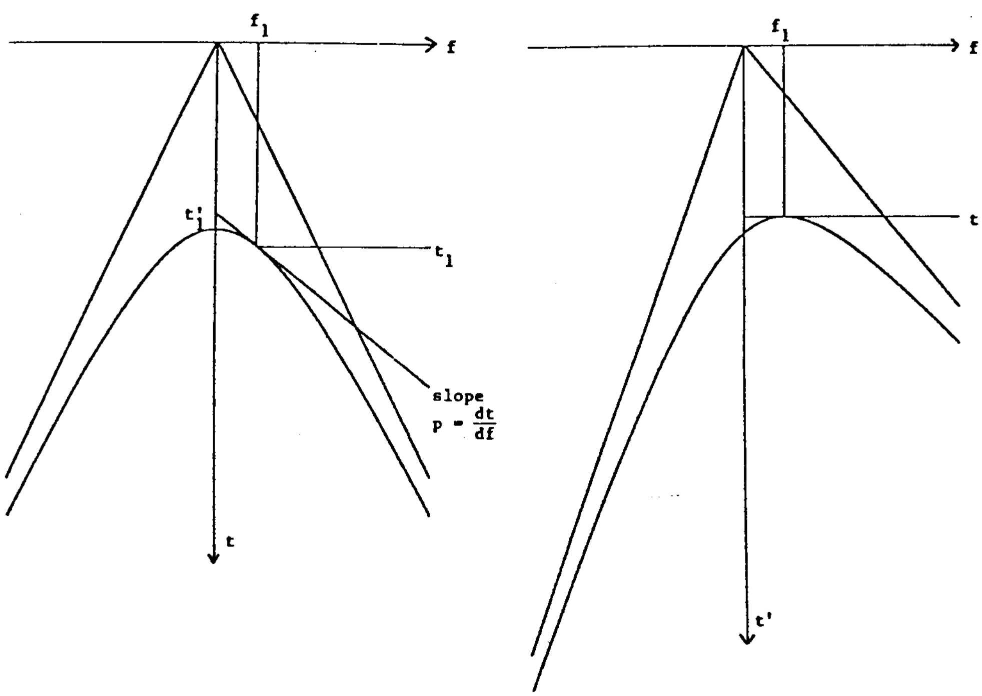

FIG. 2. Linear moveout converts the task of identifying tangencies to constructed parallel lines, to the task of locating tops of convex events.

Multiple Reflections at Vertical Incidence

All reflection seismologists are familiar with the timing and amplitude relations of vertical-incidence multiple reflections in layered media. To establish this along with some notation let us suppose that we have seafloor two-way traveltime t_1 with reflection coefficient c_1. Then the n-th multiple reflection comes at time nt_1 with reflection strength c_1^n. Suppose we have also a deeper primary reflection at traveltime depth t_2 with reflection coefficient c_2. Then we expect seafloor peglegs at time $t_2 + nt_1$ with reflection strengths $nc_2c_1^n$ (multiplied by some transmission coefficients). These familiar normal-incidence relationships apply to spherical-divergence-corrected field data at zero offset, but they do not apply at any other offset. But most if not all of our field data is recorded at non-zero offset. Normal moveout correction restores the timing relations of primaries to zero offset. But it cannot simultaneously restore zero- offset timing relations to multiples, for reasons we will consider later.

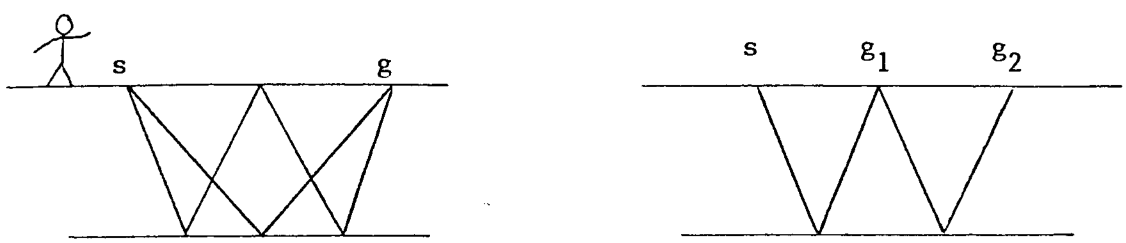

FIG. 3. Rays at constant offset (left) arrive with various angles, hence various Snell parameters. Rays with constant Snell parameter (right) arrive with various offsets. At constant p all paths have identical traveltimes.

