A PROGRAM FOR INVERSION BY T-MATRIX ITERATION

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Abstract

Inversion of a one-dimensional synthetic seismogram was attempted using one of the T-matrix iterative schemes described in this report. The results so far have been disappointing in two respects. Errors due to truncation in the frequency domain were observed, and the convergence to the actual potential appears to be slow.

Mathematical Background

The second scheme given in the paper entitled "Inversion of Seismic Data in a Laterally Heterogeneous Medium" was implemented for the special case of a one-dimensional, local, frequency-dependent potential. Some of the equations in that paper need to be modified to account for the reduction in dimensionality. With a few changes of variables equations (20a)-(20e) become

$$v(q) = \sum_{m=1}^{\infty} a^m v_{(m)}(q) \tag{1a}$$

$$v^{(1)}\left(-\frac{\omega}{v_0}\right) = \frac{-4}{(2\pi)^{\frac{1}{2}} v_0^2} D(\omega) \tag{1b}$$

$$T_1(\omega, q) = \frac{\omega^2}{(2\pi)^{\frac{1}{2}}} v^{(1)}(-q) \tag{1c}$$

$$A_m(\omega,q) = \frac{2\omega^2}{(2\pi)^{\frac{1}{2}}} \sum_{n<m} \int dq' \frac{T_n(\omega,q')v^{(m-n)}(q'-q)}{\left(\frac{\omega+i\epsilon}{v_0}\right)^2 - \left(2q' - \frac{\omega}{v_0}\right)^2} \quad (1d)$$

$$v^m\left(-\frac{\omega}{v_0}\right) = -\frac{(2\pi)^{\frac{1}{2}}}{\omega^2} A_m(\omega, \frac{\omega}{v_0}) \quad (1e)$$

$$T_m(\omega,q) = A_m(\omega,q) - A_m(\omega,\frac{\omega}{v_0}) \quad (1f)$$

so that interpolation at every iteration can be avoided. In particular, the substitution $2q \rightarrow q$ was made in v, while $q \rightarrow 2q-w/v_0$ was placed in both A and T. In the actual implementation equation (1d) is converted into a more tractable expression through the use of

$$\frac{1}{x+i\epsilon} = -i\pi\delta(x)+\frac{P}{x}$$

$$\frac{1}{x-i\epsilon} = +i\pi\delta(x)+\frac{P}{x}$$

Truncation of A and T in the q-direction still poses a problem. This problem is more severe at high ω than low, so both A and T were scaled by decaying functions of frequency. In the example below, exponential scaling functions were employed.

Results

Preliminary results seem to show very slow, if any, convergence to the actual potential. After iteration m=3, the peak value of the estimated potential has increased only a tiny fraction of difference between the Born approximation and reality. It was expected that the lobes in the potential (top of figure 1) would distort and shift a bit. This has apparently not occurred. In addition, a distressing glitch is consistently placed at t=0, implying that the algorithm is adding a constant to the spectrum. At the last iteration some high-frequency noise has been introduced by truncations. Further iterations would tend to increase that noise still further.

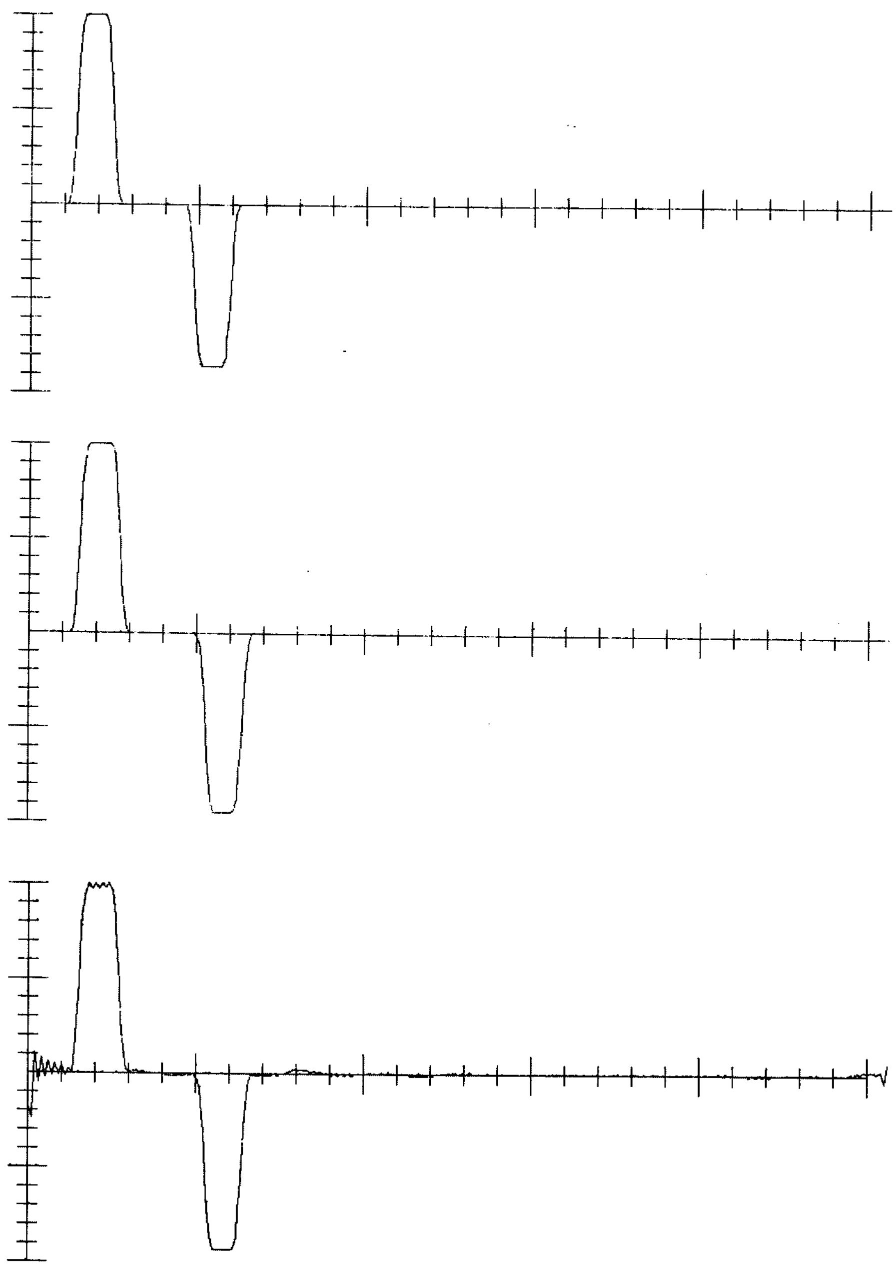

FIG. 1. Top: Actual potential, maximum = 4.32e-09 Middle: Born approximation, maximum = 4.10e-09 Bottom: Result of iteration m=3, maximum = 4.15e-09