

WAVE EQUATION SYNTHESIS AND INVERSION OF DIFFRACTED  
MULTIPLE SEISMIC REFLECTIONS

A DISSERTATION  
SUBMITTED TO THE DEPARTMENT OF GEOPHYSICS  
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FOR THE DEGREE OF  
DOCTOR OF PHILOSOPHY

By

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SEP-3

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I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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Wave Equation Synthesis and Inversion of Diffracted Multiple  
Seismic Reflections

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Stanford University, 1975

The existence of wave effects occurring with multiple seismic reflections is often characterized by diffraction travel-time hyperbolas and focused regions of the recorded wave field. More often though the wave phenomena is exhibited by more subtle variations in multiple reflection arrival time and amplitude. A theory which properly models diffracted multiple reflections is of practical importance in exploration seismology for both studying multiples and accurate reflector mapping. To a good approximation, data recorded in the reflection seismology geometry is described by the two-dimensional scalar wave equation. Assuming that the propagation is at small angles to the vertical, the wave equation may be split into two separate partial differential equations: one governing the propagation of upcoming waves and a second describing downgoing waves. The up and downward travelling wave fields are coupled at points in the earth where reflectors exist. These equations, together with their associated boundary conditions, provide the mathematical framework for synthesizing 2-D reflection seismograms from complex models. Finite difference approximations yield economical means for integrating the surface solutions.

The ability to separately compute the up and downgoing wave fields at depth allows consideration of a direct approach to inverting diffracted multiple reflections. In terms of up and downgoing waves, two fundamental principles of reflector mapping may be related to the inverse problem of both migrating primary reflections and removing multiple reflections. On the basis of these principles and the ability to downward continue the surface recordings, a consistent method of imaging reflectors, estimating reflection coefficients, and extinguishing diffracted multiple reflections may be obtained.

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