Families of Rays at Constant p

In a layered earth the complete ray path is constructed by summing the path in each layer. At vertical incidence $p = 0$ it is obvious that when a ray is in layer i its travel time t_i for that layer is independent of whatever other layers may also be traversed on other legs of the total journey. This is also true for any other fixed p. But as shown in figure 3 it is not true for a ray whose total offset Σf_i is fixed instead of p being fixed.

Likewise, for fixed p, the horizontal distance f_i which a ray travels while in layer i is also independent of other legs of the journey. Furthermore t_i + const f_i for any layer i is also independent of other legs of the journey. With multiple reflections a given layer may be crossed many times in the total path. Taking the water layer to be characterized by (t_1, f_1) then an extra leg in the water layer will have an extra time t_1 added to its time of arrival and an extra distance f_1 added to the total horizontal distance. The same constants get added whether or not the extra leg is part of a simple seafloor multiple or part of some pegleg sequence. Some paths are shown in figure 4.

It is a great advantage in data processing to have the seafloor resonance occur at a frequency which is independent of any other layers that may be involved on the total path. So the next question is how to identify and separate various p values on typical field data.

Relating Rays to Field Data

To see how to relate field data to Snell waves, begin by searching on a common-midpoint gather for all those patches of energy (tangency zones) where the hyperboloidal arrivals attain some particular numerical value of slope $p = dt/df$. These patches of energy seen on our surface observations each tell us where and when some ray of Snell's parameter p has hit the surface. Typical geometries and synthetic data are shown in figures 4 and 5. The traveltimes t_i for all these arrivals satisfy the familiar relationships which we associate with vertical incidence.

A zero-offset time t_i' less than the time t_i is found by projecting the traveltime for some patch back along the straight line of slope p to zero offset. We have $t_i' = t_i - pf_i$. The contributions to t' from each layer are additive, as they are for t and f, so the times t' also behave like the times of normal-incident multiple reflections. Three minor differences between this and the vertical-incidence case are: 1) the actual numerical values for t_1 and t_2 will change with p because of the different travel path length; 2) the reflection coefficients c_1 and c_2 will change with p because of the different reflection angle; and 3) the non-vertical-incidence case theoretically should involve shear waves but for various reasons shear waves

