# REFLECTIONS DUE TO CONTRASTS IN Q

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*Abstract*

A contrast in anelasticity at interfaces leads to reflections that may be characterized by complex, frequency-dependent reflection coefficients. If the Q factor for the two layers is different, reflections will always be present, regardless of what the values of the propagation velocities or the densities are.

*Introduction*

The conversion of mechanical wave energy into heat, as seismic waves propagate through the rocks, is a subject that is of considerable interest in reflection seismology. Since wave energy is absorbed in most rocks at a rate that is nearly proportional to the frequency (constant Q), the magnitude of the absorption limits the bandwidth of signals that can penetrate to any given depth, and thus the ultimate resolution that can be achieved in observations from the surface. This explains, for example, why shear wave exploration has failed to improve on the spatial definition obtained in conventional p-wave surveys and why higher frequencies have been observed from deep reflections in the basement in COCORP profiles than are usually observed at comparable depths in sedimentary rocks (e.g. Bloxsom, SEP-14, p. 251). It has been suggested that anomalous absorption may be associated with some types of hydrocarbon reservoirs (Sheriff, 1975; Dobrin, 1976). Strong dependence of the absorption on the saturation state has been demonstrated in laboratory experiments (Winkler and Nur, 1979).

*Reflections*

Reflection and transmission coefficients are derived for anelastic materials by matching displacements and tractions across the interfaces, just as in the elastic case. The stress at any point in a linear material may be found by convolving the strain with a modulus filter. The requirements of causality and physical realizability are satisfied when the integral of the modulus is an impedance function (see Claerbout, this report, p. ). Specializing to monochromatic plane waves at normal incidence, with an interface at $z = 0$, we have

$$\sigma(\omega) = m(\omega) * \varepsilon(\omega) = -\rho c^2 \frac{\partial U(\omega,z)}{\partial x} \tag{1}$$

where $\sigma(\omega)$ is stress, $\varepsilon(\omega)$ is strain, $U(\omega,z)$ is displacement, $m(\omega)$ is the modulus filter, $\rho$ is density, and $c$ is a velocity-like quantity defined by

$$c^2(\omega) = \frac{m(\omega)}{\rho} \tag{2}$$

Equation (1), when combined with the equilibrium equation, leads to a wave equation that has the same form as the usual wave equation, except that $c$ enters either as a filter in the time domain or as a frequency-dependent complex function in the frequency domain. Plane wave solutions to the wave equation may be written as the incident, reflected and transmitted wave displacements:

$$U_i = \exp\left[i\omega\left(t - \frac{z}{c_1}\right)\right] \tag{3a}$$

$$U_r = R \exp\left[i\omega\left(t + \frac{z}{c_2}\right)\right] \tag{3b}$$

$$U_t = T \exp\left[i\omega\left(t - \frac{z}{c_2}\right)\right] \tag{3c}$$

At the interface, $z=0$ continuity of the displacements implies that

$$U_i + U_r = U_t \tag{4}$$

or

$$T = 1 + R \tag{5}$$

Substituting Equation (1) into (2), and imposing continuity on the stresses, we get

$$\rho_1 c_1 - R\rho_1 c_1 = T\rho_2 c_2 \tag{6}$$

This combined with (5) gives

$$R = \frac{\rho_1 c_1 - \rho_2 c_2}{\rho_1 c_2 + \rho_2 c_2} \tag{7}$$

The form of $c$ depends on the particular material. The response of most rocks is well approximated by the constant $Q$ formulation (Kjartansson, 1979), where $c$ has the form

$$c = c_0 \left( \frac{i\omega}{\omega_0} \right)^{\gamma} \tag{8}$$

where $\omega_0$ is an arbitrary reference frequency, and $\gamma$ is related to $Q$ by

$$\frac{1}{Q} = \tan(\pi\gamma) \tag{9}$$

Equation (8) assumes the usual electrical engineering convention for signs of Fourier transforms; otherwise, $i$ will be replaced by $-i$ (see Claerbout, this report, p.361).

Substitution of (8) into (7) gives

$$R = \frac{\frac{\rho_1 c_{01}}{\rho_2 c_{02}} \left(\frac{i\omega}{\omega_0}\right)^{\gamma_1 - \gamma_2} - 1}{\frac{\rho_1 c_{01}}{\rho_2 c_{02}} \left(\frac{i\omega}{\omega_0}\right)^{\gamma_1 - \gamma_2} + 1} \tag{10}$$

This shows that when the $Q$ for both media are the same, the reflection coefficient is real and independent of frequency. The power series expansion for the natural logarithm, given by

$$\frac{1}{2} \ln x = \frac{1 - x}{1 + x} + \frac{1}{3}\left(\frac{1 - x}{1 + x}\right)^3 + \frac{1}{5}\left(\frac{1 - x}{1 + x}\right)^5 + \cdots \tag{11}$$

may be used to rewrite (10):

$$\frac{1}{2} \ln \left(\frac{\rho_1 c_{01}}{\rho_2 c_{02}}\right) + \frac{1}{2} (\gamma_1 - \gamma_2) \ln\left(\frac{i\omega}{\omega_0}\right) = R + \frac{1}{3} R^3 + \frac{1}{5} R^5 + \cdots \tag{12}$$

When $R$ is small, we can neglect third and higher powers of $R$. Then (12) reduces to

$$R \simeq \frac{\rho_1 c_{01} - \rho_2 c_{02}}{\rho_1 c_{01} + \rho_2 c_{02}} + \frac{1}{2} (\gamma_1 - \gamma_2) \ln \left(\frac{\omega}{\omega_0}\right)$$

$$+ i \frac{\pi}{4} (\gamma_1 - \gamma_2) \operatorname{sgn}(\omega) \tag{13}$$

Thus the reflection may be treated as a sum of two contributions, a real frequency-independent part and a frequency-dependent part that depends on the $Q$ contrast and is the "causal Hilbert" transform of the incident wave (see Claerbout and Kjartansson, this report, p.131).

*Discussion*

McDonal et al (1958) measured attenuation in water-saturated shale *in situ*. They observed Q values of about 30 for p waves and 10 for s waves. The laboratory results of Winkler and Nur (1979) indicate that Q may be an order of magnitude more sensitive than velocity to changes in conditions such as saturation or pore and confining pressures, and that p wave attenuation in partially saturated rocks may be much greater than in fully saturated or dry rocks. This raises the possibility that a substantial portion of the reflections observed in some areas are caused by changes in Q rather than elastic impedance.

## REFERENCES

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