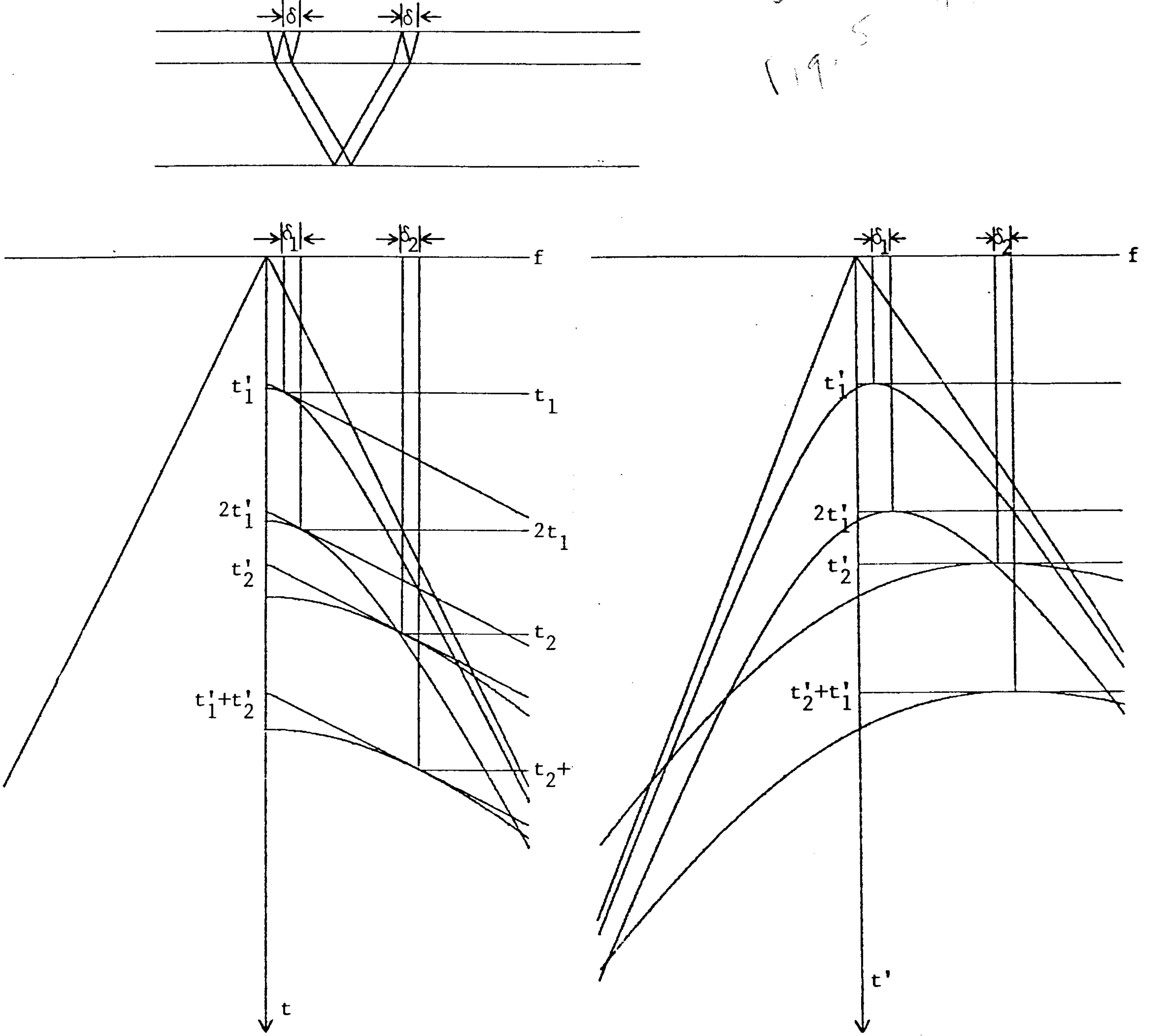

FIG. 4. A two-layer model showing the events $(t_1, 2t_1, t_2, t_2+t_1)$. Top is a ray trace. On the left is the usual data gather. On the right it is replotted with linear moveout $t' = t - pf$. Plots were calculated with $(v_1, v_2, 1/p)$ in the proportion (1,2,3). Fixing attention on the patches where data is tangent to lines of slope p, we see that arrival times are in the vertical-incidence relationships. That is, the reverberation period is fixed, and it is the same for simple multiples as it is for peglegs. This must be so because the ray trace at the top of the figure applies precisely to those patches of the data where $dt/dx = p$. Furthermore, since $\delta_1 = \delta_2$, the times $(t_1', 2t_1', t_2'+t_2')$ also follow the familiar vertical-incidence pattern.

are very rarely observed.

Given all the patches of constant p on a gather we can predict the traveltime of multiples by the familiar timing relationships. Unfortunately, the lateral location of any patch depends upon the velocity model v(z). This was considered in another chapter. It might seem to imply that you need to do velocity estimation at or before the time that you remove multiples, or that some kind of alternating bootstrap of velocity estimation along with multiple prediction and suppression is required. Luckily, the method of ***slant stacking*** which is based upon the idea of Snell waves comes to the rescue and enables us to remove multiple reflections before velocity estimation.

Slant Stacking

Slant stacking is summation over linear moveout. This paper is restricted to earth models which do not vary laterally, so it does not make any difference if we sum all shots into each geophone or if we sum all offsets into each midpoint. The first case is called ***wave stack*** and the second is called ***common-midpoint stack***. For more complicated earth models there is a substantial difference. The ***common-midpoint stacks*** do not really simulate Snell waves but they are still very useful in problems of migration and before-stack migration. The ***wave stacks*** simulate Snell waves and are useful for work on multiple reflections and work with lateral velocity variation. The procedure of slant stacking is first to do linear moveout with t'=t-pf, then to sum over the offset. In other words, you can slant stack in either of two ways: 1) sum along slanted lines in (t,f)-space; or 2) do linear moveout t' = t - pf and then sum over offset at constant t'. In either case, the entire gather P(f,t) gets converted to a single trace which is a function of t'. Let us think about what this trace actually is. We will assume that the sum over observed offsets is an adequate representation of integration over all offsets. The (slanted) integral over offset will obviously receive its major contribution from where the path of integration becomes ***tangent*** to the hyperboloidal arrivals. On the other hand, if rays carry a wavelet with no zero-frequency component, and if the arrival time curve crosses the integration curve at any fixed angle, then the contribution to the integral vanishes.

Actually we do not have line sources out of the plane of the survey so the wavefronts we would actually generate would be conical with the apex of the cone at the moving source. The major difference between the two cases is like a cylindrical-divergence amplitude correction. A minor difference predicted by wave theory would be a short wavelet with a little color and phase shift.

The strength of an arrival depends on the length of the zone of tangency. The ***Fresnel*** definition of the length of the zone of tangency is based on a half-wavelength condition. In an earth of constant velocity (but many flat layers) the width of the tangency zone would broaden with time as the hyperbolas flatten. This increase goes as $t^{\frac{1}{2}}$, which accounts for half the spherical-divergence correction. In other words, slant stacking takes us from two dimensions to one, but a $t^{\frac{1}{2}}$ remains to correct the conical wavefront of three dimensions to the plane wave of two.

Another view of slant stacking is as a sort of narrow-band-filtering operation which accepts energy at some particular Snell p value and rejects energy at other values. In the frequency domain it is closely related to what is known as dip filtering. To recognize a quantitative relationship between Snell's parameter $p = dt/df$ and the frequency domain, consider a wave field represented by the sinusoidal plane wave $\exp(-i\omega t + ik_x x + ik_z z)$. Set the phase equal to a constant and compute dt/dx at constant z. We get

$$p = \left.\frac{dt}{dx}\right)_z = \frac{k_x}{\omega} \tag{5}$$

So in the (ω,k)-plane the information about a slant stack is contained on the line $k_x = p\omega$.

Why Predictive Methods Fail on CDP Stacks

Normal moveout correction would succeed in restoring zero-offset timing relationships in a constant velocity earth, so we should ask the questions whether, in typical land and marine survey situations, $v(z)$ departs so much from constant that residual time shifts greater than a half-wavelength are routinely involved. No equations are needed to get the answer. It is generally observed that conventional common-midpoint stacking suppresses multiples because they have lower velocities than primaries. This observation alone implies that normal moveout does indeed routinely time shift multiples a half-wavelength or more out of the natural zero-offset relationships. As a result, much of the residual multiple reflection energy left in the stack does not fit the familiar vertical-incidence model. Consequently, predictive multiple suppression on a common depthpoint stack can be expected to be an exasperating undertaking. You can get rid of the vertically incident energy but the remainder will require adaptive least-squares coefficients which devour primaries as well as multiples. With marine data the moveout could be done with water velocity, but the peglegs still would not fit the normal-incident timing relationship. And the peglegs are often the worst part of the multiple-reflection problem.

Processing Sequence

The slant-stack operation on field data is fraught with complications of truncation and spatial aliasing. I'm not sure if these problems can be overcome satisfactorily in practice but in principle we begin by repeating the slant-stacking process for many separate values of p so that the (f,t)-space gets mapped into a (p,t)-space. The nice thing about (p,t)-space is that the multiple-suppression problem decouples into many separate one-dimensional problems, one for each p-value. Not only that, but you do not need to know the material velocity to solve these problems. The one-dimensional inverse problem is a classic one in geophysics with solutions published by many venerable geophysicists. After suppressing the multiples you inverse slant stack. It turns out that this is almost the same as slant stacking itself (see SEP-14, p. 81-86). Once back in (f,t)-space you estimate velocity by your favorite method.

Later chapters solve the theoretical multiple suppression problem with the simultaneous complications of non-zero offset and irregular non-planar reflectors.

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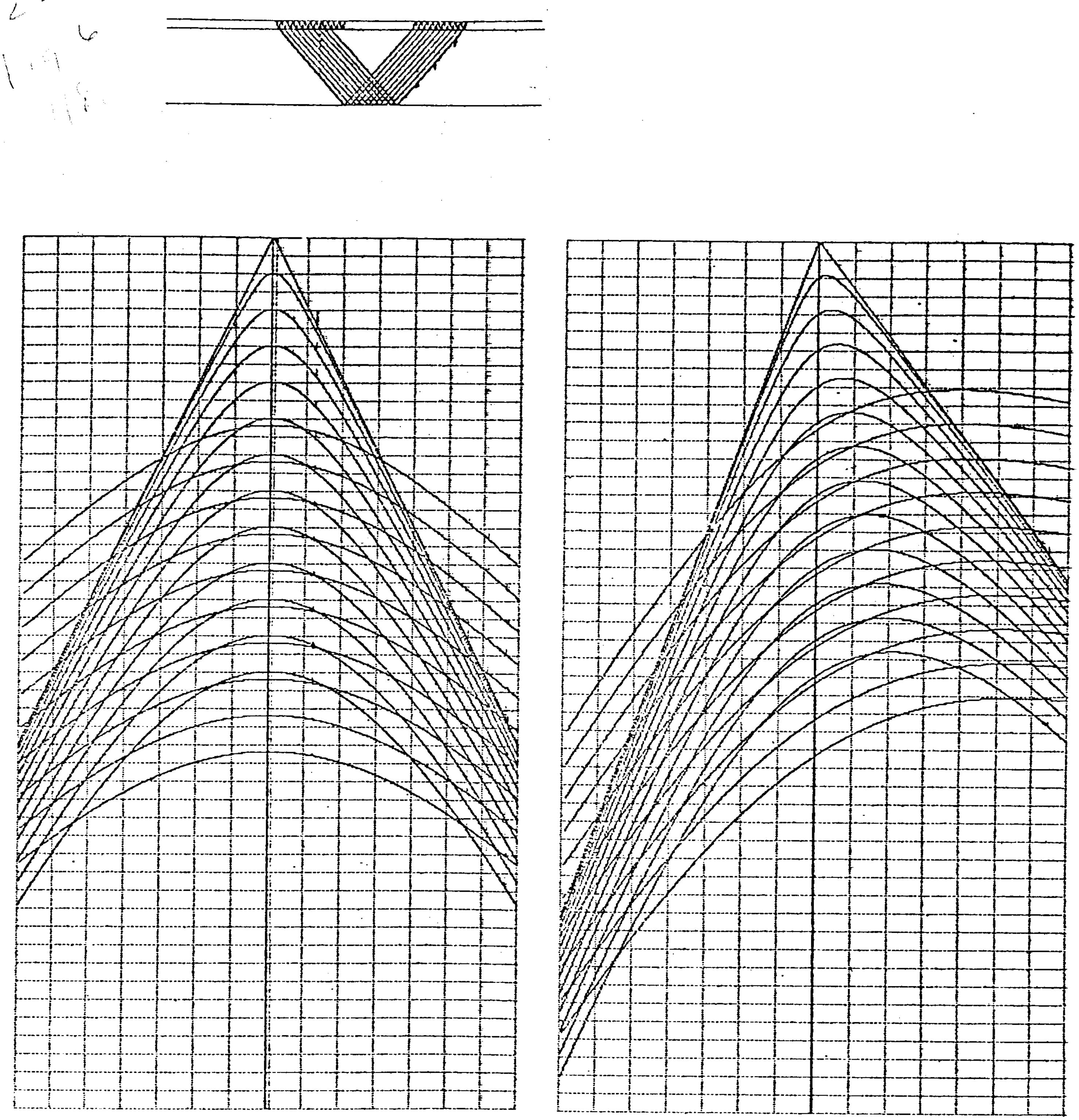

FIG. 5. This figure is the same as figure 4 but more multiple reflections are shown. This simulates much marine data. By picking the tops of all events on the right-hand frame and then connecting the picks with dashed lines, the reader will be able to verify that sea-bottom peg-legs have the same interval velocity as the simple bottom multiples. The interval velocity of the sediment may be measured from the primaries. The sediment velocity can also be measured by connecting the n-th simple multiple with the n-th peg-leg multiple.